M. Prakash Institute

Solutions of Entrance Test for students going to XI std **27 Nov 2022**

Chemistry

Useful information:
Atomic Masses: H : 1, He : 4, C : 12, O : 16, Na : 23, P : 31, Cl : 35.5

Q1) The given tables contain atomic masses of the elements that follow Dohereiner's law of triads. Using this data, find out the values of 'x' and 'y'; enter $(y - x)$ as your answer:

Table-1 :

Ca	Sr	Ba
x	88	137

Table-2:

S	Se	Te
32	79	y

Solution: 126-39=87 **Ans. 87**

Q2) When slowly heated, a sample of 6.022×10^{23} molecules of potassium chlorate ($KClO_3$) decomposes to produce a gas and a salt. What will be the mass in grams of the gas liberated?

Solution: $2\ KClO_3 \rightarrow 2\ KCl + 3O_2$

$\uparrow$ $\uparrow$

2 mole $\rightarrow$ 3 mole

$(6.022 \times 10^{23} = 1$ mole$)$ $\rightarrow$ 1.5 mole = 48 grams O_2 **Ans. 48**.

Q3) Study the following series of chemical reactions carefully:

CH_3COOH + Na_2CO_3 $\rightarrow$ Gas (X) + Byproducts

Gas (X) + $CaOCl_2$ $\rightarrow$ Gas (Y) + Byproducts

Gas (Y) + $Ca(OH)_2$ $\rightarrow$ Compond (Z) + $CaOCl_2$

Identify compound (Z) and write its molar mass.

Solution:

$CH_3COOH + Na_2CO_3 \rightarrow CO_2 \uparrow + H_2O + CH_3COONa$

(x)

$CO_2 + CaOCl_2 \rightarrow CaCO_3 + Cl_2$

(y)

$Cl_2 + Ca(OH)_2 \rightarrow CaOCl_2 + H_2O$

compound (z)

Ans. 18.

Q4) How many of the following chemicals will give only 2 H^+ ions if fully dissociated ?

$H_3PO_4, H_3PO_3, H_3BO_3, H_2CO_3, Ca(OH)_2, H_2SO_3, HCl, HNO_3, NaOH, Fe(OH)_2$

Solution: Ans. 3

Q5) What is the total number of nucleons present in one alpha particle ?

Solution: Alpha Particle is He^{2+} cation. It contains 4 nucleons. **Ans. 4**.

Q6) An element is having electronic coniguration = (2,8,7). If 'x' = group number of the element in the modern periodic table and 'y' = period number of the element in the modern periodic table, then calculate the product of 'x' and 'y'.

Solution: $17 \times 3 = 51$ **Ans. 51**.

Q7) 1755 grams of NaCl is dissolved in water to produce 15 litres of its aqueous solution. Calculate the molarity of this solution.

Solution: $30/15 = 2$ **Ans. 2**.

Q8) Methane on complete combustion with pure oxygen gas produces carbon dioxide and water. If 5 mole of methane is to be consumed in this reaction, then calculate the number of mole of oxygen gas needed for the complete combustion.

Solution: 5 mole methane will require 10 mole O_2. **Ans. 10**.

Q9) n-Butyl Alcohol ($HO - CH_2 - CH_2 - CH_2 - CH_3$) is heated at 170^oC in the presence of concentrated sulphuric acid to produce a chemical 'X' and water. What is the molecular mass of the chemical 'X' in this reaction?

Solution: Given reaction is dehydration of alcohols.
n-Butyl Alcohol + conc H_2SO_4 + 170^oC heat = Butene + H_2O.
Molar Mass of butene = 56 **Ans. 56**.

Q10) What is the total number of basic radicals among the following:
$Fe^{3+}, SO_4^{2-}, NH_4^+, MnO_4^-, Mg^{2+}, S^{2-}, ClO_3^-, NO_3^-$

Solution: Basic Radicals: Fe^{3+}, NH_4^+, Mg^{2+} **Ans. 3**.

Physics

Use following data:
Density of water = 1gm/cc
Gravitaional acceleration (g) = 10 m/s^2
Melting point of ice = 0°C
Boiling point of water = 100°C

Q.11 A certain substance has a melting point of −50°C and a boiling point of 160°C. A thermometer is designed with this liquid and its melting and boiling points are designated at 30°L and 100°L. The boiling points of water on this scale (°L) is

Solution: Let's denote by points A, B, C the temperatures −50°C, 100°C, 160°C on, say, X axis. So, $AB : BC = 150 : 60 = 5 : 2$. So, required temperature of boiling point of water on a new scale also must divide the segment joining 30°L to 100°L in the same ratio. So, answer is 80°L. **Ans. 80**.

Q.12 Two blocks of masses 0.2 kg and 0.5 kg are placed 22 m apart on a rough flat horizontal surface. The resistive force (frictional in nature) acting on each block is equal to half of its weight in magnitude. At time $t = 0$sec, blocks are pushed towards each other with equal forces of 3 N on each of the block. Find time (in sec) at which blocks collide with each other.

Solution: Resistive frictional force on block 1 is 1 N and block 2 is 2.5 N. So, net force on block 1 is $3 - 1 = 2N$ and on block 2 is $3 - 2.5 = 0.5N$. Hence accelerations are block 1: $\frac{2}{0.2} = 10\ m/sec^2$, block 2: $\frac{0.5}{0.5} = 1\ m/sec^2$. Total distance travelled by both blocks is 22 m. So, if time taken is t sec, we get $\frac{1}{2}(10)(t^2) + \frac{1}{2}(1)t^2 = 22 \Rightarrow t = 2$ sec. **Ans. 2**

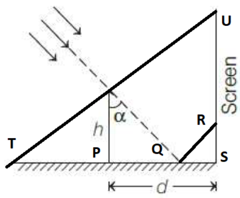
Q.13 A person goes from point P to point Q covering 1/3 of the distance with speed 10 km/h, the next 1/3 of the distance at 20 km/h and the last 1/3 of the distance at $\frac{50}{3}$ m/s. The average speed of the person (in m/s) is

Solution: Since speed required is in m/sec, let's convert all speeds to m/sec.
$10km/h = \frac{10000}{3600} = \frac{25}{9}\ m/sec$. Similarly $20km/h = \frac{50}{9}\ m/sec$.
Let the total distance be $3d$. Formula for average speed is distance covered upon total time. So, here, average speed= $\dfrac{3d}{\frac{d}{\frac{25}{9}} + \frac{d}{\frac{50}{9}} + \frac{d}{\frac{50}{3}}} = 5$. **Ans. 5**

Q.14 A solid plastic cube of side 4 cm has density of 1500 kg/m^3. It is hanging from a massless thread attached to a spring balance. Now the spring balance is held from top in such a way that the cube attached below is completely immersed in a liquid of density 1.2 g/cc. While in the liquid, the reading of the spring balance (in gram) is M. Find $5M$ and mark that number as your answer

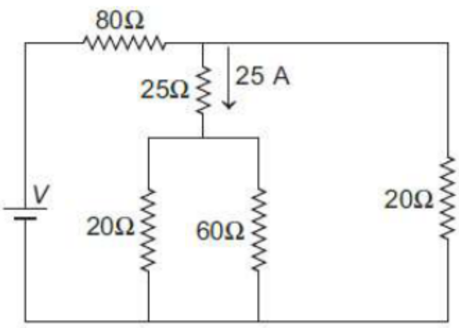
Solution: Volume of the block = 64 cm^3. Mass of the block = $64 \times 1.5 = 96$ gm. So, buoyant force by the liquid on the block (which is equivalent to the weight of the liquid of the same volume) = $64 \times 1.2 \times g$. So, net downward force on the spring balance = $(1.5 - 1.2) \times 64 \times g$. So, effective force is $\frac{1}{5}^{th}$ the weight of the cube. So, $5M$ is the mass of the cube. **Ans. 96**.

Q.15 A long horizontal mirror is next to a very tall vertical screen (see figure). Parallel light rays are falling on the mirror at an angle $\alpha = 45°$ from the vertical. If a vertical object of height $h = 24$ cm is kept on the mirror at a distance $d = 1$ m. The length (in cm) of the shadow of the object on the screen would be



Solution: Observe that ray incident at T will reach screen, so U is the uppermost part of the shadow. Ray incident at Q will reach the screen, so R is the bottom of the shadow. By symmetry, $TP = PQ = 24$ and hence $RU = TQ = 48$. **Ans. 48**.

Q.16 A current of 25 A flows through a 25Ω resistor represented by the circuit diagram. The current (in A) in 80Ω resistor is



Solution: The 20 and 60 Ω resistance below the 25Ω resistance are in parallel. So, their effective resistance is 15Ω which is in series with 25. So, we have 40Ω and 20Ω resistance in parallel. When resistances are in parallel, we know $i_1 R_1 = i_2 R_2$, which gives current in 20Ω = $50A$. So total current is 75A. **Ans. 75**.

Q.17 150 gm of ice at 0°C is mixed with 100 gm of water at temperature 80°C. The latent heat of ice is 80 cal/gm and the specific heat of water is 1 cal/gm°C. Assuming no heat loss to the environment, the amount of ice (in gm) which does not melt is

Solution: For some ice to not melt at all, the surrounding water must be at 0°. What it means is that the melted ice brought the temperature of 100 gm of water from 80 to zero °. The water will give away 100×80 calories. Since latent heat is 80 cal/gm, 100 gm of ice when melted will give this much of heat. So, 50 gm of ice does not melt. **Ans. 50**.

Q.18 A juggler tosses a ball up in the air with initial speed u. At the instant, it reaches its maximum height H, he tosses up a second ball with the same initial speed. The two balls will collide at a height $n \times H$, where n is a fraction. Calculate $72n$ and mark that as answer

Solution: Max distance travelled by a ball with initial speed u is given by $H = \frac{u^2}{2g}$. Distance travelled by second ball in time t is $ut - \frac{1}{2}gt^2$. In the same time the first ball travels $\frac{1}{2}gt^2$ distance. Sum of these distances must be H. So, we get $\frac{u^2}{2g} = ut - \frac{1}{2}gt^2 + \frac{1}{2}gt^2 = ut \Rightarrow t = \frac{u}{2g}$.
In this much time, the second ball travels $u\left(\frac{u}{2g}\right) - \frac{1}{2}g\left(\frac{u}{2g}\right)^2 = \frac{3u^2}{8g}$. So, $72n = 72\left(\frac{\frac{3u^2}{8g}}{\frac{u^2}{2g}}\right) = 54$.
Ans. 54.

Q.19 Two charges $+Q$ and $-4Q$ are located at fixed points A and B, 16 cm apart on a horizontal line as shown below. A free charge $+3Q$ is placed at point C on line AB such that it remains at rest. Find the distance of point C from point B.

Solution: It is clear that, since charge at B is more than that at A and because they are of opposite sign, C must be on the left side of A. Let the distance of C from B be x. So, its distance from A is $x - 16$ We get $\dfrac{(3Q)(4Q)}{x^2} = \dfrac{(3Q)(Q)}{(x-16)^2} \Rightarrow 4(x-16)^2 = x^2 \Rightarrow 2(x-16) = \pm x$.
But $x > 16$. So, we get $x = 32$. **Ans. 32**.

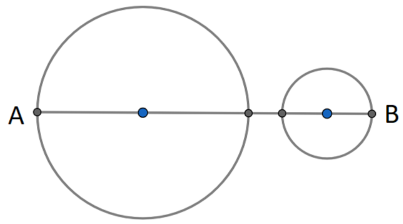
Q.20 Two point masses are kept some distance apart. First mass is smaller and the other is bigger. The gravitational force between them is 47.25 N. Now, the set up is changed. The first mass is replaced by a point mass that is four times the smaller mass. The second mass is replaced by a point mass that is eight times the bigger mass. Also the distance between them is now six times the earlier distance. Find the force (in Newton) between the new set up of masses.

Solution: Let the first set be m_1, m_2, d. So, we have $47.25 = G\frac{m_1 m_2}{d^2}$. The new set up is $4m_1, 8m_2, 6d$, so new force= $G\frac{(4m_1)(8m_2)}{(6d)^2} = \frac{8}{9}(G\frac{m_1 m_2}{d^2}) = \frac{8}{9}(47.25) = 42$. **Ans. 42**.

Maths

Q.21 What is the largest distance between a circle of diameter 15 that is centered at $(2, 2)$ and circle of diameter 5 that is centered at $(22, -19)$?

Solution:



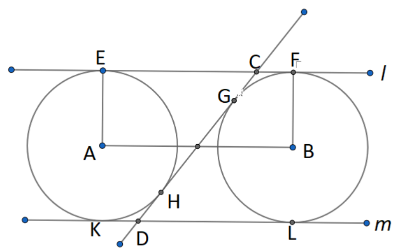
$$\begin{aligned} \text{Distance between centers} &= \sqrt{(2-22)^2 + (2-(-12))^2} \\ &= \sqrt{400+441} \\ &= \sqrt{841} \\ &= 29. \end{aligned} \tag{1}$$

$$\Rightarrow AB = 29 + \frac{15}{2} + \frac{5}{2} = 39.$$

Ans. 39

Q.22 Circles with centers at A and B, both with radius 10 units are not intersecting and the minimum distance between them is 7 units. Lines l, m are direct tangents and n is a transverse tangent. n intersects l and m at C and D. Find CD. (When both the circles are on the same side of the line then the line is called direct tangent and when two circles are on opposite sides of the line then the line is called transverse tangent)

Solution: Note:From the point external to circle if we draw two tangents to circle, then length of tangents (distance between point and point of contact) is equal.



$\Rightarrow CF = CG = DH = DK$ as both circles of 10 radius.
Also $CE = CH = DG = DL$
Note $ABFE$ is rectangle $\Rightarrow AB = EF = 10 + 10 + 7 = 27$
$CD = CG + DG = CF + CE = EF = 27$.

Q.23 Consider a sequence of integers $19, 11, -8, -19, \cdots$ where each term is equal to the term preceding it minus the term before that, except first two terms. What is the sum of first 2022 terms?

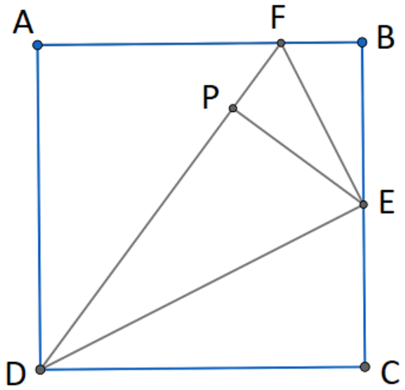
Solution: Recognise the pattern.

$$19, 11, -8, -19, -11, 8, 19, 11, -8,$$

Block of six numbers will repeat. Each block of six adds up to zero.
$2022 = 6 \times 337 \Rightarrow$ Sum of first 2022 terms is zero. **Ans. 0**

Q.24 Quadrilateral $ABCD$ is square with side length $4\sqrt{5}.E$ is midpoint of side BC. F is on side AB such that $\overline{DE}$ is angle bisector of $\angle CDF$. Find FE.

Solution: Draw $\perp$ from E on DF. Let foot be P.
Join $\overline{PC}$. Let it intersect $\overline{DE}$ at K.



Given $DC = 4\sqrt{5} \Rightarrow CE = 2\sqrt{5}$.
By Pythagoras Theorem we get $DE = 10$.
$\triangle DPE \cong \triangle DCE$ by SAA.
$\Rightarrow DP = 4\sqrt{5}EP = 2\sqrt{5}$ and $\angle PED \cong \angle CED$.
Note $\triangle EPF \cong \triangle EBF$ by RHS.
$\Rightarrow \angle FEP \cong \angle FEB$.
$\Rightarrow m\angle DEF = 90, (2x + 2y = 180 \Rightarrow x + y = 90)$
$\triangle DEF \sim \triangle DPE$.
$\Rightarrow \frac{DE}{DP} = \frac{EF}{PE} \Rightarrow EF = \frac{PE \cdot DE}{DP} = \frac{2\sqrt{5} \cdot 10}{4\sqrt{5}} = 5$ **Ans. 5.**

Q.25 Let $S = 1 + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{4 \cdot 5} + \frac{1}{4 \cdot 6} + \frac{1}{4 \cdot 7} + \frac{1}{5 \cdot 6} + \frac{1}{5 \cdot 7} + \frac{1}{6 \cdot 7} + \frac{1}{4 \cdot 5 \cdot 6} + \frac{1}{4 \cdot 5 \cdot 7} + \frac{1}{4 \cdot 6 \cdot 7} + \frac{1}{5 \cdot 6 \cdot 7} + \frac{1}{4 \cdot 5 \cdot 6 \cdot 7}$. Find S.

Solution: Note

$$\begin{aligned}
(1+a)(1+b) &= 1 + a + b + ab \\
(1+a)(1+b)(1+c) &= 1 + a + b + c + ab + bc + ca + abc \\
(1+a)(1+b)(1+c)(1+d) &= 1 + a + b + c + d + ab + ac + ad + bc \\
&+ bd + cd + abc + abd + acd \\
&+ bcd + abcd.
\end{aligned}$$

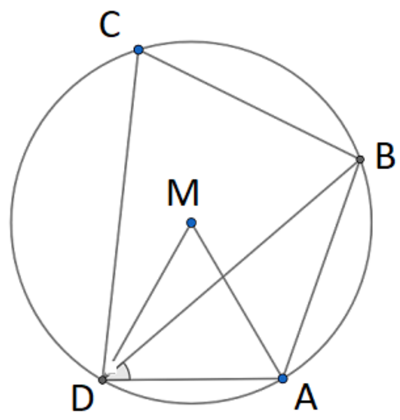
Comparing with given problem we get

$$\begin{aligned}
S &= \left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{6}\right)\left(1 + \frac{1}{7}\right) \\
&= \frac{5}{4} \times \frac{6}{5} \times \frac{7}{6} \times \frac{8}{7} = 2
\end{aligned}$$

Ans. 2.

Q.26 Quadrilateral $ABCD$ is inscribed in a circle with diameter 12. If m$\angle BDA = 40^\circ$ and $AD = 6$. If $m\angle BAD = (2X)^\circ$ report X.

Solution: Note $\triangle MAD$ is equilateral.



$$\Rightarrow m\angle DMA = 60$$

$\Rightarrow m\angle DBA = 30$ inscribed angle.

$$\Rightarrow m\angle BAD = 180 - (30 + 40) = 110$$
$$\Rightarrow \quad x = 55.$$

Ans. 55

Q.27 Find the sum of digits of constant term in the expansion of $(3x+5)^3 \times \left(2 + \frac{3}{x}\right)^2$.

Solution:

$(3x+5)^3 \left(2 + \dfrac{3}{x}\right)^2$

$= \left[(3x)^3 + 5^3 + 3(3x)(5)(3x+5)\right] \left[4 + \dfrac{9}{x^2} + \dfrac{12}{x}\right]$

$\left[(3x)^3 + 45(3)x^2 + 45(5x) + 5^3\right] \left[4 + \dfrac{12}{x} + \dfrac{9}{x^2}\right]$

Constant Term $= 45(3)x^2 \dfrac{9}{x^2} + 45(5x)\left(\dfrac{12}{x}\right) + 5^3(4)$

$= 1215 + 2700 + 500$

$= 4415$

Sum of the digits $4 + 4 + 1 + 5 = 14$

Ans. 14.

Q.28 It is given that $x^3 - 23x^2 + 167x - 385 = 0$ has one integer root α such that $8 \le \alpha \le 12$. Find the positive difference of remaining two roots.

Solution: $x^3 - 23x^2 + 167x - 383 = 0$

Checking to find α.

Attempt 1, $\alpha = 10$?

$$10^3 - 23(10)^2 + 167(10) - 383 < 0$$

Attempt 2, $\alpha = 11$?

$$11^3 - 23(11)^2 + 167(11) - 383 = 0$$

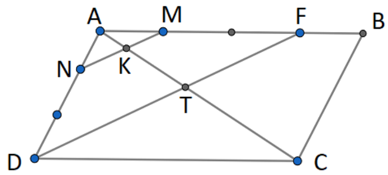
Find factor for other 2 roots.

$$x^3 - 23x^2 + 167x - 383 = (x - 11)\left(x^2 - 12x + 35\right)$$
$$x^2 - 12x + 35 = (x-7)(x-5)$$
$$\therefore 7, 5 \text{ are other roots}$$
$$\therefore \text{ Answer to be reported is } 7 - 5 = 2$$

Ans. 2.

Q.29 $ABCD$ is a parallelogram. M and N are on $\overline{AB}$ and $\overline{AD}$ respectively such that $AB = 4AM$ and $AD = 3AN$. Let K be the point of intersection of $\overline{MN}$ and $\overline{AC}$. Find $\frac{AC}{AK}$.

Solution: Let $AB = 4x, AD = 3y, AK = t$ construct $\overrightarrow{DF}$ parallel $\overrightarrow{NM}$ as shown.



$$\triangle ANM \sim \triangle ADF \Rightarrow \frac{AN}{AD} = \frac{NM}{DF} = \frac{AM}{AF} = \frac{1}{3}$$
$$\Rightarrow MF = 2x$$
$$\Delta AKM \sim \triangle AEF \Rightarrow \frac{AK}{AE} = \frac{KM}{EF} = \frac{AM}{AF} = \frac{1}{3}$$
$$\Rightarrow KE = 2t$$
$$\Delta AEF \sim \Delta CED \Rightarrow \frac{AE}{CE} = \frac{EF}{ED} = \frac{AF}{CD} = \frac{3}{4}$$
$$\Rightarrow CE = 4t \quad AC = 7$$
$$\Rightarrow AC = 7t \Rightarrow \frac{AC}{AK} = 7$$

Ans. 7.

Q.30 A model maker has clay models of sheeps, goats and cows (some of each). One buyer offers to pay Rs. 100 per each sheep, Rs 200 per each goat and Rs 400 per each cow for total of Rs 4700 . Another buyer offers to pay Rs. 135 per each sheep, Rs 265 per each goat and Rs 309 per each cow for total of Rs 5155 . How many clay models of sheeps does the model maker have?

Solution: Let

$S =$ Total Sheeps

$G =$ Total Goats

$C =$ Total Cows

$(100)S + (200)G + (400)C = 4700 \dots 1$

$(135)S + (265)G + (309)C = 5155 \cdots 2$

RHS of (2) is a multiple of 5 . Hence, C must be multiple of 5 . i.e. $C = 5, 10, 15, \dots$

(i) If $C = 5$, $(100)S + (200)G = 2700$ and $(135)S + (265)G = 3610$

That is $S + (2)G = 27$ and $(27)S + (53)G = 722$

Solving Simultenously, $S = 13, G = 7, C = 5$

For values of $C = 10$ or greater we don't get value of G positive. Hence only one solution.

Ans. 13.