M. Prakash Institute **Solutions - IX Entrance Exam**
Time: 2 Hours **20 April 2025**

1. Consider the consecutive prime numbers between 1 and 100. Suppose there are m pairs of adjacent primes with a difference of 6. What is the value of 2m?

   Solution: The pairs are $(23, 29), (31, 37), (47, 53), (53, 59), (61, 67), (73, 79), (83, 89)$ **Ans. 14.**

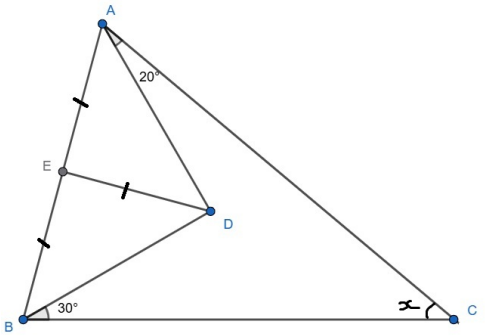
2. Nine friends went to a party. All the boys drank only coffee, and all the girls drank only tea. The price of one cup of coffee (in rupees) is 2 less than the number of girls. The price of one cup of tea (in rupees) is 2 less than the number of boys. The total money spent by the boys and the girls is in the ratio 5 : 6. Find the cost of 10 cups of coffee.

   Solution: Let the number of boys and girls be x and $9 - x$ respectively. So, price of coffee is $7 - x$ and that of tea is $x - 2$. So, we get $\dfrac{x(7-x)}{(9-x)(x-2)} = \dfrac{5}{6}$
   $\Rightarrow 42x - 6x^2 = -90 + 55x - 5x^2 \Rightarrow x^2 + 13x - 90 = 0 \Rightarrow x = -18$ or 5. So, $x = 5$. Number of girls$= 9 - 5 = 4$ so price of coffee$= 2$. **Ans. 20.**

3. If m and n are solutions of the quadratic equation $8x^2 - 25x + 18 = 0$, find the value of $8mn$.

   Solution: Product of roots $mn = \dfrac{18}{8} \Rightarrow 8mn = 18$ **Ans. 18.**

4. In the following diagram (not to scale), D is in the interior of $\triangle ABC$. E is on $\overline{AB}$ such that $AE = BE = DE$. If $m\angle DAC = 20°$, $m\angle DBC = 30°$, and $m\angle ACB = x$, find x.

   Solution: Let $\angle EAD = \angle EDA = u$ and $\angle EDB = \angle EBD = v$, so in $\triangle ADB, 2u + 2v = 180 \Rightarrow u + v = 90$
   So, In $\triangle ABC$, $u + 20 + v + 30 + x = 180 \Rightarrow x = 40$ **Ans. 40.**

5. $P = \left(\dfrac{\sqrt[3]{64}}{\sqrt[3]{32}} \times \sqrt[4]{16}\right)^3$. Find P.

   Solution: $P = \left(\dfrac{\sqrt[3]{64}}{\sqrt[3]{32}} \times \sqrt[4]{16}\right)^3 = \left(\sqrt[3]{\left(\dfrac{64}{32}\right)} \times 2\right)^3 = 2 \times 8 = 16$. **Ans. 16.**

6. 10% of 20% of 40% of 50% of 80% of 15000 is?

   Solution: $15000 \times 0.1 \times 0.2 \times 0.4 \times 0.5 \times 0.8 = 48$. **Ans. 48.**

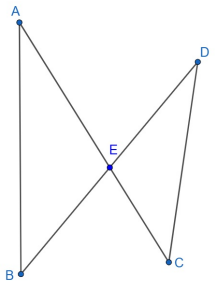
7. The average age of a family of 4 is 22 years and 6 months. The son is 2 years older than his sister. The father is 4 years older than his wife. What is the average age of the mother and son?

   Solution: Sum of all ages $= 4 \times$ (22 years and 6 months) $= 90$ years. Let the ages of mother and son be x, y respectively. So, Sister's age is $y - 2$ and father's age is $x + 4$. So, $90 = x + y + y - 2 + x + 4 \Rightarrow 2x + 2y = 88 \Rightarrow x + y = 44$. **Ans. 22.**

8. $\left[\frac{8}{7}+\frac{7}{8}\right]^2-\left[\frac{8}{7}-\frac{7}{8}\right]^2=x$, find x^2.

Solution:
$x=\left[\frac{8}{7}+\frac{7}{8}\right]^2-\left[\frac{8}{7}-\frac{7}{8}\right]^2=\left(\left(\frac{8}{7}\right)^2+2+\left(\frac{7}{8}\right)^2\right)-\left(\left(\frac{8}{7}\right)^2-2+\left(\frac{7}{8}\right)^2\right)=4.$
Ans. 16.

9. In the following diagram (not to scale), segments $\overline{AC}$ and $\overline{BD}$ intersect at E. If $m\angle BAE = m\angle ECD + 20^\circ$ and $m\angle CDE = 60^\circ$, find $m\angle ABE$.



Solution: Since $\angle AEB = \angle DEC$, we have $\angle BAE+\angle ABE = \angle CDE + \angle ECD$
$\Rightarrow \angle ABE = \angle CDE + (\angle ECD - \angle ABE) = 60 - 20 = 40.$
Ans. 40.

10. If y is a natural number that satisfies the equation below, then find y^2:
$1+\frac{y+3}{y-2}=\frac{3y-3}{6-y}$

Solution: $1+\frac{y+3}{y-2}=\frac{3y-3}{6-y}\Rightarrow\frac{y-2+y+3}{y-2}=\frac{3y-3}{6-y}$
$\Rightarrow (6-y)(2y+1)=(3y-3)(y-2)\Rightarrow -2y^2+11y+6=3y^2-9y+6$
$\Rightarrow 5y^2-20y=0\Rightarrow y=0,4$. Since y is a natural number, $y=4$. **Ans. 16.**

11. If a sum doubles in 80 months at simple interest, how many years will it take for the sum to become four times its original amount?

Solution: Interest in 80 months is equal to the original amount. Since this is simple interest, we will get same interest in next 80 months and again same interest in next 80 months, i.e. in 240 months, interest will be three times the sum, so the sum will become four times. **Ans. 20.**

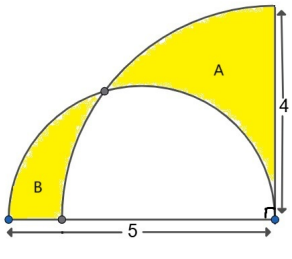
12. In a triangle, the largest angle is twice the measure of the second largest angle. One third of the largest angle is 10° more than the difference between the other two angles. Find one third of the largest angle.

Solution: Let the second largest angle be x. So the largest angle is $2x$. The smallest angle is $180-3x$. So, $\frac{2x}{3}=x-(180-3x)+10\Rightarrow x=51\Rightarrow\frac{2x}{3}=34$. **Ans. 34.**

13. If $a+b=16$, $a^2+b^2=130$, find the sum of the digits of a^3+b^3.

Solution: $a+b=16\Rightarrow a^2+2ab+b^2=256\Rightarrow ab=63$
$\Rightarrow a^3+b^3=(a+b)(a^2+b^2-ab)=(16)(130-63)=(16)(67)=1072$. **Ans. 10.**

14. A semicircle and a quarter circle are shown in the figure. The radius of the quarter circle is 4 and the diameter of the semicircle is 5. The shaded area A is inside the quarter circle but outside the semicircle. The shaded area B is inside the semicircle but outside the quarter circle. If $A-B=\frac{a\pi}{b}$, find $a+b$.



Solution: Let the area common to the quarter circle and semicicrle be C. So, $A+C=\frac{\pi}{4}(4^2)$ and $B+C=\frac{\pi}{2}(\frac{5}{2})^2\Rightarrow A-B=(A+C)-(B+C)=4\pi-\frac{25\pi}{8}=\frac{7\pi}{8}$. **Ans. 15.**

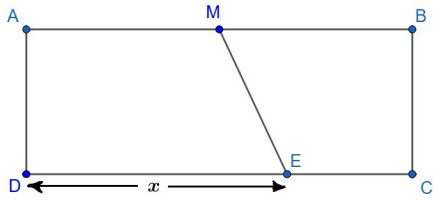
15. Let a, b, and c be numbers such that $ac = \dfrac{4}{b}$ and $b(a+c) + ac = 32$.
Find the value of $\dfrac{3}{a} + \dfrac{3}{b} + \dfrac{3}{c}$.

Solution: $\dfrac{3}{a} + \dfrac{3}{b} + \dfrac{3}{c} = 3\left(\dfrac{ab+bc+ca}{abc}\right) = 3\left(\dfrac{32}{4}\right) = 24$. **Ans. 24.**

16. A two-digit number is four times the sum of its digits and twice the product of the digits. Find the number.

Solution: $10x + y = 4x + 4y \Rightarrow 2x = y$. Using this in the second relation, we get $10x + 2x = 2x(2x) \Rightarrow x = 3 \Rightarrow y = 6$. **Ans. 36.**

17. In rectangle $\square ABCD$, $AB = 24$, $BC = 18$. M is the midpoint of $\overline{AB}$. E is on $\overline{DC}$ at a distance x from D. Find x such that the area of $\square AMED$ is twice the area of $\square BMEC$.



Solution:
Area$(\square AMED) = 2(\text{area}(\square BMEC))$
$\Rightarrow \dfrac{1}{2}\left(\dfrac{AM+ED}{2}\right)(AD) = 2\left(\dfrac{1}{2}\left(\dfrac{BM+EC}{2}\right)(AD)\right) \Rightarrow 12 + x = 2(12 + 24 - x)$
$\Rightarrow x = 20$. **Ans. 20.**

18. $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a - b\sqrt{15}$. Find $3(a-b)$.

Solution: $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \dfrac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \dfrac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$
$\Rightarrow a = 4, b = -1$. **Ans. 15.**

19. Find natural number x such that: $\sqrt{x^2 + x + 17} = 17$

Solution: $\sqrt{x^2 + x + 17} = 17 \Rightarrow x^2 + x + 17 = 289 \Rightarrow x^2 + x - 272 = 0$
$\Rightarrow x = 16, -17$. Since x is a natural number, $x = 16$. **Ans. 16.**

20. If 22 weavers weave 990 shawls in 15 days, how many weavers can weave 630 shawls in 21 days?

Solution: $\dfrac{990}{(22)(15)} = \dfrac{630}{(x)(21)} \Rightarrow x = \dfrac{(630)(22)(15)}{(990)(21)} = 10$. **Ans. 10.**