M Prakash Institute

VIII/IX std Selection Test Preparation Material

Topic 28

Rationalising Denominator

Practice Work:

1. Rationalise the denominators and simplify the following.

(a) $\dfrac{\sqrt{3}+a}{\sqrt{7}}$ (b) $\dfrac{\sqrt{6}+a}{\sqrt{5}}$ (c) $\dfrac{\sqrt{2}+a}{\sqrt{6}}$ (d) $\dfrac{3a+\sqrt{3}}{\sqrt{6}}$ (e) $\dfrac{2a+\sqrt{7}}{\sqrt{14}}$

2. Rationalise the denominator in these expressions.

(a) $\dfrac{4}{\sqrt{3}+\sqrt{7}}$ (b) $\dfrac{\sqrt{2}}{\sqrt{7}+1}$ (c) $\dfrac{\sqrt{6}}{\sqrt{6}-1}$ (d) $\dfrac{3\sqrt{2}}{\sqrt{7}-2}$

(e) $\dfrac{2\sqrt{5}}{\sqrt{5}+2}$ (f) $\dfrac{2}{\sqrt{11}-\sqrt{2}}$ (g) $\dfrac{6}{\sqrt{2}+\sqrt{5}}$

3. Rationalise the denominator of each of these fractions

(a) $\dfrac{\sqrt{2}}{\sqrt{4}-\sqrt{3}}$ (b) $\dfrac{2\sqrt{6}}{\sqrt{6}-2\sqrt{5}}$ (c) $\dfrac{4-\sqrt{3}}{8-4\sqrt{3}}$ (d) $\dfrac{\sqrt{3}}{\sqrt{5}-2}$ (e) $\dfrac{2+\sqrt{7}}{3+5\sqrt{7}}$

4. Rationalise the denominator for the following pronumerals-

(a) $\dfrac{b}{\sqrt{a}+\sqrt{b}}$ (b) $\dfrac{a}{\sqrt{a}-\sqrt{b}}$ (c) $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ (d) $\dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}$ (e) $\dfrac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}$

5. Find the value of $(2+\sqrt{2})+\dfrac{1}{(2+\sqrt{2})}+\frac{1}{(\sqrt{2}-2)}$

6. Find the value of $\left[\dfrac{7-3}{\sqrt{7}-\sqrt{3}}+\frac{5-3}{\sqrt{5}+\sqrt{3}}\right]^2-\sqrt{140}$.

7. If $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$, then find the values of a & b.

8. If $a=\dfrac{4}{3-\sqrt{5}}$, then find the values of $a+\dfrac{4}{a}$. **Answer Key:**

Practice Work:

Que no.	Answer
1	(a) $\frac{\sqrt{21}+a\sqrt{7}}{7}$ (b) $\frac{\sqrt{30}+a\sqrt{5}}{5}$ (c) $\frac{2\sqrt{3}+a\sqrt{6}}{6}$ (d) $\frac{a\sqrt{6}+\sqrt{2}}{2}$ (e) $\frac{2a\sqrt{14}+7\sqrt{2}}{14}$
2	(a) $-\sqrt{3}+\sqrt{7}$ (b) $\frac{\sqrt{14}-\sqrt{2}}{6}$ (c) $\frac{6+\sqrt{6}}{5}$ (d) $\sqrt{14}+2\sqrt{2}$ (e) $10-4\sqrt{5}$ (f) $\frac{2\sqrt{11}+2\sqrt{2}}{9}$ (g) $2\sqrt{5}-2\sqrt{2}$
3	(a) $2\sqrt{2}+\sqrt{6}$ (b) $\frac{-6-2\sqrt{30}}{7}$ (c) $\frac{5+2\sqrt{3}}{4}$ (d) $\sqrt{15}+2\sqrt{3}$ (e) $\frac{29+7\sqrt{7}}{166}$
4	(a) $\frac{b\sqrt{a}-b\sqrt{b}}{a-b}$ (b) $\frac{a\sqrt{a}+a\sqrt{b}}{a-b}$ (c) $\frac{a-2\sqrt{ab}+b}{a-b}$ (d) $\frac{a-\sqrt{ab}}{a-b}$ (e) $\frac{a\sqrt{b}+b\sqrt{a}}{a-b}$
5	2
6	12
7	a= 11; b= - 6
8	6

Topic 29

Positive Indices

Indices:

The product of a number repeatedly multiplied by itself is called a power or an exponential of the number.
The number is called the **base of the power**, and the number of times it is taken in the product is called the **index** (plural: indices) or **exponent of the power**. Some examples will make this clear.

In the product $3 \times 3 \times 3 \times 3 \times 3$, number 3 is taken five times in multiplication. This multiplication is written as 3^5.
3^5 is read as 'Three to the power 5′ or 'three raised to 5′.
In $3^5, 3$ is called as 'base', 5 as 'power' or 'index'.

2^3 is read as 'Two raised to three', 'Two to the power 3', or 'Cube of two' or 'two cube' 'Cube' in place of 'power 3' and 'Square' in the place of 'power 2' are the terms used usually.

If the power of a number is 1, it is not written. e.g. 3^1 is written as 3. While performing operations on numbers in index form, if the index of a number is not indicated, it is considered as 1. e.g. $5^4 \times 5$ is $5^4 \times 5^1$

In general, if n is a natural number, then

$a \times a \times a \times a \times \cdots \times a = a^n$
. n-times

$\mathbf{a^n}$ is called the **nth power of a** and is also read as '**a raised to the power n**'.

Writing a number with the use of 'base' and 'Index'.

eg. : Write 81 in the form of index by taking base 3.
$81 = 3^4$

Simplify:

(i) $5^2 \times 3^3$ (ii) $2^4 \times 3^2$ (iii) $3^2 \times 10^4$ (iv) $5^3 \times 2^4$

When base is negative integer:

$(-2) \times (-2) \times (-2) = -8$
$(-2) \times (-2) \times (-2) \times (-2) = 16$
. $(-2)^4 = 16$

$(-1)^{\text{An odd natural number}} = -1$ and $(-1)^{\text{An even natural number}} = 1$

i.e., $(-1)^n = \begin{cases} 1, \text{if } n \text{ is an even natural number} \\ -1, \text{if } n \text{ is an odd natural number} \end{cases}$

Simplify:

(i) $(-3) \times (-2)^3$ (ii) $(-3)^2 \times (-5)^2$ (iii) $(-2)^3 \times (-10)^3$ (iv) $(-2)^4 \times (-5)^2$

If $\frac{a}{b}$ is a rational number and n is a natural number, then

$$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\cdots\times\frac{a}{b}}_{(n\text{-times})} = \frac{\underbrace{a\times a\times a\times a\times\cdots\times a}_{(n\text{-times})}}{\underbrace{b\times b\times b\times b\times\cdots\times b}_{(n\text{-times})}} = \frac{a^n}{b^n}$$

Simplify:

(i) $\left(\frac{2}{3}\right)^2$ (ii) $\left(\frac{-3}{4}\right)^3$ (iii) $\left(\frac{-2}{5}\right)^4$

Rules of Indices :

1) $a^m \times a^n = a^{m+n}$

2) $a^m \div a^n = a^{m-n}$; (m > n)

3) $\mathrm{a}^0 = 1$

4) $(a^m)^n = a^{m\times n}$

5) $(\mathrm{a}\times\mathrm{b})^{\mathrm{m}} = \mathrm{a}^{\mathrm{m}} \times \mathrm{b}^{\mathrm{m}}$

6) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

7) $\mathrm{a}^{-\mathrm{m}} = \frac{1}{\mathrm{a}^{\mathrm{m}}}$

8) $a^m \div a^n = \frac{1}{a^{n-m}}$, where $n > m$

Practice Work:

1. Simplify using index form.

(a) $3 \times 3 \times 3 \times 3$ (b) $7 \times 7 \times 7 \times 7 \times 7 \times 7$

(c) $8 \times 8 \times 8$ (d) $2 \times x \times x \times 3 \times x$

(e) $3 \times y \times y \times 5 \times y \times y$ (f) $2 \times b \times a \times 4 \times b \times a \times a$

2. Express the following numbers as product of powers of their prime factors:

(i) 1000 (ii) 16000 (iii) 3600

3. Simplify using the first index law.

(a) $a^5 \times a^4$ (b) $x^3 \times x^2$ (c) $b \times b^5$

(d) $7\ \mathrm{m}^2 \times 2\ \mathrm{m}^3$ (e) $2s^4 \times 3s^3$ (f) $t^8 \times 2t^8$

(g) $\frac{1}{5}p^2 \times p$ (h) $\frac{1}{4}c^4 \times \frac{2}{3}c^3$ (i) $\frac{3}{5}s \times \frac{3s}{5}$

4. Simplify using the second index law.

(a) $x^5 \div x^2$ (b) $a^7 \div a^6$ (c) $q^9 \div q^6$

(d) $b^5 \div b$ (e) $\frac{y^8}{y^3}$ (f) $\frac{d^8}{d^3}$

(g) $\frac{j^7}{j^6}$ (h) $\frac{m^{15}}{m^9}$ (i) $2x^2y^3 \div x$

(j) $3r^5s^2 \div (r^3s)$ (k) $6p^4q^2 \div (3q^2p^2)$ (l) $16m^7x^5 \div (8m^3x^4)$

5. Simplify

(a) $(x^5)^2$ (b) $(t^3)^2$ (c) $(4t^2)^3$ (d) $(2u^2)^2$

(e) $\left(\frac{a^2}{b^3}\right)^2$ (f) $\left(\frac{x^3}{y^4}\right)^3$ (g) $\left(\frac{3f^2}{5g}\right)^3$ (h) $\left(\frac{3a^2b}{2pq^3}\right)^2$

6. Use appropriate index laws to simplify the following.

(a) $x^6 \times x^5 \div x^3$ (b) $x^2y \div (xy) \times xy^2$ (c) $x^4n^7 \times x^3n^2 \div (xn)$

(d) $\frac{x^2y^3 \times x^2y^4}{x^3y^5}$ (e) $\frac{m^2w \times m^3w^2}{m^4w^3}$ (f) $\frac{r^4s^7 \times r^4s^7}{r^4s^7}$

(g) $\frac{9x^2y^3 \times 6x^7y^5}{12xy^6}$ (h) $\frac{4x^2y^3 \times 12x^2y^2}{24x^4y}$ (i) $\frac{16a^8b \times 4ab^7}{32a^7b^6}$

(j) $(3m^2n^4)^3 \times mn^2$

7. If $x^4 = 3$ find the value of:

(a) x^8 (b) $x^4 - 1$ (c) $2x^{16}$ (d) $3x^4 - 3x^8$

8. Simplify and write each of the following in exponential form:

(i) $8^2 \div 2^3$ (ii) $25^4 \div 5^3$ (iii) $\frac{2^8 \times a^5}{4^3 \times a^3}$ (iv) $\frac{2^3 \times 3^4 \times 4}{3 \times 32}$

9. Find the values of n in each of the following:

(i) $(2^2)^n = (2^3)^4$ (ii) $2^{5n} \div 2^n = 2^4$ (iii) $2^{n-5} \times 5^{n-4} = 5$

(iv) $2^{n-7} \times 5^{n-4} = 1250$ (v) $5^{n-2} \times 3^{2n-3} = 135$

10. Meaning of a^b is a multiplied to a, b times. For example $a^4 = a \times a \times a \times a$.
If $775 = 5^x + 5^y + 5^z$ where x, y, z are natural numbers, find $x + y + z$.

Answer Key:

Practice Work:

Que no.	Answer
1	(a) 3^4 (b) 7^6 (c) 8^3 (d) $6x^3$ (e) $15y^4$ (f) $8a^3b^2$
2	(a) $2^3 \times 5^3$ (b) $2^7 \times 5^3$ (c) $2^4 \times 3^2 \times 5^2$
3	(a) a^9 (b) x^5 (c) b^6 (d) $14m^5$ (e) $6s^7$ (f) $2t^{16}$ (g) $\frac{p^3}{5}$ (h) $\frac{c^7}{6}$ (i) $\frac{9}{25}s^2$
4	(a) x^3 (b) a (c) q^3 (d) b^4 (e) y^5 (f) d^5 (g) j (h) m^6 (i) $2xy^3$ (j) $3r^2s$ (k) $2p^2$ (l) $2m^4x$
5	(a) x^{10} (b) t^6 (c) $64t^6$ (d) $4u^4$ (e) $\frac{a^4}{b^6}$ (f) $\frac{x^9}{y^{12}}$ (g) $\frac{27f^6}{125g^3}$ (h) $\frac{9a^4b^2}{4p^2q^6}$
6	(a) x^8 (b) x^2y^2 (c) x^6n^8 (d) xy^2 (e) m (f) r^4s^7 (g) $\frac{9}{2}x^8y^2$ (h) $2y^4$ (i) $2a^2b^2$ (j) $27m^7n^{14}$
7	(a) 9 (b) 2 (c) 162 (d) -18
8	(i) 2^3 (ii) 5^5 (iii) $(2a)^2$ (iv) 3^3
9	(i) $n = 6$ (ii) $n = 1$ (iii) $n = 5$ (iv) $n = 8$ (v) $n = 3$
10	$x + y + z = 9$

Topic 30

Negative Indices

NEGATIVE integral exponent For any non-zero rational number 'a' and a positive integer, we define

$$a^{-n} = \frac{1}{a^n}$$

For example,

(i) $2^{-4} = \frac{1}{2^4}$ (ii) $7^{-3} = \frac{1}{7^3}$ (iii) $5^{-4} = \frac{1}{5^4}$

Using the laws of exponents, simplify each of the following and express in exponential form:

(i) $3^7 \times 3^{-2}$ (ii) $2^{-7} \div 2^{-3}$ (iii) $(5^2)^{-3}$ (iv) $2^{-3} \times (-7)^{-3}$

If $\frac{a}{b}$ is any non-zero rational number and n is a positive integer, then

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$$

Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) $\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$ (ii) $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-7}{5}\right)^{2}$

Note that:

- $a^{-n} = \frac{1}{a^n}$
- $\frac{1}{a^{-m}} = a^m$
- $\frac{a^m}{a^n} = a^{m-n}$ or $\frac{1}{a^{n-m}}$

Practice Work:

1. Recall that $\frac{1}{4} = \frac{1}{2^2}$. Similarly write these using positive indices.

(a) $\frac{1}{9}$ (b) $\frac{1}{25}$ (c) $\frac{5}{16}$ (d) $-\frac{2}{27}$

2. Use index laws 1 and 2 to simplify the following. Write your answers using positive indices.

(a) $x^3 \times x^{-2}$ (b) $a^7 \times a^{-4}$ (c) $2b^5 \times b^{-9}$ (d) $3y^{-6} \times y^3$

(e) $3a^{-4} \times 2a^2$ (f) $4x^{-5} \times 3x^4$ (g) $5m^{-4} \times (-2m^{-2})$ (h) $-3a^{-7} \times 6a^{-3}$

(i) $\dfrac{2x^{-2}}{3x^{-3}}$ (j) $\dfrac{7d^{-3}}{10d^{-5}}$ (k) $\dfrac{6c^{-5}}{12c^{-5}}$ (l) $\dfrac{3b^{-2}}{4b^{-4}}$

3. Simplify the following and express your answers with positive indices.

(a) $(a^3b^2)^3 \times (a^2b^4)^{-1}$ (b) $(2p^2)^4 \times (3p^2q)^{-2}$ (c) $2\,(x^2y^{-1})^2 \times (3xy^4)^3$

(d) $\dfrac{2a^3b^2}{a^{-3}} \times \frac{2a^2b^5}{b^4}$ (e) $\dfrac{(3rs^2)^4}{r^{-3}s^4} \times \frac{(2r^2s)^2}{s^7}$ (f) $\dfrac{4\,(x^{-2}y^4)^2}{x^2y^{-3}} \times \frac{xy^4}{2x^{-2}y}$

4. Solve:

(a) $(3^{-1} \times 5^{-1})^{-1} =$ (b) $(6^{-1} - 8^{-1})^{-1} =$

5. Evaluate the following by combining fractions.

(a) $2^{-1} + 3^{-1}$ (b) $3^{-2} + 6^{-1}$ (c) $\left(\dfrac{3}{4}\right)^{-1} - \left(\dfrac{1}{2}\right)^{0}$

(d) $\left(\dfrac{3}{2}\right)^{-1} - 5\,(2^{-2})$ (e) $\left(\dfrac{4}{5}\right)^{-2} - \left(\dfrac{2^{-2}}{3}\right)$ (f) $\left(\dfrac{3}{2^{-2}}\right) - \left(\dfrac{2^{-1}}{3^{-2}}\right)^{-1}$

6. Find the value of x.

(a) $2^x = \dfrac{1}{4}$ (b) $2^x = \dfrac{1}{32}$ (c) $3^x = \dfrac{1}{27}$ (d) $\left(\dfrac{3}{4}\right)^x = \dfrac{4}{3}$

(e) $\left(\dfrac{2}{3}\right)^x = \dfrac{9}{4}$ (f) $\left(\dfrac{2}{5}\right)^x = \dfrac{125}{8}$ (g) $\dfrac{1}{2^x} = 8$ (h) $\dfrac{1}{3^x} = 81$

(i) $\dfrac{1}{2^x} = 1$ (j) $5^{x-2} = \dfrac{1}{25}$ (k) $3^{x-3} = \dfrac{1}{9}$ (l) $10^{x-5} = \dfrac{1}{1000}$

(m) $\left(\dfrac{3}{4}\right)^{2x+1} = \dfrac{64}{27}$ (n) $\left(\dfrac{2}{5}\right)^{3x-5} = \dfrac{25}{4}$ (o) $\left(\dfrac{3}{2}\right)^{3x+2} = \dfrac{16}{81}$ (p) $\left(\dfrac{7}{4}\right)^{1-x} = \dfrac{4}{7}$

7. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

8. By what number should $\left(\dfrac{5}{3}\right)^{-2}$ be multiplied so that the product may be $\left(\dfrac{7}{3}\right)^{-1}$?

Answer Key:

Practice Work:

Que no.	Answer
1	(a) $\frac{1}{3^2}$ (b) $\frac{1}{5^2}$ (c) $\frac{5}{2^4}$ (d) $\frac{2}{(-3)^3} = \frac{-2}{3^3}$
2	(a) x (b) a^3 (c) $\frac{2}{b^4}$ (d) $\frac{3}{y^3}$ (e) $\frac{6}{a^2}$ (f) $\frac{12}{x}$ (g) $\frac{-10}{m^6}$ (h) $\frac{-18}{a^{10}}$ (i) $\frac{2x}{3}$ (j) $\frac{7d^2}{10}$ (k) $\frac{1}{2}$ (l) $\frac{3b^2}{4}$
3	(a) a^7b^2 (b) $\frac{16p^4}{9q^2}$ (c) $54x^7y^{10}$ (d) $4a^8b^3$ (e) $324\frac{r^{11}}{S}$ (f) $\frac{2y^{14}}{x^3}$
4	(a) 15 (b) 24
5	(a) $\frac{5}{6}$ (b) $\frac{5}{18}$ (c) $\frac{1}{3}$ (d) $\frac{-7}{12}$ (e) $\frac{71}{48}$ (f) $\frac{106}{9}$
6	(a) -2 (b) -5 (c) -3 (d) -1 (e) -2 (f) -3 (g) -3 (h) -4 (i) 0 (j) 0 (k) 1 (l) 2 (m) -2 (n) 1 (o) -2 (p) 2
7	$\frac{1}{3}$
8	$\frac{25}{21}$

Topic 31

Linear equations - 1

Numbers and Symbols

In lower classes, we have studied numbers like $1, 2, 3, 1.2, -2.3$ as well as letters like a, b, c or x, y, z that can be used instead of numbers. These letters can be used for some known or unknown numbers. Accordingly, they are called **knowns or unknowns**. We will also come across situations in which the letters represent some particular numbers or a whole set of numbers. Accordingly, we call them constants or variables.

Numerical Expressions

Expressions of the form $3 \times 5,\ 2 + 6,\ 5 + (-4),\ 3^2 + 4^{1/2},\ \sqrt{2} + 5 \div 3$, etc. are called **numerical expressions**. Numerical expressions are made up of numbers, basic arithmetical operations $(+, -, \times, \div)$, involution (raising to a power), and evolution (root extraction).

Algebraic Expression

Expressions of the form $2x, (3x + 5), (4x - 2y), 2x^2 + 3\sqrt{\gamma}, 3x^3/2\sqrt{\gamma}$, etc. are **algebraic expressions**. $3x$ and 5 are the terms of $(3x + 5)$; $4x$ and $2y$ are the terms of $4x - 2y$. Algebraic expressions are made up of numbers, symbols, and basic arithmetical operations.

Mathematical Sentence

Two expressions joined by the equality sign $\Leftrightarrow$ or an inequality sign $(<, >, \leq, \geq)$ are **mathematical sentences**.

Example: $2 + 3 = 4 + 1, 5 \times 3 > 2 \times 4, 5 \times 6 = 10 \times 3, 15 - 8 < 3 \times 3, 3x + 5 \leq 10$, and $5x - 4 \geq 15$ are some mathematical sentences. Those which have the equality sign are **equations** or those which have an inequality sign are **inequations**.

Equation

An open sentence containing the equality sign is an **equation**. In other words, an equation is a sentence in which there is an equality sign between two algebraic expressions.

Example: $2x + 5 = x + 3, 3y - 4 = 20$, and $5x + 6 = x + 1$ are equations. Here, x and y are unknown quantities and 5,3 , 20 , etc., are known quantities.

Linear Equation

An equation, in which the highest index of the unknowns present is one, is a **linear equation**. $x + 5 = 10,\ 2x - 3 = 23,\ x + y = 10$, and $3x - 4y = 15$ are some linear equations.

Solution - A number, which when substituted for the variable in an equation, makes L.H.S. = R.H.S, is said to satisfy the equation and is called a solution or a root of the equation.

Laws of equation

1. If the same number or quantity is added to both the sides of an equation, the two sides remain equal; i.e.,**if $\mathbf{x = y}$ then $\mathbf{x + k = y + k}$.**

Example If $p - 5 = 7$ then $p - 5 + 5 = 7 + 5$ or $p = 12$.

2. If equal quantities are subtracted from both the sides of an equation then the differences are equal; i.e.. **if $\mathbf{x = y}$ then $\mathbf{x - k = y - k}$.**

Example If $x + 4 = 6$ then $x + 4 - 4 = 6 - 4$ or $x = 2$.

3. If both the sides of an equation are multiplied by the same quantity, they remain equal; i.e., **if $\mathbf{x = y}$ then $\mathbf{xz = yz}$.**

Example If $\frac{x}{5} = 6$ then $\frac{x}{5} \times 5 = 6 \times 5$ or $x = 30$.

4. If both the sides of an equation are divided by the same nonzero quantity, they remain equal; i.e., **if $\mathbf{x = y}$ then $\frac{\mathbf{x}}{\mathbf{k}} = \frac{\mathbf{y}}{\mathbf{k}}$, where $\mathbf{k \neq 0}$.**

Example If $5x = 60$ then $\frac{5x}{5} = \frac{60}{5}$ or $x = 12$.

Inequalities

The statement "8 is greater than 5" is expressed symbolically as $8 > 5$. And "3 is smaller than 5" is expressed symbolically as $3 < 5$. Such statements are called **inequalities**, and the symbols $>$ and $<$ are symbols of inequality.

Examples (i) $10 > 6$ is read as "10 is greater than 6".
(ii) $6 < 10$ is read as "6 is less than 10".
(iii) $a \geq b$ is read as "a is greater than or equal to b".
(iv) $b \leq a$ is read as "b is less than or equal to a".

• An inequality involving at least one variable, which can be substituted by some values for the statement to be true, is called an **inequation.**
• A linear inequation in x contains terms in the first degree of x only.

Examples
(i) $x > 3$, (ii) $2x + 1 < 5$, (iii) $5x - 2 \geq x + 3$ (iv) $3(x + 1) \leq x + 7$
are all linear inequations in x.

Replacement set

The set from which the values of the variable have to be selected to make an inequation true is called the replacement set, or domain of the variable, for the inequation.

Solution set

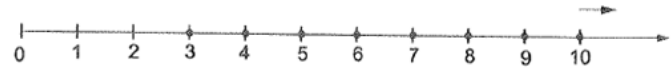
Consider the inequation $x > 3$, where $x \in N$. Here the replacement set for the inequation is the set of natural numbers.

The statement in the inequation is true if $x = 4, 5, 6, \ldots$ and false when $x = 3, 2, 1$.

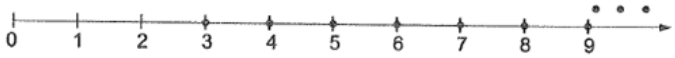
Graphical representation of the solution

Examples
(i) Consider the inequation $x + 2 \leq 6$, where $x \in W$.
Subtracting 2 from both the sides, $x + 2 - 2 \leq 6 - 2$ or $x \leq 4$.
But, the replacement set is W. So, x must be a whole number ≤ 4.
$\therefore$ the solution set is $\{0, 1, 2, 3, 4\}$.
To represent the solution set graphically, we draw a number line and mark points (thick dots) on it to represent the numbers $0, 1, 2, 3$ and 4 , as shown below.

(ii) The solution set for the inequation $x > 2$, where $x \in W$, is $\{3, 4, 5, 6, \ldots\}$.
This can be represented as follows.



or



The arrow or three dots above the number line indicate that all the subsequent integers also belong to the solution set.

Laws of inequality

Remember the following laws of inequality.

LAW 1
If the same quantity is added to or subtracted from both the sides of an inequality, the inequality does not change.

Examples
(i) If $x - 2 > 6$ then $x - 2 + 2 > 6 + 2$ or $x > 8$.
(ii) If $p + 3 < 8$ then $p + 3 - 3 < 8 - 3$ or $p < 5$.

Note: This law of inequality is similar to the corresponding law of equality. Therefore, the rule of transposition holds for inequalities as well. Thus, if $a - b > c + d$ then $a - b - d > c$ and $a > c + d + b$. In other words, a term from one side of an inequation can be shifted to the other side by simply changing its sign.

LAW 2
If both the sides of an inequality are multiplied or divided by the same positive quantity, the symbol (sign) of inequality does not change.

Examples
(i) If $\frac{x}{5} > 2$ then $\frac{x}{5} \times 5 > 2 \times 5$ or $x > 10$.

(ii) If $5m < 25$ then $\frac{5m}{5} < \frac{25}{5}$ or $m < 5$.

LAW 3
If both the sides of an inequality are multiplied or divided by the same negative quantity, the symbol (sign) of inequality is reversed, that is, $>$ becomes $<$, and $<$ becomes $>$.

Examples

(i) If $-\frac{x}{2} < 10$ then $\left(-\frac{x}{2}\right) \times (-2) > 10 \times (-2)$ or $x > -20$.

(ii) If $-6x > -24$ then $\left(-\frac{1}{6}\right) \times (-6x) < \left(-\frac{1}{6}\right) \times (-24)$ or $x < 4$.

Practice Work:

1. Solve the following equations.

(a) $4x - 11 = 49$ (b) $\frac{3y}{4} - 1 = 14$ (c) $\frac{8x}{5} + 26 = 50$

2. Solve each of the following equations. Also, verify the result in each case.

(a) $6x + 5 = 2x + 17$ (b) $2(5x - 3) - 3(2x - 1) = 9$ (c) $\frac{x}{2} = \frac{x}{3} + 1$

(d) $\frac{x}{2} + \frac{3}{2} = \frac{2x}{5} - 1$ (e) $\frac{3}{4}(x - 1) = x - 3$ (f) $3(x - 3) = 5(2x + 1)$

3. Solve the following equations and verify your answer:

(a) $\frac{2x-3}{3x+2} = -\frac{2}{3}$ (b) $\frac{2-y}{y+7} = \frac{3}{5}$ (c) $\frac{5x-7}{3x} = 2$

(d) $\frac{3x+5}{2x+7} = 4$ (e) $\frac{2y+5}{y+4} = 1$ (f) $\frac{2x+1}{3x-2} = \frac{5}{9}$

4. Solve for x :

(a) $\frac{x}{3} + \frac{2x+1}{6} = 0$ (b) $\frac{2x+3}{5} + \frac{x-1}{2} = \frac{3}{4}$ (c) $\frac{2x-3}{5} + \frac{x+4}{2} = -2$

(d) $\frac{x-2}{3} + \frac{2x+1}{12} = 2$ (e) $\frac{4x-3}{5} + \frac{x+2}{3} = -\frac{1}{4}$ (f) $\frac{x-4}{3} + \frac{2x-3}{8} = \frac{5}{6}$

5. Solve the following inequations and represent the solution set on a number line

(a) $x < 4$, x is a natural number
(b) $x \leq 3, x$ is a whole number
(c) $3 < x < 8$, x is an integer
(d) $x - 5 < 1$, x is a natural number
(e) $3x - 3 \geq 12$, x is a natural number

6. Consider Replacement set is a set of integers.
Solve for x and show the solution on a number line:

(a) $5x - 3 > 3x + 1$ (b) $2x + 1 \geqslant 4x + 7$ (c) $8x + 6 < 3x + 1$

(d) $2x + 7 > 7x + 3$ (e) $6x + 2 \leqslant 3x - 7$ (f) $x - 11 \leqslant 6x - 1$

7. Make a the subject in these equations.

(a) $a(b+1) = c$

(b) $ab + a = b$

(c) $\dfrac{1}{a} + b = c$

(d) $\dfrac{a-b}{b-a} = 1$

(e) $\dfrac{1}{a} + \frac{1}{b} = 0$

(f) $\dfrac{1}{a} + \frac{1}{b} = \frac{1}{c}$

Topic 32

Linear equations - 2

Problems leading to simple equations

To solve a word problem, we have to first frame an equation based on the condition stated in it.

Method of solving word problems

Steps 1. Read the problem carefully to determine what is required and denote the unknown quantity by x or any other symbol.
2. Use the conditions stated in the problem to frame an equation.
3. Solve the equation to obtain the value of the unknown quantity.

Following are some examples:

(i) 3 added to x is 8.
(ii) 5 subtracted from a number y is 12.
(iii) 7 less than a number x is 4.
(iv) Five times a number p is 32.
(v) A number x divided by 4 gives 3.
(vi) x multiplied by itself is 5 more than it.
(vii) The sum of a number x and twice the number y is 15.
(viii) 4 less from thrice a number m is 14.
(ix) The sum of the number x and its square is 20.

Note that:

The solution of a linear equation may be a rational number (negative, positive or fraction) but the solution of an equation, framed for a word problem has certain limitations depending upon the nature of a variable, say x.

- If x is number of persons, it cannot be negative or fractions.
- If x is age of a person, it cannot be negative.
- If x is length, it cannot be negative.
- If x is a digit, it has to be from 0 to 9.

Practice Work:

1. If 5 is subtracted from three times a number, the result is 16. Find the number.

2. Find the number which when multiplied by 7 is increased by 78.

3. Find three consecutive natural numbers such that the sum of the first and second is 15 more than the third.

4. If a number is tripled and the result is increased by 5, we get 50. Find the number.

5. A man is 4 times as old as his son. After 16 years, he will be only twice as old as his son. Find their present ages.

6. The difference between two natural numbers is 68. The quotient, when one is divided by the other is 5. Find the two numbers.

7. The breadth of a rectangle is $2/3^{\text{rd}}$ of its length. The perimeter of the rectangle is 60 cm. Find its area.

8. The sum of two numbers is 2490. If 6.5% of one number is equal to 8.5% of the other, find the two numbers.

9. 50 kg of an alloy of lead and tin contains 60% lead. How much lead must be melted into it to make an alloy containing 75% lead?

10. The distance between Kolhapur and Miraj is 60 kms. Prashant travels from Kolhapur to Miraj by bicycle with the speed of 18 km/hr and at the same time Sushant travels from Miraj to Kolhapur by bicycle with the speed of 12 km/hr. After what time they will meet each other and at what distance from Kolhapur?

11. Instead of multiplying a given number by $\frac{3}{5}$, a student divided it by $\frac{3}{5}$. His answer was 48 more than the correct answer. Then the given number was?

12. Reena's age to Teena's age is 7 : 5. Ratio of Teena's age to Seena's age is 2 : 3. Sum of ages of all three girls is 39. Find Teena's age.

13. Eleven less than, two-thirds of the sum of twice a number and nineteen, is 95. Find the number.

14. In an isosceles triangle, the length of the unequal side is 1.5 times that of equal sides. If perimeter of the triangle is 28 cm, find the lengths of all sides.

Answer Key:

Practice Work:

Q. no.	**Answer**
1	7
2	13
3	16 & 17
4	15
5	Son - 8 & Man - 32 years
6	17 & 85
7	l = 18, b = 12, A = 216
8	1411 & 1079
9	30
10	T = 2 & d = 36 km
11	15
12	10
13	70
14	8, 8 and 12 cm

Topic 33

Algebraic Expressions - 1

Addition & Subtraction

Types of algebraic expressions

Monomial:
An algebraic expression containing only one term is called a monomial.
For example, $3, 2x, 5x^2y, -6abc, \frac{3}{2}ab^2c^3$ are all monomials.

Binomial:
An algebraic expression containing two terms is called a binomial.
For example, $x+3, 5-2x, a^2-2abc, x^3+7, \frac{2}{3}x^2+xyz^2$ are all binomials. Note that $3+7$ is not a binomial, because $3+7=10$, which is a monomial.

Trinomial:
An algebraic expression containing three terms is called a trinomial.
For example, $2x-y+3, x^2+y^2+z^2, 3+xyz+x^3$ are all trinomials.
Note that $7+2x+9$ is not a trinomial, because $7+2x+9=16+2x$, which is a binomial.

Quadrinomial:
An algebraic expression containing four terms is called a quadrinomial.
For example, $a^3+b^3+c^3+3abc, a^2+b^2+c^2+5, ab+bc+ca+abc$ are all quadrinomials.

Polynomial:
An algebraic expression containing two or more terms is called a polynomial.

Like Terms:
The terms having the same literal factors are called like or similar terms.

Unlike Terms:
The terms not having same literal factors are called unlike or dissimilar terms.

• In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group.

• When a grouping symbol preceded by '- ' sign is removed or inserted, then the sign of each term of the corresponding expression is changed
(from '+' to '- ' and from '- ' to '+ ').

Practice Work:

1. Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) $4x^2$ (ii) $x^2 - 1$ (iii) $x^2 - y^2$

(iv) $3x^2 + 4y^2 + 5z$ (v) $ax^2 + bx + c$ (vi) $a^2 + b^2 + c^2 - d^2$

2. Identify the like terms in each of the following:

(i) $x^2, y^2, 2x^2, z^2$ (ii) $2xy, yz, 3x, \frac{yz}{2}$ (iii) $-2x^2y, x^2z, -yx^2, x^2y^2$

3. Evaluate each of the following expressions for $x = -2, y = -1, z = 3$:

(i) $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ (ii) $x^2 + y^2 + z^2 - xy - yz - zx$

4. Add the following algebraic expressions:

(i) $3a^2b, -4a^2b, 9a^2b$ (ii) $\frac{2}{3}a, \frac{3}{5}a, -\frac{6}{5}a$

(iii) $4xy^2 - 7x^2y, 12x^2y - 6xy^2, -3x^2y + 5xy^2$ (iv) $\frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c, \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c, \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$

5. Solve the following as per the given instructions -

From

(i) $p^3 - 4 + 3p^2$, take away $5p^2 - 3p^3 + p - 6$

(ii) $7 + x - x^2$, take away $9 + x + 3x^2 + 7x^3$

(iii) $1 - 5y^2$, take away $y^3 + 7y^2 + y + 1$

(iv) $x^3 - 5x^2 + 3x + 1$, take away $6x^2 - 4x^3 + 5 + 3x$

6. Simplify

(i) $5 + [x - \{2y - (6x + y - 4) + 2x\} - \{x - (y - 2)\}]$

(ii) $x^2 - [3x + \{2x - (x^2 - 1)\} + 2]$

(iii) $20 - [5xy + 3\{x^2 - (xy - y) - (x - y)\}]$

(iv) $85 - [12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}]$

(v) $xy - [yz - zx - \{yx - (3y - xz) - (xy - zy)\}]$

7. Add $2x^2 - 3x + 1$ to the sum of $3x^2 - 2x$ and $3x + 7$.

8. Add $x^2 + 2xy + y^2$ to the sum of $x^2 - 3y^2$ and $2x^2 - y^2 + 9$.

9. Add $a^3 + b^3 - 3$ to the sum of $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$.

10. Simplify each of the following:

(i) $x^2 - 3x + 5 - \frac{1}{2}(3x^2 - 5x + 7)$

(ii) $[5 - 3x + 2y - (2x - y)] - (3x - 7y + 9)$

(iii) $\frac{11}{2}x^2y - \frac{9}{4}xy^2 + \frac{1}{4}xy - \frac{1}{14}y^2x + \frac{1}{15}yx^2 + \frac{1}{2}xy$

(iv) $\left(\frac{1}{3}y^2 - \frac{4}{7}y + 11\right) - \left(\frac{1}{7}y - 3 + 2y^2\right) - \left(\frac{2}{7}y - \frac{2}{3}y^2 + 2\right)$

Answer Key:

Practice Work:

Q. no.	Answer
1	
2	
3	(i) $\frac{1}{6}$ (ii) 21
4	(i) $8a^2b$ (ii) $\frac{1}{15}a$ (iii) $3xy^2 + 2x^2y$ (iv) $\frac{23a}{6} - \frac{9b}{4} + \frac{53c}{20}$
5	(i) $4p^3 - 2p^2 - p + 2$ (ii) $-7x^3 - 4x^2 - 2$ (iii) $-y^3 - 12y^2 - y$ (iv) $5x^3 - 11x^2 - 4$
6	(i) $4x - 1$ (ii) $2x^2 - 5x - 3$ (iii) $-3x^2 - 2xy - 6y + 3x + 20$ (iv) $44 + 104x$ (v) $xy + 2zx - 3y$
7	$5x^2 - 2x + 8$
8	$4x^2 + 2xy - 3y^2 + 9$
9	$2a^3 - b^3 - 5$
10	(i) $-\frac{1}{2}x^2 - \frac{1}{2}x + \frac{3}{2}$ (ii) $-4 - 8x + 10y$ (iii) $\frac{167}{30}x^2y - \frac{65}{28}xy^2 + \frac{3}{4}xy$ (iv) $-y^2 - y + 12$

Topic 34

Algebraic Expressions - 2

Multiplication

Multiplying monomial by a monomial

Eg: $8a \times 3b = 24ab$

• **Rule 1:** The coefficient of the product of two monomials is equal to the product of their coefficients.

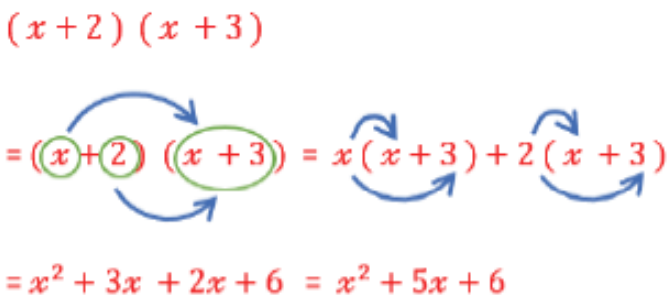
• **Rule 2:** The variable part in the product of two monomials is equal to the product of the variable parts in the given monomials.

Multiplication of monomial with binomial :

If P, Q and R are three monomials, then
(i) $P \times (Q + R) = (P \times Q) + (P \times R)$ (ii) $(Q + R) \times P = Q \times P + R \times P$

Multiplying Binomials using the Distributive Property

• To multiply a binomial by a binomial, we multiply every term of one binomial by every term of the other binomial and add the products (combine the like terms).



$$\begin{aligned}(7a + 11b)(13a + 9b) &= 7a \times (13a + 9b) + 11b \times (13a + 9b)\\ &= (7a \times 13a + 7a \times 9b) + (11b \times 13a + 11b \times 9b)\\ &= 91a^2 + 63ab + 143ab + 99b^2\\ &= 91^2 + 206ab + 99b^2\end{aligned}$$

• In mathematics, we use many formulae to find perimeter, area, etc., which are expressed as algebraic expressions.

• Some commonly used Algebraic Identities are-

a) $(a + b)^2 = a^2 + 2ab + b^2$
b) $(a - b)^2 = a^2 - 2ab + b^2$
c) $(a + b)(a - b) = a^2 - b^2$
d) $(x + a)(x + b) = x^2 + x(a + b) + ab$

Practice Work:

1. Multiply.

(a) $2x - 5y$ by $2x$ (b) $3x + 7y$ by $-3x$ (c) $-8a + 9b$ by $3a$ (d) $2x - 3y$ by $-6z$

2. Use the distributive law $(a+b)(c+d) = ac+ad+bc+bd$ to expand and simplify these algebraic expressions.

(a) $(x + 2)(x + 3)$ (b) $(x - 5)(x + 1)$ (c) $(x + 4)(x - 3)$

(d) $(2x + 1)(x - 5)$ (e) $(3x + 2)(2x - 5)$ (f) $(6x + 7)(x - 4)$

3. Expand the following

(a) $(2x - 5)^2$ (b) $(5a + 6b)^2$ (c) $(4x - 3y)^2$

4. Evaluate using identities

(a) 104^2 (b) $(99)^2$ (c) 59×61

(d) 117×83 (e) $(7.83 \times 7.83 - 1.17 \times 1.17)/6.66$

5. Given $(x + y) = 13, xy = 22$, find the value of $x^2 + y^2$

6. Given $(x - y) = 5, xy = 36$, find the value of $x^2 + y^2$

7. Find the product of $(3a + 4b)$ and $(3a - 4b)$ and verify it when $a = -1$ and $b = 1$.

8. A group of children are standing in $(3x-2)$ rows. If there are $(3x+2)$ children in each raw, how many children are standing in total?

9. Simplify

(a) $(3x - 2)(2x - 3) + (5x - 3)(x + 1)$

(b) $(5x - 3)(x + 2) - (2x + 5)(4x - 3)$

(c) $(3x + 2y)(4x + 3y) - (2x - y)(7x - 3y)$

10. Find the values of the following expressions:

(a) $16x^2 + 24x + 9$, when $x = \dfrac{7}{4}$

(b) $64x^2 + 81y^2 + 144xy$, when $x = 11$ and $y = \dfrac{4}{3}$

(c) $81x^2 + 16y^2 - 72xy$, when $x = \dfrac{2}{3}$ and $y = \dfrac{3}{4}$

11. Simplify:

(a) $2x^2\,(x^3 - x) - 3x\,(x^4 + 2x) - 2\,(x^4 - 3x^2)$

(b) $x^3y\,(x^2 - 2x) + 2xy\,(x^3 - x^4)$

(c) $3a^2 + 2(a + 2) - 3a(2a + 1)$

(d) $x(x + 4) + 3x\,(2x^2 - 1) + 4x^2 + 4$

(e) $a(b - c) - b(c - a) - c(a - b)$

Answer Key:

Practice Work:

Q. no.	Answer
1	(a) $4x^2 - 10xy$ (b) -9 $x^2 - 21xy$ (c) $-24a^2 + 27ab$ (d) $-12xz + 18yz$
2	(a) $x^2 + 5x + 6$ (b) $x^2 - 4x - 5$ (c) $x^2 + x - 12$ (d) $2x^2 - 9x - 5$ (e) $6x^2 - 11x - 10$ (f) $6x^2 - 17x - 28$
3	(a) $4x^2 - 20x + 25$ (b) $25a^2 + 60ab + 36b^2$ (c) $16x^2 - 24xy + 9y^2$
4	(a) 10816 (b) 9801 (c) 3599 (d) 9711 (e) 9
5	125
6	97
7	-7
8	$9x^2 - 4$
9	(a) $11x^2 - 11x + 3$ (b) $-3x^2 - 7x + 9$ (c) $-2x^2 + 3y^2 + 30xy$
10	(a) 100 (b) 10000 (c) 9
11	(a) $-x^5 - 2x^4 - 2x^3$ (b) $-x^5y$ (c) $-3a^2 - a + 4$ (d) $6x^3 + 5x^2 + x + 4$ (e) $2ab - 2ac$

Topic 35

Algebraic Expressions - 3

Division

Degree of a Polynomial in one variable -

In a polynomial in one variable, the highest power of the variable is called its degree.

Example

(i) $2x + 3$ is a Polynomial in x of degree 1.

(ii) $2x^2 - 3x + \dfrac{7}{5}$ is a polynomial in x of degree 2.

(iii) $\dfrac{2}{3}a^3 - \frac{7}{2}a^2 + 4$ is a polynomial in a of degree 3.

Degree of a Polynomial in two variables -

In a polynomial in more than one variable the sum of the powers of the variables in each term is computed and the highest sum so obtained is called the degree of the polynomial.

Example

$3x^4 - 2x^3y^2 + 7xy^3 - 9x + 5y + 4$ is a polynomial in x and y of degree 5, whereas $\dfrac{1}{2} - 3x + 7x^2y - \frac{3}{4}x^2y^2$ is a polynomial of degree 4 in x and y.

Constant Polynomial -

A polynomial consisting of a constant term only is called a constant polynomial.
The degree of a constant polynomial is zero.

Division of a Polynomial by a Monomial -

To divide a polynomial by a monomial, we need to divide each term of the polynomial by the monomial.

Example:

$$\frac{9m^5 + 12m^4 - 6m^2}{3m^2} = \frac{9m^5}{3m^2} + \frac{12m^4}{3m^2} - \frac{6m^2}{3m^2} = 3m^3 + 4m^2 - 2$$

Division of a Polynomial by a Binomial -

For dividing a polynomial by a binomial, we use the long division method by following the steps given.

Long Division Method

Step 1: First arrange the terms of the dividend and the divisor in the descending order of their degrees.

Step 2: Now, the first term of the quotient is obtained by dividing the first term of the

dividend by the first term of the divisor.

Step 3: Then multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

Step 4: Consider the remainder as the new dividend and proceed as before.

Step 5: Repeat this process till we obtain a remainder which is either 0 or a polynomial of a degree less than that of the

Example - Divide $(6 + x - 4x^2 + x^3)$ by $(x - 3)$

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x - 3 & x^3 - 4x^2 + x + 6 \\
 & x^3 - 3x^2 \\
 & - \quad + \\
\hline
 & -x^2 + x + 6 \\
 & -x^2 + 3x \\
 & + \quad - \\
\hline
 & -2x + 6 \\
 & -2x + 6 \\
 & + \quad - \\
\hline
 & 0
\end{array}
$$

Thus, we can say that $(6 + x - 4x^2 + x^3) \div (x - 3) = x^2 - x - 2$

The HCF of the Given Polynomials

For two given polynomials $f(x)$ and $g(x)$, $r(x)$ can be taken as the highest common factor, if:

1. $r(x)$ is a common factor of $f(x)$ and $g(x)$ and

2. Every common factor of $f(x)$ and $g(x)$ is also a factor of $r(x)$.

Practice Work:

1. Divide

(a) $x + 2x^2 + 3x^4 - x^5$ by $2x$

(b) $y^4 - 3y^3 + \frac{1}{2}y^2$ by $3y$

(c) $-4a^3 + 4a^2 + a$ by $2a$

(d) $-x^6 + 2x^4 + 4x^3 + 2x^2$ by $\sqrt{2}x^2$

(e) $5z^3 - 6z^2 + 7z$ by $2z$

(f) $\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3a^2 - 6a$ by $3a$

2. Divide and write the quotient and remainder for each.

(a) $x^2 + 5x + 6$ by $x + 2$

(b) $y^2 - 2y + 5$ by $y + 1$

(c) $m^2 - 3m + 7$ by $m - 2$

(d) $x^2 - 8x - 12$ by $x + 4$

(e) $3y^2 + 10y - 9$ by $3y - 2$

3. Rewrite the terms in proper order and then divide.

(a) $x^2 + 3x^3 - 2 + 5x$ by $x + 1$ (b) $2 - 2a^3 + a^2$ by $3 + 2a$

(c) $a + a^2 + 8$ by $a + 4$ (d) $12y + 10y^3 + 8y^4 + 15$ by $5 + 4y$

(e) $-12 + 3x^2 - 4x + x^3$ by $5x + x^2$

4. Find the value of k, if the divisor is a factor of the dividend.

(a) $2x^3 - 14x + k$ by $x + 3$ (b) $4x^3 - 12x^2 - 37x + k$ by $2x + 1$

(c) $4x^3 - 6x^2 - 4x + k$ by $2x - 1$ (d) $x^3 + x^2 - 5x + k$ by $x + 3$

5. The volume of a cube is $x^3 + 3x^2 + 3x + k$. If its length is $x + 1$, find k.

6. If $A = 3x^2 + 2x - 7$ and $B = 7x^3 - 3x + 4$, then find:

(a) $A + B$ (b) $A - B$ (c) $2A + 3B$ (d) $2A - 3B$

7. A divisor divides $2y^3 + 7y^2 + 7y + 2$ completely and the quotient is $y^2 + 3y + 2$, then find the divisor.

(a) $2y - 1$ (b) $y - 1$ (c) $y + 1$ (d) $2y + 1$

8. Determine the highest common factor of these pairs of terms.

(a) $7x$ and 14 (b) $12x$ and 30 (c) $-8y$ and 40 (d) $-5y$ and -25

(e) $4a^2$ and $2a$ (f) $12a^2$ and $9a$ (g) $-5a^2$ and $-50a$ (h) $-3x^2y$ and $-6xy$

Answer Key:

Practice Work:

Q. no.	Answer
1	(a) $2 + x + \frac{3}{2}x^3 - \frac{x^4}{5}$ (b) $\frac{y^3}{3} - y^2 + \frac{y}{6}$ (c) $-2a^2 + 2a + \frac{1}{2}$ (d) $\frac{-x^4}{\sqrt{2}} + \sqrt{2}x^2 + 2\sqrt{2}x + \sqrt{2}$ (e) $\frac{5}{2}z^2 - 3z + \frac{7}{2}$ (f) $\frac{a^3}{\sqrt{3}} + \frac{2a^2}{\sqrt{3}} + a - 2$
2	(a) Q $= x + 3$, R $= 0$ (b) Q $= y - 3$, R $= 8$ (c) Q $= m - 1$, R $= 5$ (d) Q $= x - 12$, R $= 36$ (e) Q $= y + 4$, $R = -1$
3	(a) Q $= 3x^2 - 2x + 7$, $R = -9$ (b) $Q = -a^2 + 2a - 3$, $R = 11$ (c) $Q = a - 3$, $R - 20$ (d) $Q = 2y^3 + 3$, $R = 0$ (e) $Q = x - 2$, $R = 6x - 12$
4	(a) $K = 12$ (b) $K = -15$ (c) $K = 3$ (d) $K = 3$
5	K $= 1$
6	(a) $7x^3 + 3x^2 - x - 3$ (b) $-7x^3 + 8x^2 + 5x - 11$ (c) $21x^3 + 6x^2 - 5x - 2$ (d) $-21x^3 + 6x^2 + 13x - 26$
7	Option (d)
8	(a) 7 (b) 3 (c) 8 (d) 5 (e) $2a$ (f) $3a$ (g) $5a$ (h) $3xy$

Topic 36

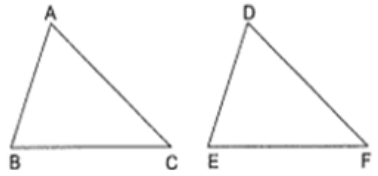
Congruence of Triangles – 1

Congruence –

Two triangles are congruent if and only if there exists a correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal or congruent.

In general terms, two triangles are said to be congruent, if in matching of their vertices, the three sides and the three angles of one triangle are respectively equal to the corresponding parts of the other triangle.

E.g.: - In $\triangle ABC$ and $\triangle DEF$,



if $A \longleftrightarrow D,\ B \longleftrightarrow E,\ C \longleftrightarrow F,$
such that $AB = DE,\ BC = EF,\ CA = FD$
and $\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$; then
$\Delta\ ABC \cong \triangle DEF$ ($\cong$ means "is congruent to")

Note: -

If two triangles are congruent then their perimeters are equal, and their areas are also equal, but two triangles with equal perimeters and / or equal areas may not be necessarily congruent.

Sufficient conditions for Congruence of two triangles -

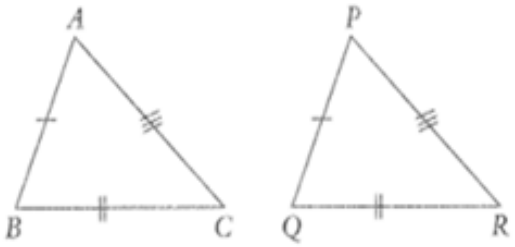
To show that the given two triangles are congruent, it is not necessary to show that all the six corresponding parts of the triangles are congruent every time. If three specific parts of one triangle are respectively congruent with the three corresponding parts of the other triangle, then the remaining three corresponding parts are also congruent with each other.

This means that **only three specific parts are sufficient to ascertain the test of congruence for the two given triangles.** Therefore, based on the study and experiments, the following tests are now considered as standard to establish the congruence of the two given triangles.

Standard Tests of Congruence -

1) S-S-S Test of Congruence -

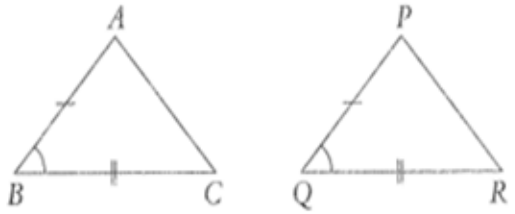
If three sides of a triangle are congruent with the three corresponding sides of the other triangle, then the two triangles are congruent.



In $\triangle ABC$ and $\triangle PQR$, If $AB = PQ, BC = QR$ and $CA = RP$,
then $\triangle ABC \cong \triangle PQR$ by S-S-S test.
$\therefore \angle A = \angle P, \angle B = \angle Q$ and $\angle C = \angle R$ $(C - P - C - T)$

2) S-A-S Test of Congruence -

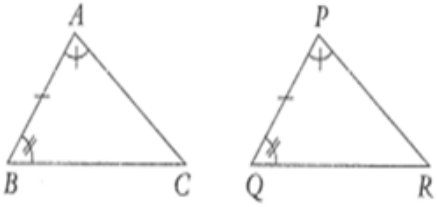
If two sides of a triangle and the angle included by them are congruent with the corresponding two sides and the angle included by them, of the other triangle, then the two triangles are congruent.



In $\triangle ABC$ and $\triangle PQR$, If $AB = PQ, BC = QR$ and $\angle ABC = \angle PQR$,
then $\triangle ABC \cong \triangle PQR$ by $S - A - S$ test.
$\therefore\ AC = PR, \angle A = \angle P$ and $\angle C = \angle R$ $(C - P - C - T)$

3) A-S-A Test of Congruence-

If two angles of a triangle and the side included by them are congruent with the corresponding two angles and the side included by them, of the other triangle, then the two triangles are congruent.

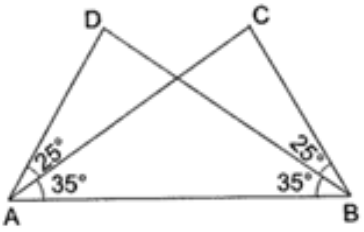


In $\triangle ABC$ and $\triangle PQR$,
If $\angle BAC = \angle QPR,\ AB = PQ$ and $\angle ABC = \angle PQR$,
then $\triangle ABC \cong \triangle PQR$ by A-S-A test.
$\therefore\ AC = PR, BC = QR$ and $\angle C = \angle R$ $(C - P - C - T)$

Practice Work:

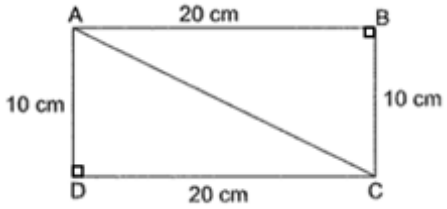
1. Name the test by which the corresponding triangles in each figure are congruent. Write the correct correspondence for the congruent triangles.

a)



$\triangle ADB \cong \Delta$ ..... by ..... test

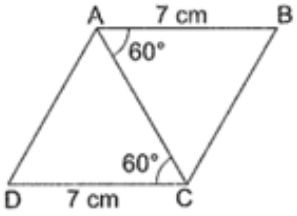
b)



$\triangle ADC \cong \triangle$ .....by.....test

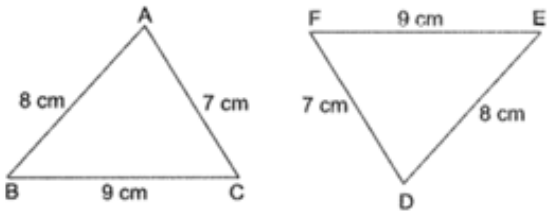
$\triangle ADC \cong \Delta$.....by ..... test

c)



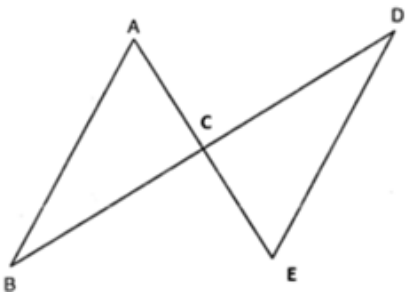
$\triangle ABC \cong \Delta$ .....by ..... test

d)



$\triangle ABC \cong \Delta$ ..... by ..... test

2. AE and BD bisect each other at C. State the test of congruence with the correct correspondence between the vertices of the given triangles.



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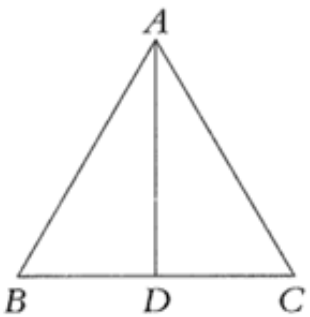
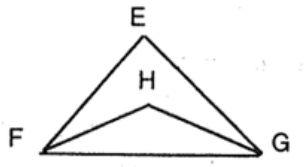
3. In $\triangle ABC$, if $AB = AC$ and AD bisects $\angle BAC$, prove that $\triangle ABD \cong \triangle ACD$



4. $\triangle EFG$ and $\triangle HFG$ are both isosceles triangles on a common base FG such that E & H lie on the same side of FG. Are triangles EHF and EHG congruent? If yes, by which test?



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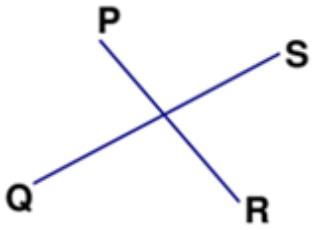
5. $\overline{PR}$ and $\overline{SQ}$ bisect each other. Prove that $\overline{PQ} \cong \overline{RS}$.



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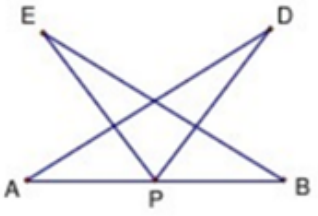
6. AB is a line segment, P is its midpoint $m\angle BAD = m\angle ABE$, $m\angle BPD = m\angle APE$. Prove that $\triangle DAP \cong \triangle EBP$



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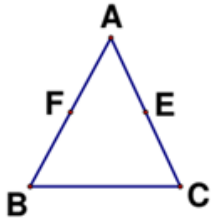
7. In $ABCD, \overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$.
Prove that (i) $\angle BAC \cong \angle DAC$ (ii) $\angle ACB \cong \angle ACD$.

8. In $\triangle ABC$, $AB = AC, F$ and E are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.
Prove that $\overline{BE} \cong \overline{CF}$.



Answer Key:

Practice Work:

Q. No.	Answer
1	(a) $\triangle BCA - ASA$; (b) $\triangle CBA - SSS$; (b) $\triangle CBA - SAS$ (c) $\triangle CDA - SAS$; (d) $\triangle DEF - SSS$;
2	SAS test
3	SAS test / ASA test
4	SSS test
5	SAS test
6	ASA test
7	SSS test
8	SAS test

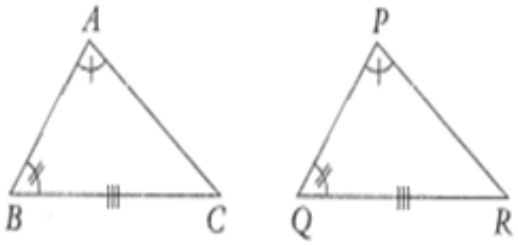
Topic 37

Congruence of Triangles – 2

Congruence –

4) S-A-A Test of Congruence -

If two angles of a triangle and the side not included by them are congruent with the corresponding two angles and the side not included by them of the other triangle, then the two triangles are congruent.

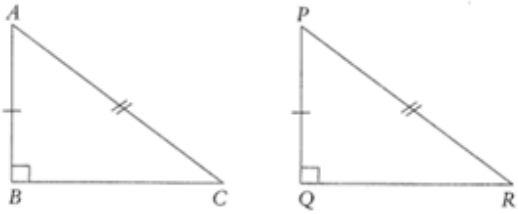


In $\triangle ABC$ and $\triangle PQR$, If $\angle BAC = \angle QPR$, $\angle ABC = \angle PQR$ and $BC = QR$,
then $\triangle ABC \cong \triangle PQR$ by S-A-A test.
$\therefore$ $AB = PQ$, $AC = PR$ and $\angle C = \angle R (C - P - C - T)$

5) R-H-S Test of Congruence -

If the hypotenuse and one side of a right-angled triangle are equal to the corresponding hypotenuse and side of the other right-angled triangle, then the two triangles are congruent.

• R-H-S Test is applicable if and only if the triangles are right-angled triangles.

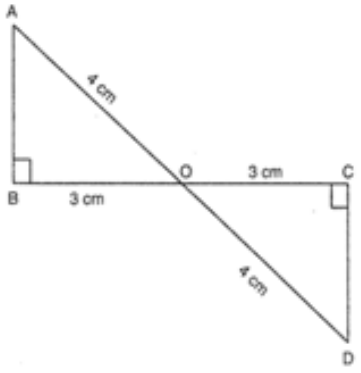


In $\triangle ABC$ and $\triangle PQR$, If $AB = PQ$, $\angle ABC = \angle PQR = 90°$ and $AC = PR$,
then $\triangle ABC \cong \triangle PQR$ by R-H-S test.
$\therefore$ $BC = QR$, $\angle BAC = \angle QPR$ and $\angle ACB = \angle PRQ$ (C-P-C-T)

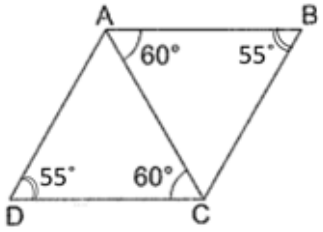
Practice Work:

1. Find the test of congruence by which the following pairs of triangles are congruent. Write the correct correspondence of the congruent triangles in each case.

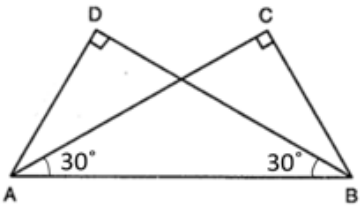
a)



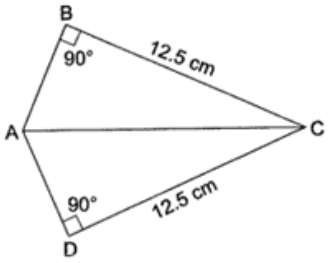
b)



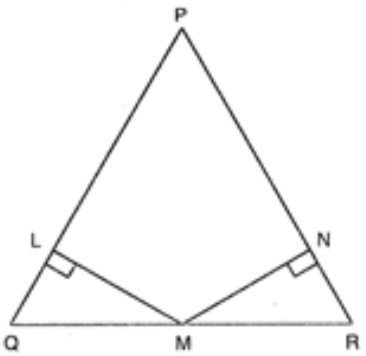
c)



d)



2. In $\triangle PQR$, M is mid-point of QR. $ML \perp PQ$ and $MN \perp PR$. If $ML = MN$, then prove that $PQ = PR$.



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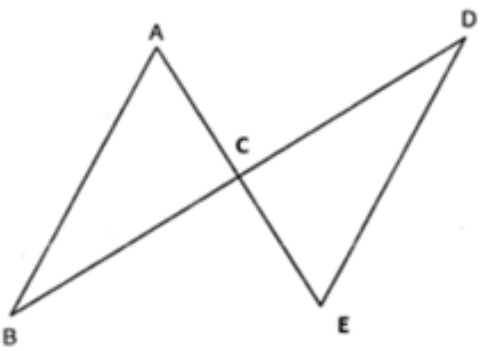
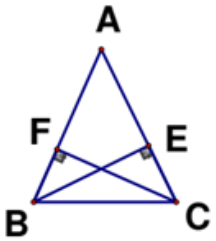
3. In the figure, if $AB//DE$ and C is mid-point of BD, then prove that BD bisects AE.



4. $ABCD$ is an isosceles trapezium with $AB//CD$. AE and BF are perpendiculars drawn to CD. Prove that $\triangle AED$ and $\triangle BFC$ are congruent.

5. In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{BE}$ and $\overline{CF}$ are altitudes. Prove that $\overline{BE} \cong \overline{CF}$.



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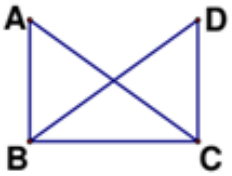
6. Given $\overline{AC} \cong \overline{DB}$, $m\angle ABC = m\angle DCB = 90$. Prove that $\overline{AB} = \overline{DC}$.



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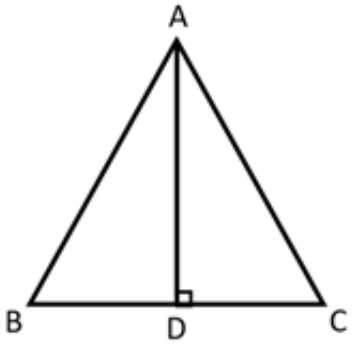
7. In the given figure, $\triangle ABC$ is an isosceles triangle. AD is an altitude on BC. Prove that $\triangle ABD \cong \triangle ACD$.



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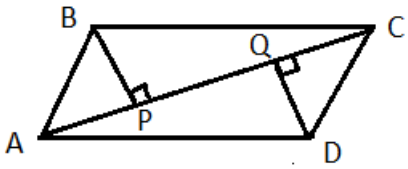
8. $\square ABCD$ is a parallelogram. AC is diagonal. Prove that B & D are equidistant from AC. i.e., prove that $BP = DQ$.



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Answer Key:

Practice Work:

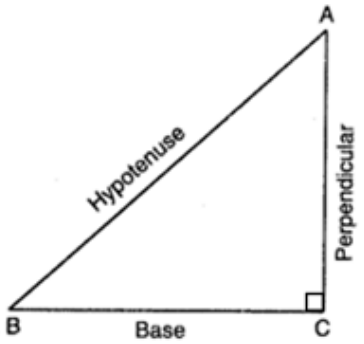
Q. No.	Answer
1	(a) $\triangle ABO \cong \triangle DCO-$ RHS test; (b) $\triangle ADC \cong \triangle CBA$ - SAA test; (c) $\triangle ADB \cong \triangle BCA$ - SAA test; (d) $\triangle ABC \cong \triangle ADC$ - RHS test.
2	RHS test
3	SAA test / ASA test
4	RHS test / SAA test
5	SAA test
6	RHS test
7	RHS test / SAA test
8	SAA test

Topic 38

Pythagoras Theorem - 1

Theorem -

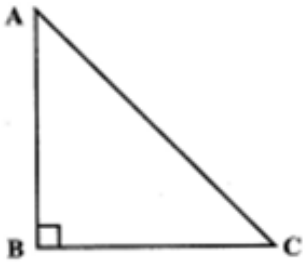
In a right-angled triangle, the square of the hypotenuse equals to the sum of the squares of its remaining two sides.



Example:
In the given right-angled triangle $ABC, \angle ACB = 90°$, then
as per the Pythagoras theorem,
$(AB)^2 = (BC)^2 + (AC)^2$
Or Hypotenuse^2 = Base^2+ Perpendicular^2
Or $c^2 = a^2 + b^2$; where $c = AB,\ a = BC$ and $b = AC$.

Converse -

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right-angled triangle with the angle opposite to the first side as the right-angle.



Example:
In the given triangle ABC, if $(AC)^2 = (BC)^2 + (AB)^2$, then $\triangle ABC$ is a right-angled triangle with $\angle ABC = 90°$.

Pythagorean Triplets -

In a group of three positive integers, if the square of the largest integer is equal to sum of squares of the remaining two integers, then the group of three integers is called a Pythagorean Triplet.

E.g.: $-(3,4,5),(6,8,10),(5,12,13),(8,15,17)$ are some examples of Pythagorean triplets.

• Here the largest integer is the measure of the hypotenuse of the right-angled triangle so formed and the other two integers indicate measures of the perpendicular sides of the triangle.

Important properties to remember -

• Isosceles triangle -

The altitude from the apex angle bisects the base forming two equal right-angled triangles.

• Equilateral triangle -

The altitudes also bisect the opposite bases thus forming six equal right-angled triangles.

• Rectangle -

Each diagonal divides the rectangle into two equal right-angled triangles.

• Square & Rhombus -

The diagonals bisect each other at right angles thus forming four equal right-angled triangles.

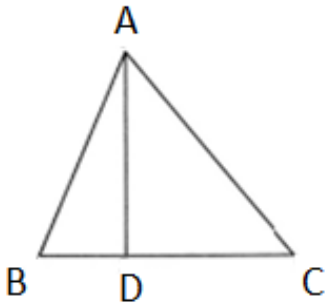
• Kite -

The longer diagonal is the perpendicular bisector of the shorter diagonal thus forming two pairs of equal right-angled triangles.

Practice Work:

1. $\triangle ABC$ is acute angled triangle. AD is an altitude (perpendicular) from vertex A on side BC. Point D is on side BC. $BD = 5,\ DC = 9,\ AC = 15$. Find AB.

2. In $\triangle ABC,\ D$ is the foot of the altitude from A on segment BC. If $AD = 24,\ AC = 30$ and $BC = 28$. Find AB.



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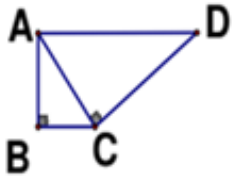
3. If $13,\ x + 24,\ 85$ is a pythagoras triplet, then what is the value of x ?

4. In the given diagram, $m\angle ABC = 90°$ and $m\angle ACD = 90°$. If $AB = 4, BC = 3$ and $AD = 13$, find length of $\overline{CD}$



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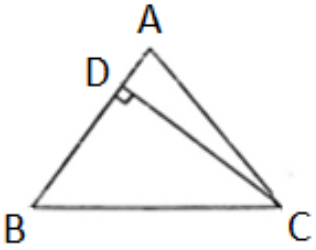
5. In $\triangle ABC$, segment CD is the altitude on side AB drawn from the vertex C. If $AB = 39$, $AC = 41$ and altitude $CD = 40$, then find the length of segment BC.



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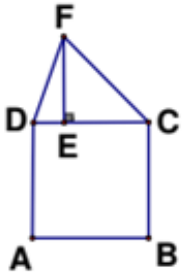
6. $\triangle ABC$ is right angled at $B \cdot \dfrac{BC}{AB} = \frac{2}{3}$ and $AC = \sqrt{52}$. Find BC and AB.

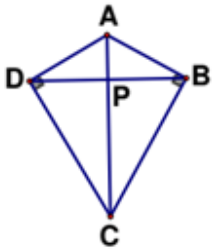
7. In $\triangle ABC$, $B = 90°$. AD is the angle bisector with D on $\overline{BC}$. P is the foot of altitude from D on $\overline{AC}$. If $AB = 40$, $AC = 50$, find PD.

.

8. In $\square ABCD$, is a square $\overline{FE} \perp \overline{DC}$. $\overline{DE} = 2$ cm, $\overline{FC} = 10$ cm, $FE = 6$ cm. Find perimeter of the square.



9. In $\square ABCD,\ B = D = 90°,\ AD = AB = 60.\ CD = BC = 80.$ Diagonals AC and BD intersect at P. If $AP = 36$, find (i) PC and (ii) BD.



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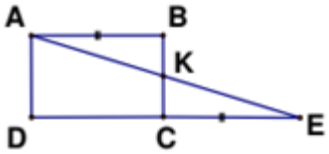
10. As shown in the figure, $\square ABCD$ is a rectangle. $AB = CE.\ AB = 12,\ AD = 10.$ Find KE.



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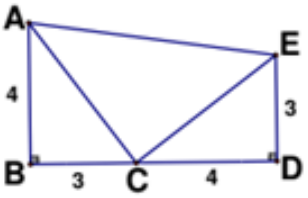
11. Use following figure and Find $(AE)^2$.



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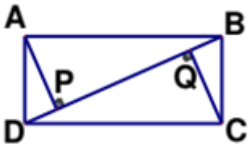
12. $\square ABCD$ is a rectangle with $AD = 60, AB = 80$ as shown. P and Q are feet of altitude on diagonal $\overline{BD}$ from A and C respectively. If $AP = 48$, find PQ.



Answer Key:

Practice Work:

Q. No.	Answer	Q. No.	Answer
1	13	7	13.33
2	26	8	40
3	60	9	64, 96
4	12	10	13
5	50	11	50
6	4, 6	12	28

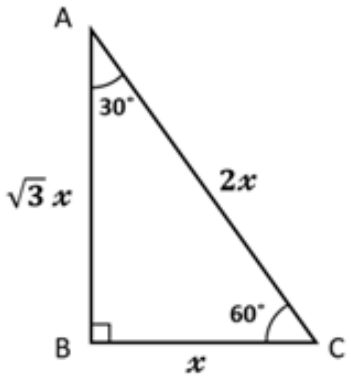
Topic 39

Pythagoras Theorem - 2

Theorem of $30^\circ - 60^\circ - 90^\circ$ triangle :

The side opposite to the 30° angle is half the measure of the hypotenuse and the side opposite to the 60° angle is $\sqrt{3}/2$ times the hypotenuse.

Example:

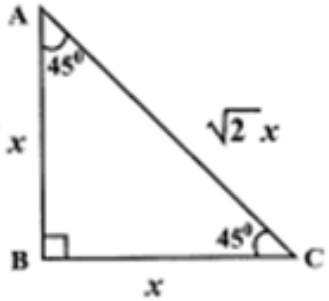


In $\triangle ABC$, if $\angle B = 90^\circ,\ \angle A = 30^\circ,\ \angle C = 60^\circ$ and $AC = 2x$,
then $BC = \frac{1}{2}AC$ and $AB = \frac{\sqrt{3}}{2}AC$
Hence, $BC = x$ and $AB = \sqrt{3}x$

Theorem of $45^\circ - 45^\circ - 90^\circ$ triangle :

In an isosceles right-angled triangle, the hypotenuse measures $\sqrt{2}$ times the congruent sides.

Example:



In $\triangle ABC$, if $\angle B = 90^\circ,\ \angle A = \angle C = 45^\circ$, and $AB = BC = x$, then $AC = \sqrt{2}x$

Practice Work:

1. In $\triangle ABC,\ m\angle ABC = 90°,\ m\angle ACB = 60°$ and $AC = 8$. Find BC & AB.

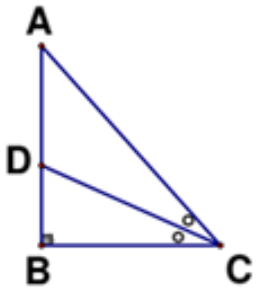
2. In $\triangle XYZ,\ m\angle XYZ = 45°,\ XY = 8$, and $YZ = 6$. Find XZ^2.

3. In $\triangle PQR,\ PR = 6,\ PQ = 10,\ m\angle PRQ = 90°$. Find the length of the median PS.

4. Manish started walking from point A to the east and reached at point B; 25 m distant from A, he turned to the right and reached at point C in the south, at 24 m. Again, he turned to the right and reached at point D at 18 m in the west. What is the distance of point A from point D ?

5. The height of an equilateral triangle is 12 cm. Find its area.

6. In $\triangle ABC,\ B = 90°,\ C = 60°$. CD is the angle bisector of $\angle C$ such that D is on $\overline{AB}$. If $AD = 10,\ BD = 5$, find AC^2.



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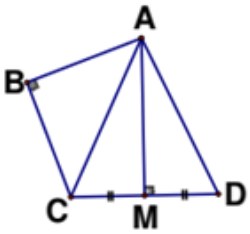
7. In the given figure, M is midpoint of $CD,\ \angle D = 60,\ \angle B = 90.\ AD = 20,\ AB = 12$. Find perimeter of $\square ABCD$.



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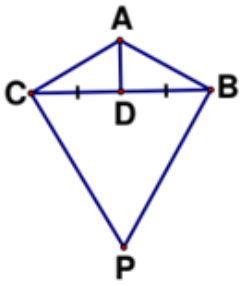
8. In $\triangle ABC,\ AB = AC,\ D$ is the midpoint of $\overline{BC}\cdot\ \triangle BCP$ is an equilateral triangle outside $\triangle ABC$ as shown.
If $AB = 25,\ AD = 24$, find the perimeter of $\triangle BCP$.



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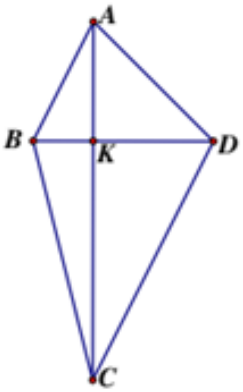
9. In $\square ABCD$, diagonals AC and BD meet at K and $BD \perp AC$ at K.
$BD = 3BK$, $CK = 2AK$, $AK = 2BK$, $AB^2 = 125$. Find AD^2, BC^2, BD, AC and CD^2.



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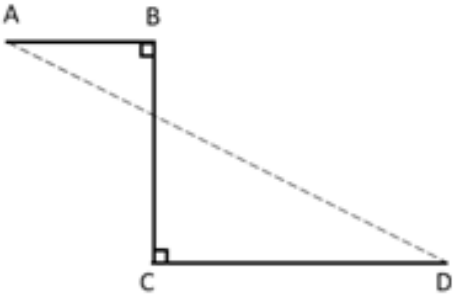
10. In the given figure, $AB = 21$ cm. $BC = 16$ cm. $DC = 42$ cm. Find AD.



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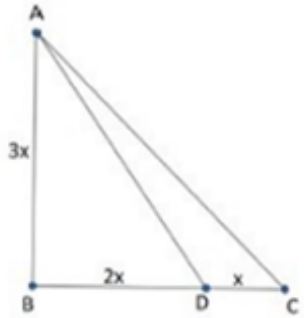
11. $\triangle ABC$ is right angled triangle as shown. $DC = x$, $DB = 2x$, $AB = 3x$, if $AC = 3\sqrt{26}$ find AD.



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12. In isosceles triangle ABC, $AB = AC = 65$ and $\overline{AD}$ is perpendicular to $\overline{BC}$. P is a point on $\overline{BC}$ such that $AP = 51$. If $PB = PC = 20$ find BC.

.

Answer Key:

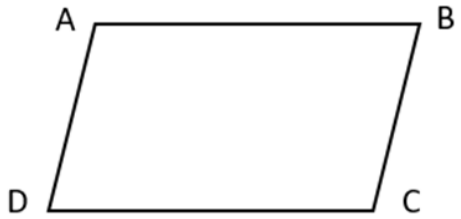
Practice Work:

Q. No.	Answer	Q. No.	Answer
1	$BC = 4,\ AB = 4\sqrt{3}$	7	68
2	$100 - 48\sqrt{2}$	8	42
3	$\sqrt{52}$	9	200, 425, 15, 30, 500
4	25	10	65
5	$48\sqrt{3}$	11	13
6	300	12	32

Topic 40

Parallelograms

Definition - A quadrilateral is a parallelogram if both its pairs of opposite sides are parallel.

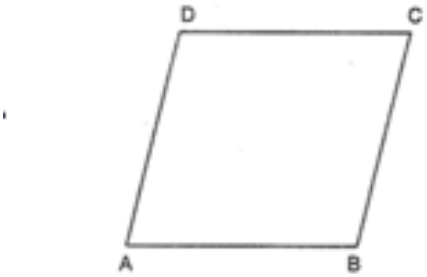


In the figure above, $AB//CD$ and $BC//AD$, hence $\square ABCD$ is a parallelogram.

Properties of Parallelogram :
- Opposite sides are equal.
- Opposite angles are equal.
- Adjacent angles are supplementary.
- Diagonals bisect each other.

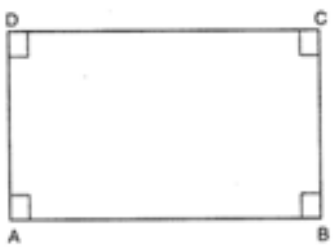
Special Types of parallelograms:

Rhombus:
A parallelogram having all sides equal is called a rhombus.



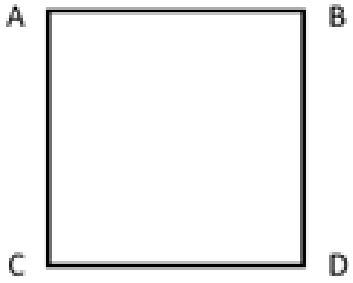
In the given figure, if in the parallelogram $ABCD, AB//DC, AD//BC$ and $AB = BC = CD = DA$, then $\square ABCD$ is a rhombus.

Rectangle:
In a parallelogram, if one angle is a right angle, then it is a rectangle.



In the given figure, if in the parallelogram $ABCD, AB//DC, AD//BC$ and $\angle A = 90°$, then $\square$ABCD a rectangle.

Square:
In a rhombus, if one angle is a right angle, then it is a square.
OR
In a rectangle, if all sides are equal, then it is a square.

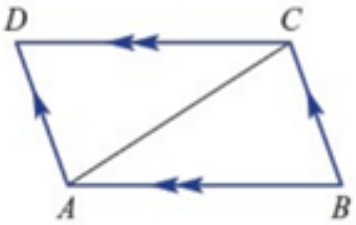


In the given figure, if in the rhombus $ABCD$, $\angle A = 90°$, then $\square ABCD$ is a square.
OR
if in rectangle $ABCD$, $AB = BC = CD = DA$, then $\square ABCD$ is a square.

• Since a rhombus, a rectangle and a square are special types of parallelograms, all the properties of a parallelogram are true for each of them.

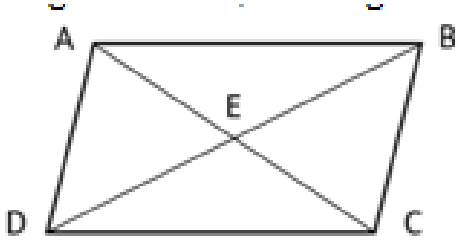
Practice Work : -

1) In the given figure, □ ABCD is a parallelogram. Prove that opposite sides are equal.

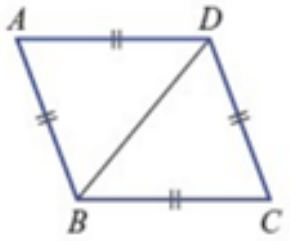


2) Prove that in a parallelogram the opposite angles are equal.

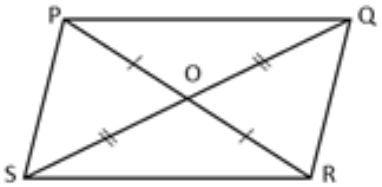
3) Prove that in the given parallelogram $ABCD$, the diagonals bisect each other.



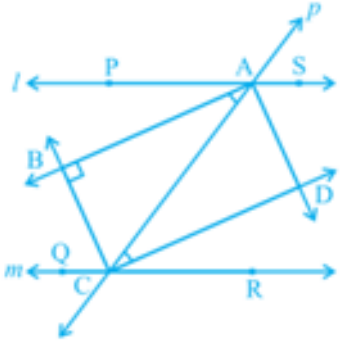
4) Prove that in a rhombus (special parallelogram), the diagonals bisect the interior angles.



5) The diagonals of quadrilateral PQRS bisect each other, hence prove that □ PQRS is a parallelogram.

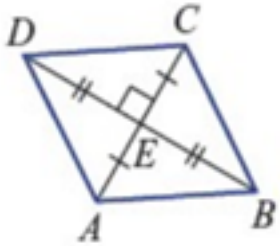


6) Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of the interior angles is a rectangle.

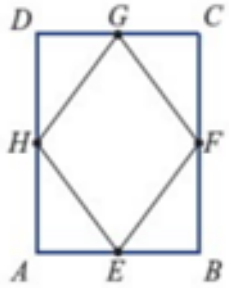


7) Prove that if the diagonals of a quadrilateral bisect each other and are of equal length then it is a rectangle.

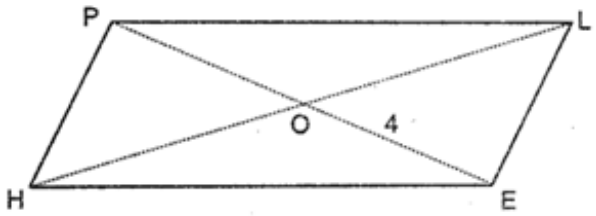
8) In the given quadrilateral $ABCD$, if the diagonals bisect each other at right angles, then prove that it is a rhombus.



9) In this diagram, E, F, G and H are mid-points of AB, BC, CD and DA respectively and $ABCD$ is a rectangle. Prove that $EFGH$ is a rhombus.



10) HELP is a parallelogram.lf OE=4 cm and HL is 5 cm more than PE. Find OH.



Topic 41

Perimeter and Area-1

Perimeter: The perimeter of any given closed figure is the sum measure of all its sides.

1. Perimeter of Triangle = Sum measure of all its sides.
2. Perimeter of square = 4 × side.
3. Perimeter of rectangle = $2(l + b)$

Area : The area of any plane figure is the surface enclosed by its sides. Area is measured in square units i. e. Square centimetres $\left(\mathrm{Cm}^2\right)$ or Square metres (m^2) or in square units.

Formulae for finding the area of different figures

1. Triangle:
(i) When the base and height (altitude) are known

Area = $\frac{1}{2}$ × base × height.

(ii) When the sides of the triangle are known

Area = $\sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$; s = semiperimeter.

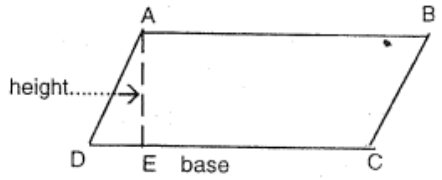
(iii) When the triangle is an equilateral triangle and its side is known.
Area = $\frac{\sqrt{3}}{4}\times$ side ^{2}
height of an equilateral triangle = $\frac{\sqrt{3}}{2}\times$ side

2. Square Area = $(side)^2$

3. Rectangle Area = length × breadth

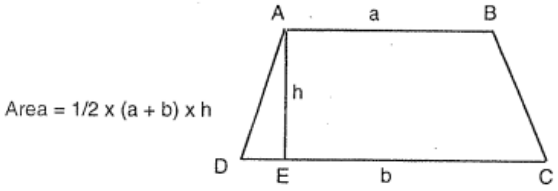
4. Parallelogram: When the distance between the parallel sides (i.e.height) is known.

Area = base × height
Area = $DC \times AE$



5. Trapezium:

Area = $\frac{1}{2}\times$ (sum of the parallel sides)× distance between the parallel sides.



6. Rhombus:

Area $= \frac{1}{2}\times$ (Product of diagonals)

Practice Work:

1. The base of a right angled triangle is 4 units and the hypotenuse is 5 units. What is the area of the triangle?

2. The side of an equilateral triangle is 8 cm. Find its area.

3. The area of a square is equal to that of a rectangle. The length of the rectangle is 12 more and the breadth is 8 less than the side of the square. Find area of square.

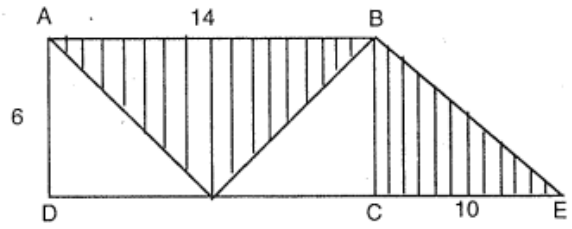
4. A room is 10 m long and 6 m broad. Find the cost of covering the floor with carpet 75 cm wide at Rs. 10 per metre.

5. The sides of a parallelogrom are 20 cm and 15 cm. The distance between the 20 cm sides is 18 cm. Find the distance between 15 cm sides.

6. The perimeter of a rhombus is 100 cm and one of its diagonals is 40 cm. Find the area of the rhombus.

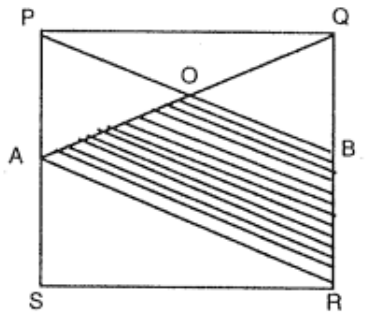
7. Find the area of the shaded portion in the adjoining figure.

$\square ABCD$ is a rectangle.



8. The area of a rectangle is 196 sq.cm. The length is 4 times the breadth.
Find the perimeter of the rectangle.

9. In the adjoining figure $PQRS$ is a square of side $8cm$. A and B are the midpoints of side PS and QR respectively Find the area of the shaded portion $AOBR$.



10. What is the area of a rectangular hall in (metres $)^2$, if it requires 60 tiles of length 300 cm and breadth 200 cm ?

Answer key:

Q	1	2	3	4	5	6	7	8	9	10
A	6	$16\sqrt{3}$	576	800	24	600	72	70	24	360

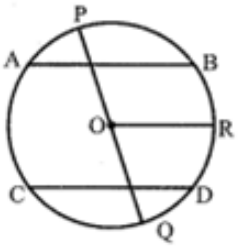
Topic 42

Circle-1

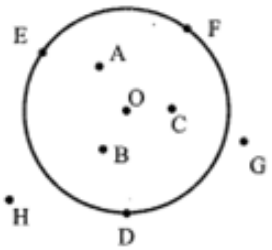
Circle : A circle is a set of points in a plane which are equidistant from a fixed point in the plane.

Radius and Diameter :
1) All radii of a circle arc of same length.
2) A circle has infinite number of radii. (Radius = r)
3) A circle has infinite number of equal diameters. (diameter = d)
4) Length of a diameter of a circle is twice the length of its radius.

Chord of a circle :
1) A line joining any two points on the circle is called chord of the circle.
2) If a chord passes through the centre of a circle then the chord is called a diameter of the circle.
3) Diameter is the longest chord of a circle.
4) A circle has infinite number of chords.
seg OR, seg OP, seg OQ - Radii of circle.
seg AB, seg CD, seg PQ - Chords of circle.
seg PQ - diameter of circle. Point O - Centre of circle.
If radii of two circles arc equal, then the circles arc called congruent circles. Every circle is congruent to itself. For congruent circles : $r_1 = r_2$

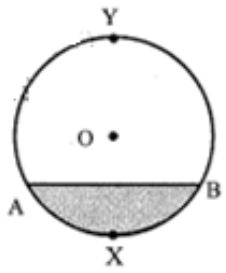


Circular region : A circle divides a plane in three parts.
1) Interior of a circle 2) Exterior of a circle 3) The circle itself.

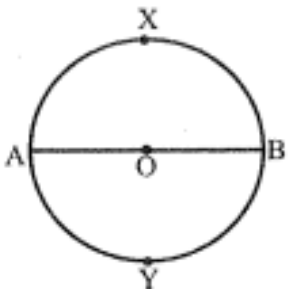


Circular region is formed by the interior of a circle and the circle.
Points in the interior of circle : point A, point B, point C, point O
Points in the exterior of circle : point H, point G.
Points on the circle: point D, point E, point F.

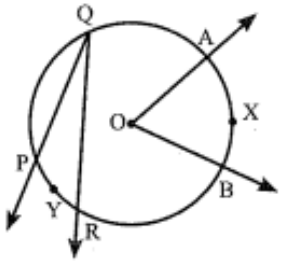
Segment of a circle :
Due to a chord, a circular region is divided into two parts.
Each part is called **segment of the circle**.
The part of the circle which contains the centre is called
major segment and opposite part is called **minor segment**.
The shaded portion AXB is minor segment and the unshaded portion AYB is major segment of the circle.



The semicircular segment : The two segments formed by a diameter of the circle arc called **semicircular segments.**
In the figure, segment AXB and segment AYB are two semicircular segments formed by diameter AB.



The Arc of a circle :
The intercepted arc
arc AXB and arc PYR are intercepted by ∠AOB and ∠PQR respectively.



Minor arc and Major arc :

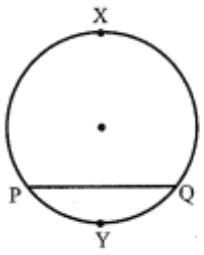


Fig. No. I

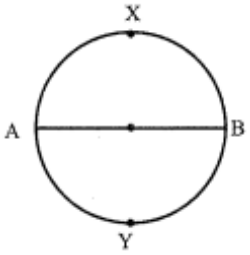
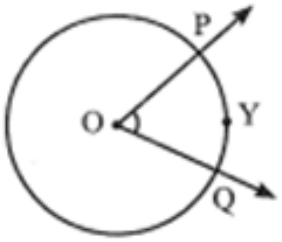


Fig. No. II

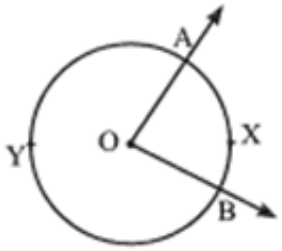
In the I^{st} figure, arc PXQ is major arc and arc PYQ is a minor arc.
In the II^{nd} figure, arc AXB and arc AYB are semicircular arcs.

The central angle :
An angle having the centre of the circle as its vertex is called a **central angle.**
In the adjacent figure $\angle POQ$ is a central angle.



The measurement of an arc:

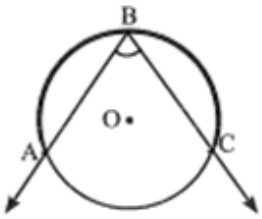
1) The measure of a minor arc : The measure of the central angle corresponding to a minor arc is the measure of the minor arc. $m\angle AOB = m(arc\ AXB)$



2) The measure of a major arc:
Measure of a major arc $= 360° -$ measure of the corresponding minor arc.

3) The measure of a semicircular arc : The measure of a semicircular arc is $180°$.

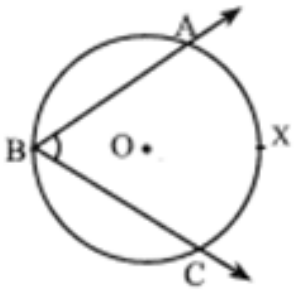
Inscribed angle : If vertex B of $\angle ABC$ is on the arc ABC and the end points of this arc on the arms of $\angle ABC$ then $\angle ABC$ is inscribed in the arc ABC.



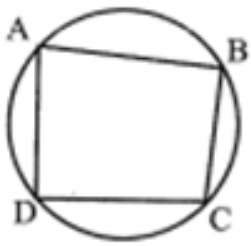
The relation between inscribed angle and intercepted arc :
The measure of an inscribed angle is half the measure of the arc intercepted by it.

Measure of inscribed angle $= \dfrac{Measure\ of\ intercepted\ arc}{2}$

$\therefore m\angle ABC = \dfrac{m(arc\ AXC)}{2}$



Cyclic Quadrilateral



If all vertices of a quadrilateral arc on a circle, then it is called as **cyclic quadrilateral**.
In the adjoining fig. □ABCD is a cyclic quadrilateral.
The opposite angles of a cyclic quadrilateral arc supplementary.
m∠DAB + m∠BCD = 180°
m∠ADC + m∠ABC = 180°

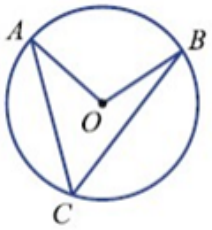
Practice Work:

1. Prove that the perpendicular drawn from centre to a chord of a circle bisects the chord.

2. Prove that congruent chords in the same circle or congruent circles are at an equal distance from the centre of the circle.

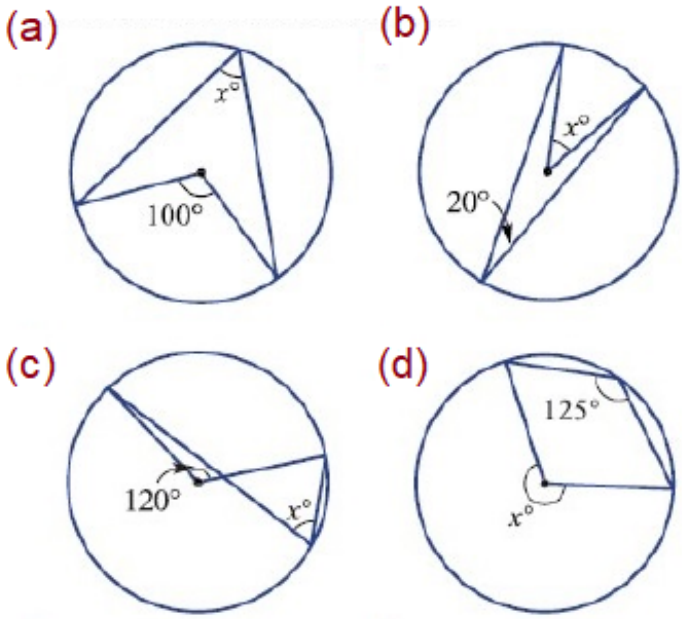
3. Prove that congruent chords in the same circle or congruent circles form congruent angles at the centre of the circle.

4. For this circle O is the centre.

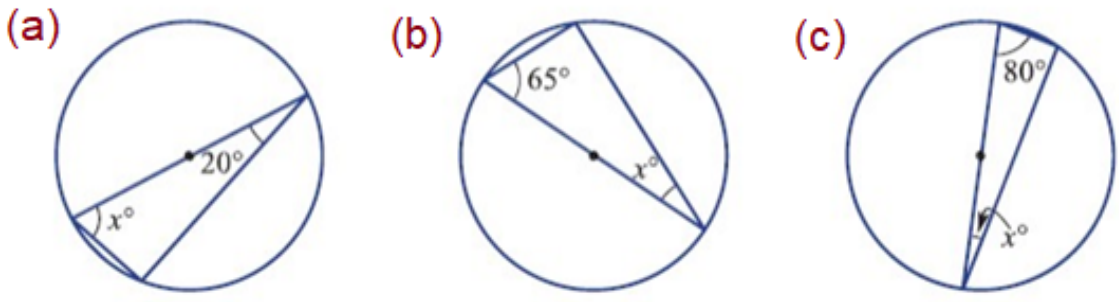


a. Name the angle at the centre of the circle.
b. Name the angle at the circumference of the circle.
c. If $\angle ACB = 40°$ find $\angle AOB$
d. If $\angle AOB = 122°$ find $\angle ACB$

5. Find the value of x in these circles.

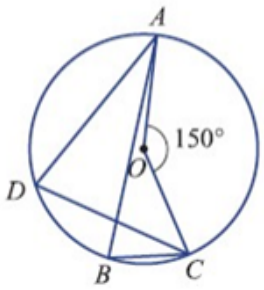


6. Find the value of x in these circles.

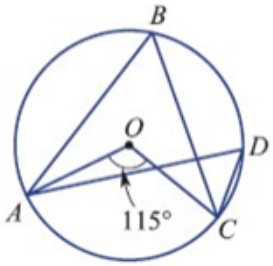


7.

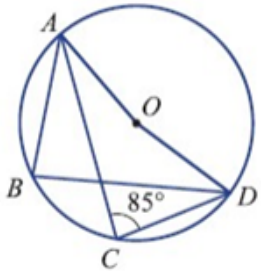
(a) Find $\angle ADC$ and $\angle ABC$.



(b) Find $\angle ABC$ and $\angle ADC$.

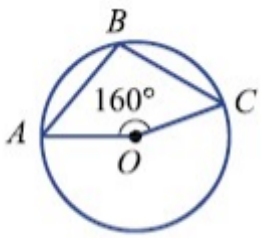


(c) Find $\angle AOD$ and $\angle ABD$.

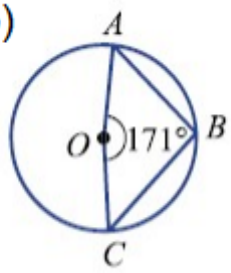


8. Find $\angle ABC$.

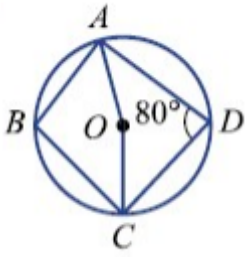
(a)



(b)



(c)



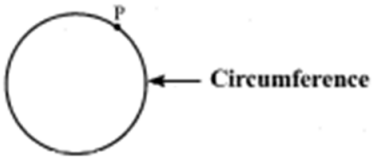
Answer key:

CW	Ans
1.	-
2.	-
3.	-
4.	a. $\angle AOB$ b. $\angle ACB$ c. 80 d. 61
5.	a. 50 b. 40 c. 60 d. 250
6.	a. 70 b. 25 c. 10
7.	a. 75,75 b. $57.5, 57.5$ c. 170,85
8.	a. 100 b. 94.5 c. 100

Topic 43

Circle–3

Circumference and area of a circle :
Circumference :
The length of circular ring is the circumference of the circle.



Relation between circumference and diameter :
The ratio of the circumference of any circle to its diameter, is a constant and this constant is an irrational number. The Greek letter ' π ' is used as a symbol for this constant.

($\pi = \frac{22}{7}$ or 3.14 , are taken for convenience)

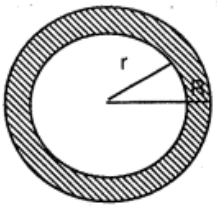
$\therefore \dfrac{Circumference}{Diameter} = \pi$

If radius = r, diameter = d and circumference = c

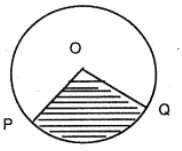
1) $\dfrac{c}{d} = \pi$ 2) $c = \pi d$, 3) $d = \dfrac{c}{\pi}$, 4) $r = \dfrac{c}{2\pi}$, 5) $r = \dfrac{d}{2}$

Area of a circle:
Area of a circle is the region bounded by the circumference of the circle.
If ' r ' is the radius of the circle then Area of the circle = πr^2

• Area of a circular track (or circular ring) = $\pi\,(R^2 - r^2)$
R = external radius and r = internal radius (see figure)



Area of a Sector:
A sector of circle is the surface enclosed between any two radii and arc intercepted by the end points of the radii.
The shaded portion (OPQ) in the figure is a sector.



Area of a Sector = $\dfrac{x^0}{360} \times \pi r^2$ (where x = angle subtended at the centre by the arc PQ)

Area of a segment:

The surface of a circle, enclosed between an arc and the chord joining the end points of the arc, is called a segment. The shaded portion in the adjoining figure shows a segment.

Area of segment $= \left[\frac{x}{360}\left(\pi r^2\right) - A\Delta OAB\right]$

Revolutions

One revolution by a circular wheel is equal to circumference of that wheel.

Number of revolutions made by a circular wheel $= \frac{Distance}{Circumference}$

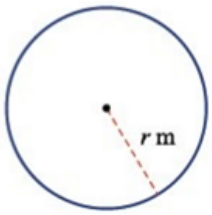
Practice Work:

1. If a circle has radius r cm, find the following, rounding to two decimal places where necessary.

a) The circumference of a circle if $r = 7$

b) A rule for r in terms of the circumference C

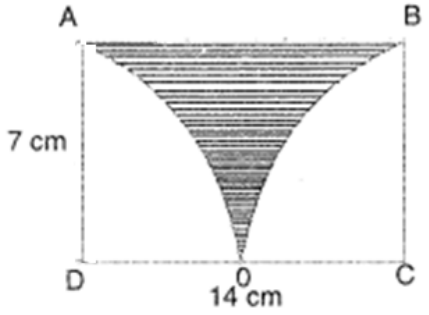
c) The radius of a circle with a circumference of 66 m



2. The internal and the external radii of a circular ring are 3 cm and 10 cm respectively. Find the area of the circular ring.

3. From the given figure calculate the area of the shaded portion

4. Figure $ABCD$ is a rectangle. Side $AD = 7$ cm, side $DC = 14$ cm. O is the midpoint of DC. Find the area of the shaded portion.



5. Find the cost of fencing a semi-circular garden of radius 14 m at $Rs.10$ per metre.

6. From a circular aluminium sheet of radius 14 cm, a sector of angle 45° is removed. Find the percentage of the area of the sector removed.

7. If the radius of a wheel is 35 cm, then find how many times it should rotate to cover a distance of 33 m.

8. Find the length of an arc of a sector whose central angle is 45° and radius is 56 cm.

9. If the radius of a wheel is 42 cm and it makes 20 rotations in 1 h, then find the time taken by the wheel to cover a distance of 528 m.

10. The radius of a circular card board is 20 cm. A strip of width 4 cm is removed all along its border. The percentage of the area of the strip removed is .....

Answer key:

	Practice Work
1.	a. 44 b. $r = \frac{C}{2\pi}$ c. 10.5
2.	286
3.	198
4.	21
5.	720
6.	12.5
7.	15
8.	44
9.	10
10.	36

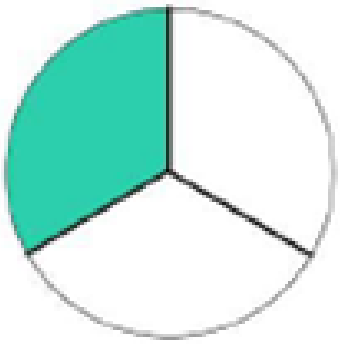
Topic 44

Factorization-1

The process of writing a given algebraic expression as the product of two or more factors is called factorization.

Factors: If an algebraic expression is written as the product of numbers or algebraic expressions, then each of these numbers and expressions are called the factors of the given algebraic expression and the algebraic expression is called the product of these expressions.

Factorisation is the reverse process of expansion.



When we **expand** an expression we **remove its brackets.**

When we **factorise** an expression we **insert brackets.**

Greatest Common Factor (GCF) OR Highest Common Factor (HCF):

The greatest common factor of two or more monomials is the product of the greatest common factors of the numerical coefficients and the common letters with smallest powers.

To factorise an algebraic expression involving a number of terms, we look for the HCF of the terms. We write it in front of a set of brackets. We then identify the contents of the brackets.

If the given expression is the difference of two squares, then to factorize it, we use the formula:

$$\left(a^2 - b^2\right) = (a + b)(a - b)$$

Practice Work:

1) Find the greatest common factor ($GCF/H\check{C}F$) of the following polynomials:

a. $2x^2$ and $12x^2$ b. $6x^3y$ and $18x^2y^3$

c. $7x, 21x^2$ and $14xy^2$ d. $42x^2yz$ and $63x^3y^2z^3$

e. $12ax^2, 6a^2x^3$ and $2a^3x^5$ f. $9x^2, 15x^2y^3, 6xy^2$ and $21x^2y^2$

g. $4a^2b^3, -12a^3b, 18a^4b^3$ h. $6x^2y^2, 9xy^3, 3x^3y^2$

2) Factorize-

(a) $12x^3y^4 + 16x^2y^5 - 4x^5y^2$ (b) $18a^3b^2 + 36ab^4 - 24a^2b^3$

3) Factorize each of the following algebraic expressions:

a. $6x(2x - y) + 7y(2x - y)$ b. $2r(y - x) + s(x - y)$

c. $7a(2x - 3) + 3b(2x - 3)$ d. $9a(6a - 5b) - 12a^2(6a - 5b)$

e. $a(x - y) + 2b(y - x) + c(x - y)^2$ f. $-4(x - 2y)^2 + 8(x - 2y)$

g. $x^3(a - 2b) + x^2(a - 2b)$ h. $(2x - 3y)(a + b) + (3x - 2y)(a + b)$

4) Factorize each of the following expressions:

a. $qr - pr + qs - ps$ b. $p^2q - pr^2 - pq + r^2$

c. $1 + x + xy + x^2y$ d. $ax + ay - bx - by$

e. $x^3 - y^2 + x - x^2y^2$ f. $6xy + 6 - 9y - 4x$

g. $x^2 - 2ax - 2ab + bx$ h. $x^3 - 2x^2y + 3xy^2 - 6y^3$

5)

a) Show that $4 - (x + 2)^2 = -x(x + 4)$ by factorising the left-hand side.

b) Now factorise these:

i. $9 - (x + 3)^2$ ii. $16 - (x + 4)^2$

iii. $25 - (x - 5)^2$ iv. $25 - (x + 2)^2$

v. $49 - (x - 1)^2$ vi. $100 - (x + 4)^2$

6) Factorize each of the following expressions:

a. $(2a - b)^2 - 16c^2$ b. $(x + 2y)^2 - 4(2x - y)^2$

c. $3a^5 - 48a^3$ d. $a^4 - 16b^4$

e. $x^8 - 1$ f. $64 - (a + 1)^2$

g. $75a^3b^2 - 108ab^4$ h. $x^5 - 16x^3$

i. $\frac{50}{x^2} - \frac{2x^2}{81}$ j. $256x^5 - 81x$

k. $a^4 - (2b + c)^4$ l. $(3x + 4y)^4 - x^4$

Answer Key:

Practice Work:

Q No	Answer
1)	a. $2x^2$ b. $6x^2y$ c. $7x$ d. $21x^2yz$ e. $2ax^2$ f. $3x$ g. $2a^2b$ h. $3xy^2$
2)	a) $4x^2y^2\,(3xy^2 + 4y^3 - x^3)$ b) $6ab^2\,(3a^2 + 6b^2 - 4ab)$
3)	a. $(2x - y)(6x + 7y)$ b. $(x - y)(s - 2r)$ c. $(2x - 3)(7a + 3b)$ d. $3a(3 - 4a)(6a - 5b)$ e. $(x - y)(a - 2b + cx - cy)$ f. $4(x - 2y)(2 - x + 2y)$ g. $x^2(a - 2b)(x + 1)$ h. $5(a + b)(x - y)$

Q No	Answer
4)	a. $(q-p)(r+s)$ b. $(p-1)\,(pq-r^2)$ c. $(1+x)(1+xy)$ d. $(x+y)(a-b)$ e. $(x-y^2)\,(x^2+1)$ f. $(2x-3)(3y-2)$ g. $(x-2a)(x+b)$ h. $(x-2y)\,(x^2+3y^2)$
5) b)	a) $-x(x+6)$ b) $-x(x+8)$ c) $-x(x-10)$ d) $(7+x)(3-x)$ e) $(6+x)(8-x)$ f) $(14+x)(6-x)$
6)	a. $(2a-b+4c)(2a-b-4c)$ b. $5x(-3x+4y)$ c. $3a^3(a-4)(a+4)$ d. $(a+2b)(a-2b)\,(a^2+4b^2)$ e. $(x-1)(x+1)\,(x^2+1)\,(x^4+1)$ f. $-(a+9)(a-7)$ g. $3ab^2(5a+6b)(5a-6b)$ h. $x^3(x-4)(x+4)$ i. $2\left(\frac{5}{x}+\frac{x}{9}\right)\left(\frac{5}{x}-\frac{x}{9}\right)$ j. $x(4x+3)(4x-3)\,(16x^2+9)$ k. $(a-2b-c)(a+2b+c)\,(a^2+(2b+c)^2)$ l. $8(x+2y)(x+y)\,[(3x+4y)^2+x^2]$

Topic 45

Factorization-2

Identifying Perfect Squares-

Notice that

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a-b)^2 = a^2 - 2ab + b^2.$$

(a^2, b^2: perfect squares)

So, a perfect square must be one of these two forms. It must contain **two squares and a middle term** of $\pm 2ab$.

For example,

$x^2 + 10x + 25$	and	$x^2 - 10x + 25$
$= x^2 + 2 \times 5 \times x + 5^2$		$= x^2 - 2 \times 5 \times x + 5^2$
$=(x+5)^2$		$= (x-5)^2$

• If the given expression is the difference of two squares, then to factorize it, we use the formula:

$$\left(a^2 - b^2\right) = (a+b)(a-b)$$

• If the given expression is in the form $x^2 + x(a+b) + ab$, then we factorize it in the form

$$(x+a)(x+b)$$

For example:
i) Factorise: $x^2 + 11x + 24$
We need to find two numbers with sum 11 and product 24 . Pairs of factors of 24 :

Factor product	1×24	2×12	3×8	4×6
Factor sum	25	14	11	10

this one

The numbers we want are 3 and 8 .
So, $x^2 + 11x + 24$
$=(x+3)(x+8)$

ii) Factorise:
$x^2 - 7x + 12$
sum $= -7$ and product $= 12$
$\therefore$ the numbers are -3 and -4
So, $x^2 - 7x + 12$
$= (x-3)(x-4)$

Practice Work :

1) Find all perfect squares of the form:

a. $x^2 + \square + 1$ b. $x^2 + \square + 4$ c. $x^2 + \square + 16$
d. $4x^2 + \square + 1$ e. $9x^2 + \square + 4$ f. $16x^2 + \square + 81$
g. $4x^2 + \square + c^2$ h. $x^2 + \square + 4d^2$ i. $a^2c^2 + \square + 4$

2) Find two numbers which have:

a. product 8 and sum 6 b. product 14 and sum 9
c. product 21 and sum 10 d. product -6 and sum 5
e. product -7 and sum -6 f. product -22 and sum -9
g. product 16 and sum -10 h. product 24 and sum -11

3) Factorisc:

a. $x^2 + 3x + 2$ b. $x^2 + 10x + 24$ c. $x^2 + 11x + 18$
d. $x^2 - 11x + 18$ e. $x^2 - 14x + 33$ f. $x^2 - 10x + 24$
g. $x^2 - 3x - 18$ h. $x^2 + 6x - 27$ i. $x^2 - x - 30$

4)Find two consecutive integers such that the square of the smaller plus twice the larger is 50.

5)Two numbers have a sum of 9 and the sum of their squares is 153. Find the numbers.

6) The product of two consecutive integers is 156. Find the integers.

7) A rectangle has sides which differ in length by $3cm$. If the area is $154cm^2$, find the perimeter of the rectangle.

8) The sum of the squares of three consecutive integers is 110.
a. Let the smallest integer be x and find a quadratic equation involving x.
b. Solve the quadratic equation and hence write down the three integers.
c. This time let the middle integer be x and find the quadratic equation involving x.
d. Solve the equation in c and hence write down the three integers.
e. Which method is better, starting with the smallest being x or the middle number being x ? Why? **Answer key:**

Practice Work:

Q. No	Ans.
1)	a. x^2+2x+1 and x^2-2x+1 b. x^2+4x+4 and x^2-4x+4 c. $x^2+8x+16$ and $x^2-8x+16$ d. $4x^2+4x+1$ and $4x^2-4x+1$ e. $9x^2+12x+4$ and $9x^2-12x+4$ f. $16x^2+72x+81$ and $16x^2-72x+81$ s. $4x^2+4cx+c^2$ and $4x^2-4cx+c^2$ h. $x^2+4dx+4d^2$ and $x^2-4dx+4d^2$ i. $a^2c^2+4ac+4$ and $a^2c^2-4ac+4$
2)	a. 2 and 4 b. 2 and 7 c. 3 and 7 d. 6 and -1 e. -7 and 1 f. -11 and 2 g. -2 and -8 h. -8 and -3
3)	a. $(x+1)(x+2)$ b. $(x+6)(x+4)$ c. $(x+2)(x+9)$ d. $(x-2)(x-9)$ e. $(x-11)(x-3)$ $f.(x-6)(x-4)$ g. $(x-6)(x+3)$ h. $(x+9)(x-3)$ i. $(x-6)(x+5)$
4)	6 and 7 or -8 and -7
5)	12 & -3
6)	12 and 13 or -13 and -12
7)	50 cm
8)	a. $x^2+2x-35=0$ b. $-7,-6,-5$ or $5,6,7$ c. $x^2=36$ d. $-7,-6,-5$ or $5,6,7$ e. Starting with the middle term being x.

Topic 46

Simultaneous equations-1

Consider the following equation,

$4x + 7y = 29 \cdots (1)$

Can we solve this equation, based on what we learned in Linear equations?

We can't. Solving this would just lead to - one variable expressed in terms of other.

Now, consider another equation $x + 3y = 11 \cdots (2)$

By considering the 2^{nd} equation, now we will be able to solve x and y. These are called as **Simultaneous equations.**

Thus, in order to get the solutions for two unknowns, one equation is not sufficient, we need two equations.
In general, to get solution for ' n ' unknowns we need ' n ' equations.

Simultaneous equations:

When two or more equations are satisfied by the same values of the unknown quantities, they are called simultaneous equations.
$4x + 7y = 29 \cdots (1)$
$x + 3y = 11 \cdots (2)$

Equations (1) and (2) are simultaneous equations.

Methods for Solving Simultaneous Equations:

1. Elimination method:

The process by which we get rid of either of the unknown quantities is called Elimination and it must be solved in different ways according to the nature of the equations given.
Example (1) solve

$4x + 7y = 29 \cdots (1)$
$x + 3y = 11 \cdots (2)$

To eliminate x, we multiply (2) by 4 so as to make the coefficients of x in both the equations equal. This gives

$4x + 12y = 44 \cdots \cdots (3)$

Now subtract (1) from (3)

$5y = 15 \Rightarrow y = 3$

Substituting value of y in (1) or (2), x can be obtained.

$\therefore x + 3 \times 3 = 11 \therefore x = 2$

2. Substitution method:

Step 1) Get value of one unknown in terms of other unknown from any of the equations.
Step 2) Substitute value obtained in step 1 in other equation thus obtaining an equation in one unknown.
Step 3) Solve the equation and get the solution for one unknown.
Step 4) Substitute the value obtained in any of the given equations and obtain other unknown.

Example: $x + 2y = 13 \cdots (1)$

$3x + y = 14 \cdots (2)$

From (1), we get

$x = 13 - 2y \cdots (3)$

Substitute (3) in (2)

$\therefore 3(13 - 2y) + y = 14$

$\therefore 39 - 6y + y = 14$

$\therefore -5y = -25$

$\therefore \text{y} = 5 \cdots (4)$

Substitute (4) in (2)

$\therefore 3x + 5 = 14$

$\therefore 3x = 9 \Rightarrow x = 3$

$\therefore$ solution of (1) and (2) is $x = 3, y = 5$

Practice Work:

Q1. Write the equation that results when:

a) $2x - 3y = 5$ is multiplied by 2

b) $x + 3y = 7$ is multiplied by -3

c) $2x + 5y = 1$ is multiplied by 4

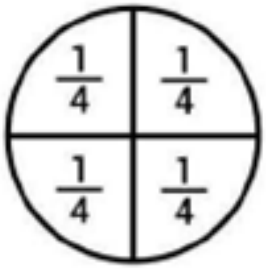
d) $3x - 2y = 8$ is multiplied by -2

e) $5x - y = 2$ is multiplied by 5

f) $-2x + 5y = -1$ is multiplied by -1

Q2. Solve the following equations simultaneously using the elimination method

Q3. Solve the following pairs of simultaneous equations using the substitution method.



Answers Key:

Practice Work

Q1	a. $4x - 6y = 10$ b. $-3x - 9y = -21$ c. $8x + 20y = 4$ d. $-6x + 4y = -16$ e. $25x - 5y = 10$ f. $2x - 5y = 1$
Q2.	a. $x = 4, y = 1$ b. $x = 3, y = -2$ c. $x = -2, y = 5$ d. $x = -3, y = -2$ e. $x = 7, y = -1$ f. $x = 3, y = -5$ g. $x = 6, y = -5$ h. $x = -3, y = 8$ i. $x = 3, y = -4$
Q3.	a. $x = 7, y = 3$ b. $x = -1, y = -5$ c. $x = 1/2, y = 4$ d. $x = 2, y = 4$ e. $x = 1, y = -1$ f. $x = -2, y = -12$ g. $x = 5, y = -2$ h. $x = 0, y = -4$

Topic 47

Simultaneous equations-2:

Application of Simultaneous Equations to Practical Problems:

Here we will solve word problems.

The 1^{st} task is to convert the word problem into an equation.

Then get two equations if there are two unknowns.

Then we use any of the methods learnt ealier to solve the given simultaneous equations.

Solve and get the values of the two unknown variables.

Example: Half the sum of two numbers is 20, and three times their difference is 30, find the numbers.

Let the greater number be x and smaller number be y.

It is given that half the sum of these numbers is 20.

Means the sum is $(x + y)$, half of the sum is $(x + y)/2$ which is 20.

$\therefore 1/2(x + y) = 20 \Rightarrow x + y = 40 \cdots (1)$

Now, three times their difference is 30.

Means the difference is $(x - y), 3$ times their difference is $3(x - y)$ which is 30.

$\therefore 3(\mathrm{x} - \mathrm{y}) = 30 \Rightarrow \mathrm{x} - \mathrm{y} = 10 \cdots (2)$

Solving (1) and (2) we get $x = 25$ and $y = 15$

Practice Work:

Q.1) Find two equations in x and y and solve algebraically.

a) Two numbers have a sum of 201 and a difference of 37. Find the numbers.

b) The difference between two numbers is 84 and their sum is 278.
What are the numbers?

c) One number exceeds another by 11. The sum of the two numbers is 5. What are the numbers?

d) The larger of two numbers is four times the smaller number and their sum is 85.
Find the two numbers.

e) Find two integers such that three times the smaller is 33 more than twice the larger, and twice the smaller plus five times the larger equals 250 .

Q.2) Form simultaneous equations and solve using any method.

a) Two hammers and a screwdriver cost a total of \$34, but a hammer and three screwdrivers cost a total of \$32. Find the price of each type of tool.

b) Four adults and three children go to a theatre for \$74. Two adults and five children

at the theatre are charged \$58. Find the price of an adult's ticket and a child's ticket.

c) Three blankets and a sheet cost \$190 altogether, whereas two sheets and a blanket cost a total of \$100. Find the cost of one blanket and one sheet.

d) Seven apples and eight oranges cost \$4.06 altogether. At the same shop three apples and one orange cost of total of \$1.06. Find the cost of an apple and the cost of an orange.

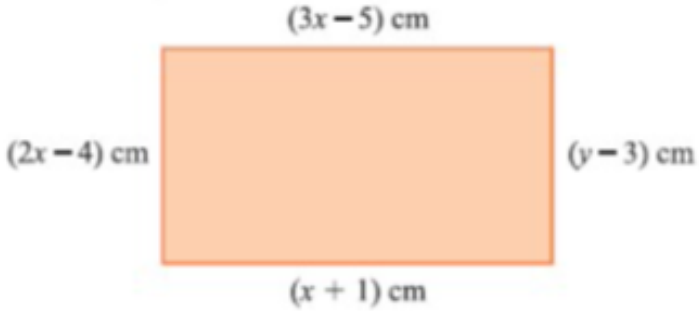
e) A purse contains \$3.75 in 5-cent and 20-cent coins. If there are 33 coins altogether, how many of each type of coin are in the purse?

f) I collect only 50-cent and \$1 coins. My collection consists of 43 coins and their total value is \$35. How many of each coin type do I have?

Q.3) Milk is sold in either 1 L or 2 L cartons. If a delicatessen owner orders 120 L of milk and receives 97 cartons, how many cartons of each size did they receive?

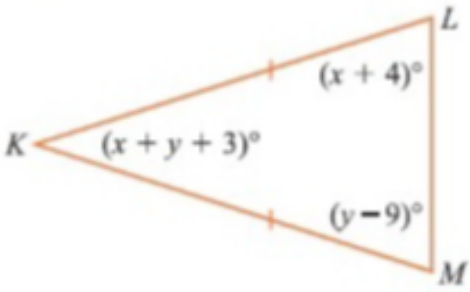
Q.4) A baker's basket contains bread rolls and sandwich loaves. The loaves weigh 750 g and the rolls 150 g. If the total number of loaves and rolls is 16 and their total weight is 6 kg, how many rolls and how many loaves are in the basket?

Q.5) Find x and y in this rectangle.



Q.6) An equilateral triangle has sides of length $(3x - y)$ cm, $(x + 5)$ cm and $(y + 3)$ cm. Find the length of each side.

Q.7) KLM is an isosceles triangle. Find x and y and hence find the measure of the angle at K.



Q.8) Twelve years ago Jane was five times as old as Anne. In 3 years time Anne will be half Jane's age. How old is each girl at the moment?

Q.9) Nine years ago a mother was three times as old as her son. In eight years from now the sum of their ages will be 78. How old are they today?

Q.10) Temperature in Centigrade (C) and Fahrenheit (F) are related to each other by formula $C = \frac{5}{9}(F - 32)$. If the temperature recorded in both Centigrade and Fahrenheit shows the same number then the number is

Answers

Practice Work

Q. no.	**Answer**
Q1.	a. 119 and 82 b. 181 and 97 c. 8 and -3 d. 68 and 17 e. 35 and 36
Q2.	a. 14 and 6 b. 14 and 6 c. Blanket = 56 and Sheet is = 22 d. Apple = 0.26 and Orange = 0.28 e. 19×5 cents and 14×20 cents f. 16×50 cents and 27×1 dollar
Q3.	74-1 liter and $23 - 2$ liter
Q4.	10 and 6
Q5.	$x = 3, y = 5$
Q6.	12 cm
Q7.	$x = 39, y = 52$ and $K = 94$
Q8.	Jane = 37 , Anne = 17
Q9.	Mother = 42 , Son = 20 .
Q10.	$-40°$

Topic 48

Average:

The average value for a set of given numbers is found by adding all the numbers and dividing the sum by number of numbers.

$\therefore$ Average $= \dfrac{\text{Sum of all numbers}}{\text{Total Number of numbers}}$.

e.g. Average of $1, 3, 5, 7, 9 = \dfrac{(1+3+5+7+9)}{5} = \dfrac{25}{5} = 5$

Key Points:

Average is also called as "Mean" in Statistics.

Total number of numbers $\times$ Average = Sum of all numbers in the data.

e.g. : If the average of 8 numbers is 15 , find the sum of those numbers.
Sum $= 8 \times 15 = 120$.

If we multiply the average (15) by Number of Numbers (8) we get the sum of all the numbers (120)

Average of ' n ' consecutive numbers $= \dfrac{\textbf{First Term + Last Term}}{\textbf{2}}$

e.g. Average of 1 to 10 $= (1+10)/2 = 5.5$

• The average of three, five or seven natural consecutive numbers is the middle number.

e.g. if we have 5 consecutive numbers as $1, 2, 3, 4, 5$; their average is 3.

$\dfrac{1+2+3+4+5}{5} = \dfrac{15}{5} = 3$ (Which is middle term of $1, 2, \underline{3}, 4, 5$)

• The average of consecutive three, five or seven even or odd numbers is the middle number.

e.g. $\dfrac{3+5+7+9+11}{5} = \dfrac{35}{5} = 7$ (which is middle term of $3, 5, \underline{7}, 9, 11$)

e.g. $\dfrac{12+14+16}{3} = \dfrac{42}{3} = 14$ (which is middle term of $12, \underline{14}, 16$)

Average value is the balancing point (fulcrum) of the set.

• If more numbers in the set are small, the average tends towards it.

e.g. $1, 2, 3, 4, 45$.

Sum $= 1+2+3+4+45 = 55$

$\therefore$ Average $= 11$ (tending towards the lower end)

• Similarly, if more numbers are big, the average tends towards the bigger end.

e.g. average of $5, 41, 42, 43, 44$

Average $= 175/5 = 35$.

Note that the average value may or may not be one of the numbers in the set.

• If the average and all the values in the set are given, we can find the unknown.

Eg : The average of 4 numbers $1, 2, 3$ and ' x ' is 4 . Find the value of ' x '.
Solution: The given numbers are $1, 2, 3, x$
$\therefore$ Average $= (1 + 2 + 3 + x)/4 = 4$.
Solving, we get $x = 10$

• If the numbers in a group are decreased or increased by same value then their average also decreases or increases by that value.

e.g. 15, 20, 25 have their average 20.

1) If they are increased by 5 each. We get the numbers $20, 25, 30$. and the average of $20, 25, 30 = 25$ (i.e. $20 + 5$)

2) If they are decreased by 5 each. We get the numbers $10, 15, 20$. and the average of $10, 15, 20 = 15$. (i.e. $20 - 5$)

Speed:

In case of average speed the distance covered is taken into consideration. If only the different speeds are added and then divided by the number of speeds the answer will be wrong.

$\therefore$ Average speed $= \dfrac{\text{Total Distance}}{\text{Total Time}}$

e.g.
A scooterist is going to a place 240kms. away. He travels 120 kms. at the average speed of 60kms/hr. and the remaining 120kms. at the average speed of 40kms./hr. His average speed for the entire journey is

Part 1: 120kms at 60 km/hr. Travel time = 2 hours.

Part 2: 120kms at 40 km/hr. Travel time = 3 hours.

Total distance = 240kms. and Total time = 5 hours.

$\therefore$ Average Speed $= 240/5 = 48$kms/hr.

[Note: For Speed, you must NOT calculate Average as $(60 + 40)/2 = 50$kms/hr. Don't take averages of speed]

Practice Work:

Q1. Find the average of $11, 18, 22, 14, 15$.

Q2. The average of three numbers is 24. If one of the number is 18 and the other is 19, find the third.

Q3. The average of 10 numbers is 7. If each number is multiplied by 12, then the average of the new set of numbers will be _____.

Q4. The average of 7 consecutive odd numbers is 45. Find the biggest of all.

Q5. The average of two numbers is M. If one number is N, the other number is_____.

Q6. The average score of five students is 35 runs. If the average score of the first three students is 41, find the average score of the last two.

Q7. The average age of 7 members of a family is 18 years. If the head of the family is excluded the average age of the rest of the members would be 5 years less. Find the age of the head of the family.

Q8. The average age of a class of 24 boys is 14 years 7 months. A new boy joins the class and the average age now becomes 14 years 6 months. What is the age of the new boy?

Q9. If average of ' m ' and ' n ' is 20 ; of ' n ' and ' y ' is 25, of m, n, and y is 23 then what is average of m and y ?

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	16
Q2.	35
Q3.	84
Q4.	51
Q5.	2M-N
Q6.	26 runs
Q7.	48 years.
Q8.	12 years 6 months.
Q9.	24.

Topic 49

Direct & Inverse Proportion - 1

Variation/ Proportion

Different quantities are related to each other in different ways. We shall first study the relationship between two quantities. If one quantity varies, then another quantity that is related to the first can vary in a number of ways. We shall study two such ways of variation.

Types of Variation / Proportion:
There are two types of variation/ proportion.

1) Direct Proportion
2) Inverse Proportion

In this lecture, we will learn about Direct Proportion.

Direct Proportion

If two quantities are related to each other such that an increase (or decrease) in the first quantity results in a corresponding proportionate increase
(or decrease) in the second quantity, then the two quantities are said to vary directly with each other.

Note: In Direct Proportion, the ratio of the two quantities always remains constant.

Example: at constant speed, distance covered varies directly as time, i.e., the longer the time for which the body' travels, the greater will be the distance covered. This is expressed as, distance travelled $\propto$ time.

Consider cost of 1 kg of Sugar is Rs. 50 , what is the cost of 3 kg of Sugar? It is Rs. 150.

Observe that as the weight of Sugar increases, the cost also increases in the same manner. Note that the ratio however remains constant.

Now, suppose a Car travels with a mileage of 40Kms with 2 Litres of petrol, then how will it travel with 10 Liters of petrol? The car can travel 200Kms.

Observe that change in one quantity (Independent variable x) leads to change in other quantity (dependent variable y).

In both the above examples, we find that as the value of x increases, value of y also increases in the same manner i.e. y is directly proportional to x denoted by $\mathbf{y \alpha x}$, such that the ratio y/x does not change; it remains constant (say k)

We say that x **and** y **are in direct proportion, if** $\mathbf{y \alpha x}$ **or** $\frac{\mathbf{y}}{\mathbf{x}} = \mathbf{k}$ **or** $\mathbf{y = kx}$**.**

You can calculate and check that value of **k** in the first example is 50, while in the second example is 20.

Example 1 In which of the following tables, a and b vary directly. Also, find the constant of variation if a and b are in direct variation.

(i)

a	4	7	21	28
b	12	21	63	84

(ii)

a	2.5	5	7.5	10	15
b	10	20	30	40	60

(iii)

a	1	2	3	4	5
b	2	1	6	3	2/5

Solution (i) We have,

$$\frac{4}{12} = \frac{7}{21} = \frac{21}{63} = \frac{28}{84} = \frac{1}{3}.$$

Thus, the ratio of the corresponding values of a and b is constant and is equal to $\frac{1}{3}$. Hence, a and b are in direct variation with the constant of variation equal to $\frac{1}{3}$.

(ii) We have,

$$\frac{2.5}{10} = \frac{5}{20} = \frac{7.5}{30} = \frac{10}{40} = \frac{15}{60} = \frac{1}{4}$$

This shows that the ratio of the corresponding values of a and b is constant and is equal to $\frac{1}{4}$. Thus, a and b vary directly. The constant of variation is $\frac{1}{4}$.

(iii) It is evident from the table that the ratio of the corresponding values of a and b is not constant. So, a and b are not in direct variation i.e. they do not vary directly.

Practice Work

Q1 If x and y vary directly, find the missing entries in the following table:

(i)

x	2.5	...	...	21
y	5	8	24	...

(ii)

x	...	9	15	...
y	3	4.5	7.5	13.25

Q2. A car travels 432 km on 48 litres of petrol. How far would it travel on 20 litres of petrol?

Q3. Ajay bought 12 books for Rs. 1560 ,what is the cost of 7 books?

Q4. Neeta takes 125 mins to walk 100 meters. What distance can she cover in 315 mins?

Q5. A worker is paid Rs 210 for 6 days work. If his total income of the month is Rs 875, for how many days did he work?

Q6. An electric pole, 14 meter high casts a shadow of 10 meters. Find the height of tree that casts a shadow of 15 meters.

Q7. A Car is travelling at a speed of 70 km/hr. What would be the distance travelled by the car in 24 mins?

Q8. Rita types 2070 words in $1\frac{1}{2}$ h. How long will she take to type 3450 words? How many words will she type in 3 hours?

Q9. If 25 men can dig a canal of length 62.5 m, how many men will be required to dig a similar canal of length 155 m ?

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	i) 4, 12 and 42; ii) 6 and 26.5
Q2.	180 kms
Q3.	Rs 910
Q4.	252 meters
Q5.	25 days
Q6.	21 meters
Q7.	28 kms
Q8.	i) $2\frac{1}{2}$ hrs (or 150 mins); 4140 words
Q9.	62 Men

Topic 50

Direct & Inverse Proportion - 2

Inverse Proportion : If two quantities are related to each other such that an increase (or decrease) in the first quantity results in a corresponding proportionate decrease (or increase) in the second quantity, then the two quantities are said to vary inversely with each other. i.e. the two quantities are said to be in Inverse Proportion.

Note : In Inverse Proportion, the product of the two quantities always remains constant.

E.g. 1) 2 Students take 20 minutes to arrange chairs, now if 4 students do the same work, will the time taken increase or decrease? The same work can now be done in 10 mins. Observe that as the number of people doing the work increases, the time taken decreases.

2) Now, suppose Amit can go to school, which is 2 kms from his house by walking or cycling. His walking speed is 4 km/ hour, while cycling speed is 12 km/ hour. Observe that, by walking he would take 30 mins, while with cycling he takes 10 mins. As the speed increases the time taken to travel the same distance decreases.

Observe that the change in one quantity (Independent variable x) leads to change in the other quantity (dependent variable y), in an inverse way.

In both the above examples, we find that as the value of x increases, the value of y decreases in the same manner or vice-versa.

i.e. y is inversely proportional to x, denoted as $\mathbf{y \alpha 1/x}$, such that their product $\mathbf{yx}$ does not change; it remains constant (say k).

Therefore, we say that x and y are in inverse proportion, if $\mathbf{y \alpha \frac{1}{x}}$ or $\mathbf{yx = k}$ or $\mathbf{y = k/x}$.

We can calculate and check that value of k.

In the first example (no. of people) x (time taken) = person-minutes i.e. 40 is constant. While in the 2^{nd} example (Speed) x (time taken)= Distance i.e. 2 kms is constant.

Example 1) In which of the following tables a and b vary inversely:

(i)

a	8	16	32	256
b	32	16	8	1

(ii)

a	9	18	2	12
b	8	4	30	6

(iii)

a	2	16	8	4
b	40	5	10	20

Solution: We know that if a and b vary inversely, then the product ab remains same for all values of a and b.

(i) Here,

$$8 \times 32 = 256, 16 \times 16 = 256, 32 \times 8 = 256 \; and \; 256 \times 1 = 256.$$

Clearly, the products of the values of a and the corresponding values of b are fixed. So, a and b vary inversely.

(ii) We have,

$$9 \times 8 = 72, 18 \times 4 = 72, 2 \times 30 = 60 \; and \; 12 \times 6 = 72.$$

Since the products of the values of a and the corresponding values of b are not fixed. So, a and b do not vary inversely.

(iii) We have,

$$2 \times 40 = 80, 16 \times 5 = 80, 8 \times 10 = 80 \; and \; 4 \times 20 = 80.$$

Clearly, the products of the values of a and the corresponding values of b are fixed. S_0, a and b vary inversely.

Practice Work

Q1. If a and b vary inversely, fill in the blanks:

(i)

a	8	2	...	5	1
b	10	...	20	...	80

(ii)

a	16	32	8	128
b	4	...	...	0.5

Q2.If 52 men can do a piece of work in 35 days, in how many days 28 men will do it?

Q3. Shalu cycles to her school at an average speed of 12 km/hr. It takes her 20 minutes to reach the school. If she wants to reach her school in 15 minutes, what should be her average speed?

Q4. 120 men had food provision for 200 days. After 5 days, 30 men died due to an epidemic. How long will the remaining food last?

Q5. 6 Pipes are required to fill a tank in 1 hour 20 mins. How long will it take if only 5 pipes are used?

Q6. If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Q7. A car can finish a certain journey in 10 hours at the speed of 48 km/hr. By how much should its speed be increased so that it may take only 8 hours to cover the same distance?

Q8. Three spraying machines working together can finish painting a house in 60 minutes. How long will it take for 5 machines of the same capacity to do the same job?

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	i) 40,4 and 16 ii) 2 and 8
Q2.	65 days
Q3.	16 km/hr
Q4.	260 days
Q5.	96 mins.
Q6.	24 workers
Q7.	12 km/hr
Q8.	36 mins

Topic 51

Time, Speed & Distance - 1

Speed: The distance travelled / covered by an object in unit time is called Speed. The units of speed are km/hr and meter/sec.

Speed $= \frac{Distance}{Time}$ OR Distance = Speed x Time OR Time $= \frac{Distance}{speed}$

• If two bodies travel with the same speed, then the distance covered varies directly as time, and it is written as **Distance** $\propto$ **Time**.

• If two bodies travel for the same period of time, the distance covered varies directly as the speed, i.e.**Distance** $\propto$ **Speed.**

• If two bodies travel the same distance, time varies inversely as speed, i.e., **Time** $\propto \frac{\mathbf{1}}{\mathbf{Speed}}$.

Important Conversion factor:

1 km/hr = 5/18 m/s

i.e. To convert km/hr to m/s multiply by 5/18. Why?

$\frac{1 \text{ km}}{1\text{hour}} = \frac{1000 \text{ m}}{3600 \text{ s}} \quad \frac{5 \text{ m}}{18\text{sec}}$

• Similarly, 1 m/s = 18/5 km/hr

i.e. To convert m/s to km/hr multiply by 18/5.

Important Cases:

1. When a train crosses a pole (or a standing man):

Distance covered to pass the pole = Length of the train.

2. When a train crosses a bridge (or platform or tunnel)

Distance covered to pass the bridge = Length of the train + Length of the bridge

3. When two trains go in the same direction on parallel tracks.

Distance = Length of train 1 + Length of train 2.

Relative speed = Speed of the faster train - Speed of the slower train.

4. When two trains go in opposite direction

Distance = Length of train 1+ Length of train 2 .

Relative speed:- Speed of train 1+ Speed of train 2.

Practice Work :

Q1. a. Convert 150 m/sec to km/hr.

b. Convert 72 km/hr to m/sec

Q2. A freight train 1 KM long is travelling at a speed of 20 KMPH. It enters a tunnel one KM long at 1 pm. In how many minutes does the rear of the train emerge from the tunnel ?

Q3. A train runs at the rate of 90 km/hr. The speed of the train in meters per second is

Q4. A train, 300 meters in length overtakes a telephone post in 15 seconds. Find the speed of the train.

Q5. With a speed of 60 km/hr a train crosses a pole in 30 sec. The length of the train in meters is

Q6. A train 250 m. in length crosses a bridge 350 m. in length in 20 seconds. Find the speed of the train.

Q7. Two trains of lengths 400 m. and 350 m. are running in opposite directions on parallel tracks. They cross each other in 30 sec. If one of them has a speed of 50 km/hr. Find the speed of the other.

Q8. A car takes 6 hours to cover a journey at the speed of 45 km/hr. At what speed (in km/hr) must it travel in order to complete the journey in 5 hours?

Q9. A is cycling at the speed of 10 km/hr. B is cycling at the speed of 8 km/hr. Both start moving simultaneously from two places 1 km apart in the same direction. How far (in kms) will A have cycled before he overtakes B ?

Q10. Two trains with speeds 60 km/hr and 75 km/hr are running in the same directions on parallel tracks. The faster train takes 1 min. 30 sec. to cross over the other train. If the faster train is 225 m in length, find the length of the other train.

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	a) 540 km/hr b) 20 m/sec
Q2.	6 mins.
Q3.	25 m/sec
Q4.	72 km/hr or 20 m/sec
Q5.	500 meters
Q6.	108 km/hr
Q7.	40 km/hr
Q8.	54 km/hr
Q9.	5 kms
Q10.	150 m

Topic 52

Time, Speed & Distance - 2

Average Speed:

In the previous lecture, we learnt the co-relation between Speed, Time and Distance. However, here we assumed that people or objects always move at the same pace all the time.

In reality, however, things and people mostly do not have uniform speeds. They pick up speed and slow down during the course of their travel. So, we calculate their average speed by dividing the total distance covered by a body by the total time it takes to cover the distance.

$\therefore$ Average speed $= \dfrac{\text{Total Distance}}{\text{Total Time}}$

e.g. A scooterist is going to a place 240 kms away. He travels 120 kms at the average speed of 60 kms/hr and the remaining 120 kms at the average speed of 40kms/hr. His average speed for the entire journey is

Part 1: 120kms at 60 km/hr. Travel time = 2 hours.

Part 2: 120kms at 40 km/hr. Travel time = 3 hours.

Total distance = 240kms. and Total time = 5 hours.

$\therefore$ Average Speed $= \dfrac{\text{Total distance}}{\text{Total time}} = 240/5 = 48$kms/hr.

[Note: For calculating Average Speed, we must NOT calculate the average of the speeds given]

Relative Speed:

The relative speed of two moving bodies is the rate at which the distance between them increases or decreases.

Important Points to remember:

1. For two bodies moving in the same direction, relative speed = difference of their speeds.
2. For two bodies moving in opposite directions, relative speed = sum of their speeds.
3. Time taken for one moving train to pass by another $= \dfrac{\text{sum of the lengthsof the trains}}{\text{relative speed}}$.
4. If the speed of a boat in still water be x km/h and the speed of a stream be y km/h,

i. When a boat floats along the current

Relative Speed of the boat = Actual speed of boat (x) + Speed of the current (y)

ii. When a boat floats against the current

Relative Speed of the boat = Actual Speed of boat (x) − Speed of the current (y)

Practice Work :

Q1. The road distance between Pune and Delhi is 2850 km. A train left Pune on Sunday at 5.00 a.m When will it reach Delhi at the average speed of 30 km/hr ?

Q2. A man travels on a car from x to y at a speed of 77 km/hr and returns at 33 km/hr from y to x. Find the average speed of the journey.

Q3. A car completes a certain journey in 8 hours. It covers half the distance at 40 km/hr and the rest at 60 km/hr. The length of the journey in km. is

Q4. Speed of the boat is 6kmph and that of the current is 2kmph. The boat travelled 32 kms with the current and returned to its starting point. Find average speed of the boat in the total journey.

Q5. A woman walks a certain distance in 84 min. She covers two thirds of it at 4 km/hr and the rest at 5 km/hr. Find the total distance she covered.

Q6. In a river from point A to B, a boat takes 4 hours to travel. Same boat takes 6 hours to come back from B to A, assuming speed of boat is constant. If a wooden block is put in the same river at A, how much time in hrs. it will take to reach B?

Q7. Virat runs twice as fast as he walks. He travels from his house to school by walking some distance and by running some distance. On Monday his walking time is twice his running time and reaches the school in 30 minutes. On Tuesday his running time is twice his walking time. Find the time in minutes he takes to reach the school on Tuesday.

Q8. A car travels from point A to point B. Car travels first half the distance at a constant speed of 30 km/hr. What should be the speed of the car in second half of the journey so that total distance AB is travelled at an average speed of 40 km/hr.

Q9. The speed of a boat in still water is 9 km/hr and the speed of a stream is 3 km/hr. Find the time taken by the boat to cover (i) 24 km upstream (ii) 42 km downstream.

Q10. A boat can travel at 9 km/hr upstream and at 12 km/hr downstream. Find the speed of the boat in still water and the speed of the stream.

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	Thursday 4 am
Q2.	46.2 km/hr
Q3.	384 kms
Q4.	5.33 kms/hr
Q5.	6 kms
Q6.	24 hours
Q7.	24 mins
Q8.	60 km/hr
Q9.	i) 4 hours; ii) 3 1/2 hours.
Q10.	speed of boat is still water=10.5 km/hr ; speed of the stream= 1.5 km/hr

Topic 53

Time and Work - 1

We should recollect the fundamentals of variation (direct and inverse) here.

1. **When the number of days is constant, work and men are directly proportional to each other,** i.e., if the work to be done increases, more number of men are required to complete the work in the same number of days.

2. **When the number of men is constant, work and days are directly proportional,** i.e., if the work increases, then more days are required, provided the work is to be completed by the same number of men.

3. **When the work is constant, number of men and days are inversely proportional,** i.e., if the number of men increases, less days are required to complete the same work and vice-versa.

The concept of man-days is very important and useful here. The number of men multiplied by the number of days.required to complete the work will give the number of man-days. Here work is measured in terms of man-days. The total number of man-days representing a specific task will remain a constant. So, if we change one of the variables, i.e., men or days, then the other will change accordingly, so that their product will remain constant (remember from our knowledge of variation, two variables whose product is a constant are said to he inversely proportional to each other). The two variables 'men' and 'days' are inversely proportional to each other.

Scenario 1: More people are deployed to do the same work OR If the same work needs to be done in lesser number of days.

Different Methods to solve these problems.

If 12 men can plough a field in 6 days, how many men will be required to plough the field in 4 days?

Unitary method

6 days are required to have the field ploughed by 12 men.

$\therefore$ 1 day is required to have the field ploughed by 12×6 men.

$\therefore$ 4 days are required to have the field ploughed by $\dfrac{12 \times 6}{4}$ men $= 18$ men.

Thus. 18 men will be required to plough the field in 4 days.

Proportion method

Clearly, this is a case of inverse variation because less men will require more days and more men will require less days to plough the field. From the problem, 12 men are needed to plough the field in 6 days. Let x men be needed to plough the field in 4 days.

Then, (ratio of 6 to 4) = (inverse ratio of 12 to x) = (ratio of x to 12).

$\therefore \quad \dfrac{6}{4} = \dfrac{x}{12} \quad$ or $\quad 4x = 6 \times 12 \quad$ or $\quad x = \dfrac{6 \times 12}{4} = 18.$

Thus, 18 men will be required to plough the field in 4 days.

Person-days method:

12 men can plough the field in 6 days, so it is 12*6 = 72 person-days work.

If the work is to be completed in 4 days, then 72 / 4 = 18 men will be required.

Note: The quantum (or volume) of the work is remains constant:
72 person-days (12*6) or (18*4).

Scenario 2: Two people working together at different speed.

Different Methods to solve these problems.

Unitary Method:

As we all know that the amount of work done by a person varies directly with the time taken by him (her) to complete it. Thus, if a man can complete a piece of work in 10 days, then by unitary method, we can say that in one day he will do only $\frac{1}{10}$ th part of the total work. On the other hand, if a man completes $\frac{1}{10}$ th of the work in one day, then he will take 10 days to complete the work. Thus, we obtain the following rules:

Rule 1: If a person X completes a piece of work in n days, then work done by person X in

one day is $\left(\frac{1}{n}\right)^{th}$ part of the work.

Rule 2: If a person X completes $\left(\frac{1}{n}\right)^{th}$ part of the work in one day, then person X will take n days to complete the work.

Example

Vijay completes a certain work in 15 days. Ajay takes 10 days to complete the same work. In how may days will they complete the same work, if both of them work together.

Explanation : In such problems first find one day's work of each of them, then add their one day's work. Then find the answer.

Vijay's one day's work $= \frac{1}{15}$

Ajay's one day's work $= \frac{1}{10}$

Vijay's + Ajay's one day's work $= \frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}$

Total time taken by both of them working together = 6 days

Alternative method:

A can do a piece of work in 10 days & B can finish the same work in 15 days. If they work together, in how many days will they complete the work?

As. $\frac{\text{Total work}}{\text{speed}} = \text{Time required.}$

Let speed of A be a and B be b.

If total work is W, then.

$10 = \frac{W}{a}$ and $15 = \frac{W}{b}$

$\therefore \quad a = \frac{W}{10}$ and b $= \frac{W}{15}$

Speed of $(a + b) = \frac{W}{10} + \frac{W}{15}$

$\therefore$ Time required $= \frac{W}{\frac{W}{10} + \frac{W}{15}}$

$= \frac{W(150)}{W(10 + 15)}$

$= \frac{150}{25} = 6$ days

<u>Scenario 3</u>: Three people working together at different speed.

A, B, C can complete a job working individually in 10, 20 and 60 days respectively. If they work together how many days will they take?

Let W be the work.

Speed of A = a $= \frac{W}{10}$

B = b $= \frac{W}{20}$

C = c $= \frac{W}{60}$

$\therefore$ Speed of A, B, C together $= \frac{W}{10} + \frac{W}{20} + \frac{W}{60}$

$\therefore$ Time required $= \frac{W}{\frac{W}{10} + \frac{W}{20} + \frac{W}{60}}$

$= \frac{W(60)}{6W + 3W + W} = \frac{60W}{10W}$

=6 days.

Practice Work :

Q1. 10 workers can build a wall in 8 days. How long would it take for 16 workers to build the same wall ?

Q2. 3 Girls can do a piece of work in 2 days. How long will a girl take to do it?

Q3. Rakesh can do a piece of work in 20 days. How much work can he do in 4 days?

Q4. 6 men can complete the electric fitting in a building in 7 days. How many days will it take if 21 men do the job?

Q5. Reema weaves 35 baskets in 25 days. In how many days will she weave 55 baskets?

Q6. If 12 boys earn Rs 840 in 7 days, what will 15 boys earn in 6 days?

Q7. A and B can do a piece of work in 72 days, B and C can do it in 120 days and A and C can do it in 90 days. A alone can do it in_____

Q8. A piece of work can be done by 6 men and 5 women in 6 days or 3 men and 4 women in 10 days. It can be done by 9 men and 15 women in how many days?

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	5 days
Q2.	6 days
Q3.	20% or $1/5th$
Q4.	2 days
Q5.	39 2/7 days
Q6.	Rs. 900
Q7.	120 days
Q8.	3 days

Topic 54

Time and Work - 2

Scenario 4 : Increasing efficiency.

The more efficient a person is, the lesser time he will take to complete a given work.

e.g. A person can do certain task in 10 days. He was then sent for a training program, which increased his efficiency by 25%. Now, he is able to do the same task in_____ days.

Work done in 1 day $= 1/10^{th}$ of the task $= 0.1$

With 25% increase in efficiency,

Now work done in 1 day $= 1/10^{th} \times 1.25 = 0.1 * 1.25 = 0.125$ (or $1/8^{th}$).

Therefore, he is now able to do the same task in 8 days.

Scenario 5 : People joining and leaving the work

A can do a piece of work in 25 days and B can finish it in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the remaining work?

We have,

A can finish the work in 25 days, B can finish the work in 20 days.

$\therefore$ A's 1 day's work $= \frac{1}{25}$, B 's 1 day's work $= \frac{1}{20}$.

So, $(A+B)$'s 1 day's work $= \frac{1}{25} + \frac{1}{20} = \frac{4+5}{100} = \frac{9}{100}$

$\therefore$ $(A+B)$'s 5 day's work $= 5 \times \frac{9}{100} = \frac{9}{20}$

Remaining work $= 1 - \frac{9}{20} = \frac{11}{20}$

The remaining work is done by B.

$\therefore$ Complete work is done by B in 20 days.

$\therefore$ $\frac{11}{20}$ of the work is done by B in $\left(20 \times \frac{11}{20}\right)$ days $= 11$ days.

Hence, the remaining work is done by B in 11 days.

Scenario 6 : Pipes and Water Tank

As you know that a water tank is always connected with two types of pipes. One which fills it up and the other which empties it out. The pipe which fills up the water tank is called an inlet and the one which empties it is called an outlet.

Let an inlet fill up a water tank in 8 hours. Then-in one hour it fills up $\frac{1}{8}$ th part of it. We can also say that the work done by inlet in 1 hour is $\frac{1}{8}$. Similarly, if an outlet empties out a water tank in 6 hours, then in one hour it empties out $\frac{1}{6}$ th part of the water tank

i.e. the work done by the outlet in one hour is $\frac{1}{6}$.

NOTE: The work done by the inlet is always positive whereas the work done by the outlet is always negative.

Practice Work :

Q1. A person is able to do a task in 46 days. After a training program he is 15% more efficient. In how many days can he now do the same task ?

Q2. A can do a piece of work in 40 days and B in 45 days. They work together for 10 days and then B goes away. In how many days will A finish the remaining work?

Q3. A and B can individully do a piece of work in 20 and 15 days respectivly. They work together for 2 days and then A goes away. In how many days will B finish the remaining work?

Q4. 'A', 'B' and ' C ' together complete a work in 15 days.' B ' takes 60 days to complete the work, ' C ' takes 40 days to complete the work. How many days will ' A ' require to complete the same work?

Q5. A and B can do a piece of work in 30 days. While B and C can do the same work in 24 days. C and A in 20 days. They all work for 10 days when B and C leave. How may days will A take to finish the work?

Q6. A water tank can be filled by a tap in 6 hours and emptied by an outlet pipe in 8 hours. How long will it take to fill the water tank if both the tap and the pipe are opened together?

Q7. Tap A can fill a tank in 4 hours while tap B can empty it in 6 hours. If both taps are opened together at the same time when the tank is empty, then find the number of hours required to fill the tank completely.

Q8. Pipe A can fill an empty tank in 6 hours and pipe B in 8 hours: If both the pipes are opened and after 2 hours pipe A is closed, how much time B will take to fill the remaining tank?

Q9. A tank can be filled by two taps A and B in 12 hours and 16 hours respectively. The full tank can be emptied by a third tap in 8 hours. If all the taps be turned on at the same time, in how much time will the empty tank be filled up completely?

Q10. Tap A can fill a tank in 6 hours and tap B can fill the same in 8 hours. If tap A, B and C can fill the tank in 3 hours, how long will tap C alone take to fill the tank?

Answer Key:

Practice Work:

Q. NO	**Answer**
Q1.	40 days
Q2.	21 1/9 days
Q3.	23/2 days
Q4.	40 days
Q5.	18 days
Q6.	24 hours
Q7.	12 hours
Q8.	3 and 1/3 hours or 3 hours and 20 mins.
Q9.	48 hours
Q10.	24 hours