M Prakash Institute

VIII/IX std Selection Test Preparation Material

Topic 1

Basic Properties of Numbers :

Types of Numbers:

• **Natural Numbers:** Common counting numbers. Eg: $1, 2, 3, \ldots$.

• **Whole Numbers:** The set of Natural Numbers with the number 0 adjoined.

Eg : $0, 1, 2, 3, \ldots$

• **Integers:** Whole Numbers with their opposites

(negative numbers) adjoined. Eg: ...., $-3, -2, -1, 0, 1, 2, 3 \ldots$.

Prime and Composite numbers

• **Prime Number:** A natural number greater than 1 which has only 1 and itself as factors. Eg: 2,3,5, 7, ...

• **Composite Number:** A natural number greater than 1 which has more than two factors.

Even and Odd numbers

• Even numbers: Divisible by 2 without remainders; end in $0, 2, 4, 6$, or 8 .

• Odd numbers: Not evenly divisible by 2 ; end in $1, 3, 5, 7$, or 9 .

• Co prime numbers: Two numbers are co-prime if they have no common factor other than 1.

Number line:

- A **number line** is a visual representation of numbers on a straight line.

- Every point of a number line is assumed to correspond to a real number, and every real number to a point.

- The number associated with the point is called its 'co-ordinate'

- The value of a number increases from left to right.

- Distance between any two points on the number line is the positive difference in their co-ordinates

Rational numbers: A rational number is a number which can be expressed as a ratio of p/q where q is not equal to 0. The numerator and denominator of a rational number will be integers.

Eg. $\left(\frac{N}{D}\right.$ form). Eg: : $-\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, 0, +\frac{1}{4}, +\frac{1}{3}, +\frac{1}{2}, \ldots\ldots$

Properties of Rational Numbers:

Let a, b, c be any rational numbers, then $a = \frac{p}{q}, b = \frac{r}{s}, c = \frac{l}{m}$

where $p, q, r, s, l,$ m $\in I, q \neq 0, s \neq 0, m \neq 0$.

• $a + b = b + a$ (addition is commutative)

• $a \times b = b \times a$ (multiplication is commutative)

• $(a + b) + c = a + (b + c)$ (addition is associative)

• $(a \times b) \times c = a \times (b \times c)$ (multiplication is associative)

• $a + 0 = a = 0 + a$ (0 is additive identity)

• $a \times 1 = a = 1 \times a$ (1 is multiplicative identity)

• $a + (-a) = 0 = (-a) + a$ ($-a$ is additive inverse of a)

• $a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$ ($\frac{1}{a}$ is multiplicative Inverse of $a(a \neq 0)$)

• $a \times (b + c) = a \times b + a \times c$(left) and $(b + c) \times a = b \times \mathrm{a} + c \times a$ (right)
(distributive over addition).

• All integers are rational numbers as $3 = \frac{3}{1}$. 0 is also rational number as $0 = \frac{0}{1}$.

• If a and b are two rational numbers where $a < b$ then the rational number $\frac{a+b}{2}$ always lies between a and $b, a < \frac{a+b}{2} < b$

• **Opposites Numbers:**
The same number with opposite signs are called 'Opposite numbers'.
They are equidistant from '0' on the number line.
Their sum is always '0' Eg : $(+7 - 7) = 0$)
Symbol used: '-'. Answer is called **'Difference'**

Basic Operations on Numbers:

Addition: Sum or total of given numbers. Eg: $2 + 2 = 4$
Symbol: ' + '. Answer is called 'Sum'

Subtraction : Difference in two numbers. Eg: $3 - 2 = 1$

Multiplication: Repeated addition.
Eg: $2 + 2 + 2$ (3 times) $= 2 \times 3 = 6$
Symbol used : 'x '. Answer is called **'Product'**
Other terms : The number that multiplies : **'Multiplier'**
The number that gets multiplied : **'Multiplicand'**

Division: Repeated subtraction.
Eg: $12 - 3 - 3 - 3 - 3$ (4times) $= 0$ So, $12 \div 3 = 4$
Symbol used: '$\div$'. Answer is called : **'Quotient'**
Other terms : The number that divides : **'Divisor'**
The number that gets divided : **'Dividend'**
The integer "left over" after division : **'Remainder'**
Dividend = (Divisor $\times$ Quotient) + Remainder
When the divisor completely divides the dividend, remainder is '0'.

What is BODMAS Rule?

BODMAS stands for Bracket, Of Division, Multiplication, Addition and Subtraction.

The BODMAS is used to explain the order of operation of a mathematical expression. In some regions, the BODMAS is also known as PEDMAS which stands for Parentheses, Exponents, Division, Multiplication, Addition, and Subtraction.

According to BODMAS rule, the brackets have to be solved first followed by powers or roots (i.e. of), then Division, Multiplication, Addition and at the end Subtraction. Solving any expression is considered correct only if the BODMAS rule or the PEDMAS rule is followed to solve it.

Vinculum: A straight horizontal mark placed over two or more members of a compound mathematical expression and equivalent to parentheses or brackets around them.

Practice Work:

1. $16 \times 5 \div [\{12 \div (4-2)\} - 2]$

2. $18 - [8 \div 2\{8 \div \overline{4 \times 2}\}]$

3. $8 \times 3 + [63 \div \{18 \div 3(9 - 17 + 5 \times 2)\}]$

4. $78 - [5 + 3 \text{ of } (25 - 2 \times 10)]$

5. A and B are two distinct numbers on a number line. If the number associated with the point A is as given below and the distance between them is given, find the number/s associated with the point B.

(a) $A = -4$ Distance $A \to B = 5$ (b) $A = 8$ Distance $A \to B = 3$

(c) $A = 1$ Distance $A \to B = 6$ (d) $A = 0$ Distance $A \to B = 7$

(e) $A = -3$ Distance $A \to B = 9$

6. What must be subtracted from -1 to get -19 ?

7. Find the smallest integer 'n' such that $5 \times 12 \times n$ is the product of 3 consecutive integers.

8. A hall 10 ft. by 4 ft. is to be paved with square tiles. What is the least number of tiles required to cover the floor?

9. 'K' is an even number. What is the 3 rd odd number before 'K' in serial order?

10. The cost of an eraser and sharpener is Rs. 4.50. If the cost of sharpener is Rs. 1.00 more than that of eraser, find the cost of the eraser.

Answer Key

Practice Work :

Que no.	Answer
1	20
2	14
3	45
4	58
5	(a) 1, -9 (b) 11, 5 (c) 7, -5 (d) 7, -7 (e) 6, -12
6	18
7	1
8	10
9	K-5
10	1.75

Topic 2

Basic Properties of Numbers

Irrational numbers

The square root symbol or square root sign is a mathematical symbol, denoted by '$\sqrt{\ }$'. This symbol is known as radical, in words.

The number present under the root is called the radicand.

Radicand: The number you are finding root of.

Radical sign: The $\sqrt{}$ symbol that means "root of".

Degree: The number of times the radicand is multiplied by itself. 2 means square root, 3 means cube root. After that they are called the 4th root, 5th root and so on. If this is missing, it is assumed to be 2 - the square root.

Irrational Numbers: All numbers which cannot be written as fractions.
Eg: $-\sqrt{3}, -\sqrt{2}, +\sqrt{2}, +\sqrt{3} \cdots ..$
Irrational numbers consist of non-terminating and non-recurring decimals. Irrational numbers are also known as surds.

Real Numbers: The set of Rational Numbers with the set of Irrational Numbers adjoined.

Computation of Numbers

• **'From' and 'Between' numbers:**

'From' numbers include the tail end numbers of the given set

Eg: There are 4 numbers 'from' 2 to 5

How to find? (For consecutive numbers)

Number of numbers = (Last number -1st number) +1

'Between' numbers exclude the tail end numbers of the given set
Eg: There are 2 numbers 'between' 2 and 5

How to find? (For consecutive numbers)

Number of numbers = (Last number -1st number) -1

Practice Work:

1. State true or false:

(a) Every natural number is a whole number but every whole number is not a natural number.
(b) Every natural number is an integer but every integer is not a natural number.
(c) All rational and irrational numbers are real numbers.
(d) On a real number line, numbers decreases towards the right and increases towards the left.
(e) Every rational number is a whole number but every whole number is not a rational number.

2. Choose the correct word from the words given in underlined to make the sentence true.

(a) A number that cannot be expressed as a fraction is a rational / irrational number.

(b) A surd is an irrational number that uses a root/square symbol.

(c) The decimal representation of a surd is a terminating/recurring/non recurring decimal.

(d) $\vee 25$ is a surd/rational number.

3. In a bundle of notes of Rs.50 denominations, there are notes numbered from 17401 to 17485 arranged serially. What is the total amount in the bundle?

4. On real number line, distance between point A and point B is 6 units. The bigger number is 3/5 th of the smaller number. Find the smaller integer.

5. When certain number is divided by 13, quotient is 22 and remainder is 12. If the same number is divided by 22, what will be the remainder?

6. Sum of 5 consecutive odd integers is 15. Find the largest integer.

7. By using the digits 2,3 and 5, how many three- digit numbers greater than 500 can be formed, such that one digit occurs only once in each number?

8. What is the sum of all even numbers between 75 and 150.

9. a, b, c, d are four distinct prime numbers. $ab = 65, bc = 221, abcd = 47515$, find $a + b + c + d =?$

10. How many numbers above 3000 can be formed using the digits $3, 1, 5, 0$, such that each digit appears only once in each number?

Answer Key

Practice Work :

Que no.	Answer
1	(a) True (b) True (c) True (d) False (e) False
2	(a) Irrational (b) Root (c) Non recurring (d) Rational
3	4250
4	-15
5	12
6	7
7	2
8	4144
9	78
10	12

Topic 3

Divisibility, HCF, LCM

What is divisibility?

When we say a number is divisible by another number, we mean that a number is getting divided equally and the remainder is always zero. For example, 261 is divisible by 9, since $261 \div 9 = 29$.

Some divisibility tests can be done by looking at the ones digit.
A number is:

- **divisible by 2** - If the ones digit is even $(2, 4, 6, 8, 0)$
- **divisible by 5** - If the ones digit is 5 or 0
- **divisible by 10** - If the ones digit is 0

Some divisibility tests can be done by adding all the digits together.
A number is:

- **divisible by 3** - If the sum of the digits is divisible by 3
- **divisible by 9** - If the sum of the digits is divisible by 9

ex. 87 is divisible by 3 because $8 + 7 = 15$ and $1 + 5 = 6$ and 6 is divisible by 3

ex. 261 is divisible by 9 because $2 + 6 + 1 = 9$ and
9 is divisible by 9

Some less common divisibility tests:
A number is:

- **divisible by 4** - If the last two digits are divisible by 4
- **divisible by 8** - If the last three digits are divisible by 8
- **divisible by 6** - If it follows the rules for 2 and 3
- **divisible by 12** - If it follows the rules for 3 and 4

ex. 432 is divisible by 4, 6 & 8. It is also divisible by 12, since it is divisible by 3 & 4.

- **divisible by 11** - If sum of the digits in odd places is equal to sum of the digits in the even places or if their differnce is divisible by 11 then the given number is divisible by 11.

ex. 1,012 is divisible by 11 because $+1 - 0 + 1 - 2 = 0$ and 0 is divisible by 11 ex. 616 is divisible by 11 because $+6 - 1 + 6 = 11$ and 11 is divisible by 11

GCD and LCM

Greatest of the common factors of two numbers is known as **Greatest Common Divisor (G.C.D.)** or **Highest Common Factor (H.C.F.)**

e.g. Let us consider all the divisors of 56 and 24.
Divisors of 56 : $1, 2, 4, 7, 8, 14, 28, 56$.
Divisors of 24 : $1, 2, 3, 4, 6, 8, 12, 24$.
Common divisors : $1, 2, 4, \underline{8}$.
$\therefore$ G.C.D. of 56 and 24 = 8

Least of the common Multiples of two numbers is known as **Least Common Multiple (L.C.M.)**

e.g. Let us consider the multiples of 15 and 20
Multiples of 20 : $20, 40, \underline{60}, 80, 100, \underline{120}, 140, 160, 180, 200, \ldots$
Common multiples : $60, 120, 180, \ldots$
$\therefore$ L.C.M. of 15 and 20 = 60.

e.g. Find the L.C.M and G.C.D. of 108 and 180
$108 = \underline{2} \times \underline{2} \times \underline{3} \times \underline{3} \times 3$
$180 = \underline{2} \times \underline{2} \times \underline{3} \times \underline{3} \times 5$
L.C.M. $= \underline{2} \times \underline{2} \times \underline{3} \times \underline{3} \times 3 \times 5 = 540$
L.C.M. = product of common factors $\times$ product of uncommon factors
G.C.D. = product of common factors $= \underline{2} \times \underline{2} \times \underline{3} \times \underline{3} = 72$

The G.C.D. of two consecutive even numbers is 2 and the L.C.M of them is half of their product.

e.g. 1) G.C.D. of 10 and 12 = 2

2) L.C.M. of 10 and 12 $= \frac{1}{2}(10 \times 12) = 60$

The G.C.D. of two consecutive odd numbers is 1. and the L.C.M. of them is equal to their product.

L.C.M. = product of common factors $\times$ product of uncommon factors.
G.C.D. = product of common factors.

L.C.M. = G.C.D. $\times$ product of uncommon factors.

i.e. **Product of uncommon factors** $= \frac{\text{L.C.M.}}{\text{G.C.D.}}$

Methods of finding G.C.D.

1) Divisor method :

e.g. Find G.C.D. of 12 and 15.
Divisors of 12 : $1, 2, 3, 4, 6, 12$
Divisors of 15 : $1, 3, 5, 15$
Common divisors $= 1, 3$
$\therefore$ Greatest Common Divisor (G,C.D.) = 3

2) Prime factorization method :

e.g. Find G.C.D. of 12 and 15
$12 = 2 \times 2 \times \underline{3}$
$15 = \underline{3} \times 5$
Product of common factors = 3
$\therefore$ G.C.D. =3

3) Euclidean method : (Division algorithm)

e.g. Find G.C.D. of 12 and 15.

$$\begin{array}{r} 1 \\ 12\overline{)15} \\ -\ \underline{12} \\ 3 \end{array} \qquad \begin{array}{r} 4 \\ 3\overline{)12} \\ -\ \underline{12} \\ 00 \end{array}$$

Here, last divisor 3 exactly devides 12. Hence G.C.D = 3

$\therefore$ G.C.D. of 15 and 12 is 3.

Methods of finding L.C.M.

1) Prime factorization method 2) Method by writing multiples.

1) Prime factorization method : Horizontal arrangement

Method 1; e.g. Find L.C.M. of 12,15 and 18.

Solution :

$12 = 2 \times 2 \times \underline{3}$

$15 = \underline{3} \times 5$

$18 = 2 \times \underline{3} \times 3$

L.C.M. $= \underline{3} \times 2 \times 2 \times 5 \times 3 = 180 \left\{ \begin{array}{c} \text{Product of common and} \\ \text{uncommon factors.} \end{array} \right\}$

Prime factorization method : Vertical arrangement

Method 2 : e.g. Find L.C.M. of 12,15 and 18.

3	12	15	18
2	4	5	6
2	2	5	3
3	1	5	3
5	1	5	1
	1	1	1

$\therefore$ L.C.M. $= 3 \times 2 \times 2 \times 3 \times 5 = 180$

2) Method by writing multiples :

e.g. Find L.C.M. of $2, 3, 4$

Solution : Multiples of 2 : $2, 4, 6, 8, 10, \boxed{12}, 14, 16, 18, 20, 22, \boxed{24}, 26, 28, 30,$

. $32, 34, \boxed{36}, 38, 40, 42, 44, \ldots$

Multiples of 3 : $3, 6, 9, \boxed{12}, 15, 18, 21, \boxed{24}, 27, 30, 33, \boxed{36}, 39, \ldots$

Multiples of 4 : $4, 8, \boxed{12}, 16, 20, \boxed{24}, 28, 32, \boxed{36}, 40, \ldots$

Common multiples $= \underline{12}, 24, 36, \ldots$

$\therefore$ L.C.M. $= 12$

Relation between GCD and LCM

Product (a, b)**=GCD** $(a, b)\times$ **LCM** (a, b)

We can also write the above formula in terms of GCD and LCM.

GCD (a, b) = Product (a, b)/ LCM (a, b)

LCM (a, b) = Product (a, b)/GCD(a, b)

LCM (a, b)/ **GCD** (a, b) = **Product of uncommon factors of** a **and** b

Practice Work:

1. Check whether:
(a) 7704 is divisible by 3
(b) 76736 is divisible by 9
(c) 171853 is divisible by 11

2. How many 3 digit natural numbers are divisible by 6?

3. In a bag full of marbles, each marble has a number from 1 to 145. Check how many numbers are divisible by 3,4 and 12.

4. An 8-digit number $3681m42n$ is divisible by 72. Find the value of $m + n$.

5. n is a natural number divisible by 3. $n^2 = 5k9824$. Find the value of k.

6. Find GCD and LCM of the following sets
(a) 12,26 (b) 13,14 (c) 15,25 (d) 36,144 (e) 21,35

7. Find the LCM and HCF of the following pairs of integers and verify that
LCM $\times$ HCF= product of the two numbers.
(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

8. There is a circular path around a sports field. Sonia takes 18 minutes to run one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

9. The numbers 72 and $1 * 6$ have HCF as 18. Find the digit in place of the *

10. The LCM of two numbers is 320 , and the product of the two numbers is 10240. Find the larger number of the two ....

Answer Key

Practice Work :

Que no.	**Answer**
1	(a) Yes (b) No (c) Yes
2	150
3	48, 36, 12
4	12
5	8
6	(a) 2, 156 (b) 1, 182 (c) 5, 75 (d) 36, 144 (e) 7, 105
7	(a) 13, 182 (b) 2, 23460 (c) 6, 3024
8	36
9	2
10	320 or 160

Topic 4

Fractions

What is a Fraction?

• The meaning of the word 'fraction' is a part of the whole.
The whole may be a single object or a group of objects. All the parts must be equal.

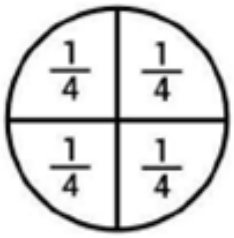
• In Maths, a fraction is written in the form: $\frac{\text{Numerator}}{\text{Denominator}} = \frac{N}{D}$

• The 'Denominator' will tell how many parts of a whole are made. The 'Numerator' will tell how many parts of these are considered.

For e.g. $\frac{1}{3}$ means 3 parts of the whole are made & one of it is considered.

• When the N & D are same the value of a fraction is one. It is a whole.

For e.g. $\frac{4}{4}$ means 4 parts out of 4 so $\frac{4}{4} = 1$ (whole)



The '0 ' - '1' game
- When the 'D' of the fraction = 1, it is a whole number.
- When the 'N' of the fraction is '0', it is = 0
- It is not allowed to take 'D' as '0'
CAN YOU DECODE THESE CONVENTIONS USING THE DEFINITION OF 'N' AND 'D'?

TYPES OF FRACTIONS :

PROPER - IMPROPER FRACTION

• When $N < D$, it is called a 'Proper fraction'. It is always less than one.

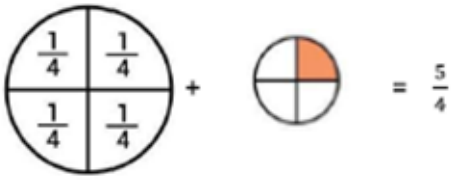
• When $N > D$ it is called an 'Improper fraction'. It is always greater than 1.

MIXED FRACTION

• Improper fraction written in the form of a whole number and a proper fraction it is

called a 'Mixed Fraction'.

For e.g. $\frac{1}{4}$: proper fraction; $\frac{5}{4}$: improper fraction; $1\frac{1}{4}$: mixed fraction.



INTER - CONVERSION OF MIXED & IMPROPER FRACTIONS.

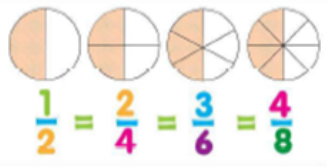
The same number can be expressed as improper fraction or mixed fraction.

For e.g. $1\frac{1}{4} = \frac{(4\times1)+1}{4} = \frac{5}{4}$

EQUIVALENT FRACTIONS

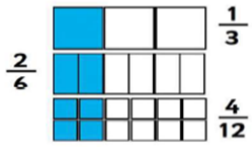
• Fractions that are equal in values are called Equivalent Fraction.

For e.g. $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$, etc. all are equivalent to half.



• When any fraction is multiplied or divided by the same number in the N & the D both. We get an equivalent fraction.

For e.g. $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$ $\quad \frac{4}{12} \div \frac{2}{2} = \frac{2}{6}$. So, $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$ are all equivalent fractions.



LOWEST FORM OF A FRACTION:

• When a given fraction cannot be further reduced to smaller equivalent fraction, we get the lowest form of the fraction.

For e.g. $\frac{4}{8} \div \frac{2}{2} = \frac{2}{4}$; $\quad \frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$

$\frac{1}{2}$ cannot be further reduced. So it is the lowest form of all these fractions

LIKE FRACTIONS & UNLIKE FRACTIONS:

Fractions with same denominators are called **like fractions.**

Thus $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{8}{15}$ are all like fractions.

Fractions with different denominators are called **Unlike fractions.**

Thus $\frac{7}{13}, \frac{4}{15}, \frac{31}{23}, \frac{8}{25}$ are unlike fractions.

SIGN OF A FRACTION:

• When both the 'N' and 'D' of a fraction are positive or negative, the fraction is said to be positive.

For eg: Both $\frac{2}{3}$ and $\frac{-2}{-3}$ are positive and have the same value

• When any one of the 'N' and 'D' of the fraction is negative, the fraction is said to be negative.

For eg: Both $\frac{2}{-3} = \frac{-2}{3}$ are negative and have the same value.

OPERATIONS ON FRACTIONS

Addition & subtraction of fractions:

• **For fractions with same Ds :** Add /Subtract their Ns and keep the D as it is

For e.g. $\frac{3}{9} + \frac{1}{9} = \frac{4}{9}$ and $\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

• **For fractions with different Ds:**

N of the answer: Sum /Difference of cross multiplications of Ns $\times$ Ds

D of the answer: Product of Ds of the two fractions.

For e.g. $\frac{2}{3} + \frac{1}{2} = \frac{4+3}{6} = \frac{7}{6} = 1\frac{1}{6}$ and $\frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$

Is this the same as finding equivalent fractions with same 'D' and add?

• Sometimes the denominators are multiples / factors of one another or some common number. In such cases, we find the LCM of these denominators for simplifying the calculations.

e.g., $\frac{7}{12} + \frac{5}{4} = \frac{7+15}{12} = \frac{22}{12} = \frac{11}{6}$; $\frac{3}{10} - \frac{2}{15} = \frac{9-4}{30} = \frac{5}{30} = \frac{1}{6}$

Multiplication of fractions:

• Multiply the Ns. of the two fractions to get the numerator of the product and the Ds. to get the Denominator of the product.

For e.g. $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

• Multiplication of a Fraction by a Whole Number

To multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same. To multiply a mixed fraction to a whole number, first convert the mixed fraction to an improper fraction and then multiply.

$3 \times \frac{1}{2} = \frac{3\times1}{2} = \frac{3}{2}$ $\quad\quad$ $\frac{2}{3} \times 5 = \frac{2\times5}{3} = ?$

$2 \times \frac{5}{3} = \frac{2\times5}{3} = \frac{10}{3}$ $\quad\quad$ $3 \times \frac{8}{7} = ? 4 \times \frac{7}{5} = ?$

$3 \times 2\frac{5}{7} = 3 \times \frac{19}{7} = \frac{57}{7} = 8\frac{1}{7}$ $\quad\quad$ $2 \times 4\frac{2}{5} = 2 \times \frac{22}{5} = ?$

Division of fractions:

• Write the reciprocal of second fraction and multiply with the first fraction.

For e.g. $\frac{4}{3} \div \frac{2}{3} = \frac{4}{3} \times \frac{3}{2} = 2$

Meaning of division: $10 \div 5$

Divide '10' in '5' parts : Each part is $= 2$

OR

Divide '10' such that each part is '5'. We get 2 parts

$10 \div 5 = 10 \times \frac{1}{5} = 2$

Division by Fraction = Multiplication by its reciprocal

So, $\frac{4}{3} \div \frac{2}{3} = \frac{4}{3} \times \frac{3}{2} = 2$

CAN WE SEE THAT

When we divide $\frac{4}{3}$ such that each part $= \frac{2}{3}$, we get 2 parts

• **Note that the final answer is always to be written in the lowest form**

Practice Work:

1. Find:

(a) $\frac{3}{14} + \frac{6}{14}$ (b) $\frac{7}{16} + \frac{2}{16}$ (c) $\frac{4}{7} - \frac{1}{2}$ (d) $1 - \frac{5}{8}$

(e) $5 - 3\frac{1}{8}$ (f) $2\frac{4}{5} - 1\frac{1}{3}$ (g) $1\frac{1}{2} + 2\frac{1}{6}$ (h) $2\frac{1}{3} + 3\frac{3}{4}$

2. Calculate:

(a) $\frac{3}{4} \times \frac{5}{6}$ (b) $\frac{4}{7} \times \frac{1}{2}$ (c) $\frac{7}{12} \div \frac{3}{4}$ (d) $\frac{2}{3} \div 4$

(e) $2 \div \frac{3}{4}$ (f) $1\frac{1}{2} \div \frac{5}{8}$ (g) $\left(2\frac{1}{3}\right)^2$ (h) $\left(1\frac{1}{2}\right)^3$

3. In which of the following operations, the answer by using BODMAS and by not using BODMAS is same?

(a) $3 \times 4 + 5$ (b) $3 \times 4 \div 2$ (c) $5 - 3 \times 2$ (d) $3 + 9 \div 3$

4. Simplify the following

(a) $\left\{\frac{1}{2}+\frac{2}{3}+\frac{3}{4}\right\}-\frac{5}{12}=?$

(b) $\frac{5}{8}+\frac{9}{21}=?$

(c) $\left\{\frac{5}{7}\times\frac{28}{15}\times\frac{9}{8}\times 3\right\}\div\frac{18}{4}=?$

(d) $\left\{4+\frac{1}{4}\right\}\times\left\{1+\frac{1}{3}\right\}=?$

(e) $\frac{-28}{41}\times\frac{123}{-105}=?$

(f) $\frac{4\frac{1}{6}-2\frac{1}{3}}{7\frac{1}{8}+6\frac{3}{4}}$

5. If $\frac{3}{5}$ a number is 81 , find the number.

6. 4 fractions having equal denominators have their numerators as 2,4,5,1 respectively. If the sum of the fractions is 4, find the denominator.

7. Simplify $\frac{1}{24\times 50}+\frac{1}{50\times 26}-\frac{1}{26\times 24}=?$

8. What should be added to $5\frac{3}{7}$ to get 12?

9. Mihir eats $\frac{1}{4}$th of a cheesecake and later eats $\frac{2}{5}$th of the remaining cheesecake. What fraction remains?

10. Let $P=\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{2020}\right)$. Report sum of the digits of P.

Answer Key

Practice Work :

Que no.	Answer
1	(a) $\frac{9}{14}$ (b) $\frac{9}{16}$ (c) $\frac{1}{14}$ (d) $\frac{3}{8}$ (e) $\frac{15}{8}$ (f) $\frac{22}{15}$ (g) $\frac{11}{3}$ (h) $\frac{73}{12}$
2	(a) $\frac{5}{8}$ (b) $\frac{2}{7}$ (c) $\frac{7}{9}$ (d) $\frac{1}{6}$ (e) $\frac{8}{3}$ (f) $\frac{12}{5}$ (g) $\frac{49}{9}$ (h) $\frac{27}{8}$
3	(b)
4	(a) $\frac{3}{2}$ (b) $\frac{59}{56}$ (c) 1 (d) $\frac{17}{3}$ (e) $\frac{4}{5}$ (f) $\frac{44}{333}$
5	135
6	3
7	0
8	$\frac{46}{7}$
9	$\frac{9}{20}$
10	5

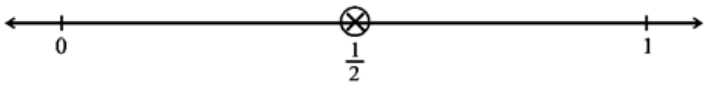
Topic 5

FRACTIONS

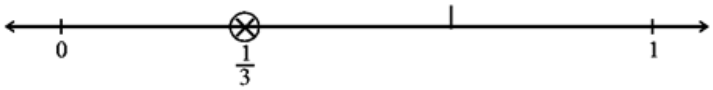
Fraction on the Number Line

• You have learnt to show whole numbers like $0, 1, 2 \ldots$ on a number line.

• We can also show fractions on a number line. Let us draw a number line and try to mark $\frac{1}{2}$ on it. We know that $\frac{1}{2}$ is greater than 0 and less than 1, so it should lie between 0 and 1. Since we have to show $\frac{1}{2}$, we divide the gap between 0 and 1 into two equal parts and show 1 part as $\frac{1}{2}$.



• Suppose we want to show $\frac{1}{3}$ on a number line. Into how many equal parts should the length between 0 and 1 be divided? We divide the length between 0 and 1 into 3 equal parts and show one part as $\frac{1}{3}$.



• Can we show $\frac{2}{3}$ on this same number line?

COMPARISION OF FRACTIONS :

Method 1: When N or D are equal:

• When the D of two fractions is same, greater the N greater is the fraction.

For e.g. $\frac{3}{4} > \frac{1}{4}$

• When the N of two fraction is same, smaller the D greater is the fraction.

For e.g. $\frac{1}{4} > \frac{1}{6}$

Method 2: When $N \neq D$

(i) Forming equivalent fractions and comparing the 'N's.

For e.g. Compare: $\frac{2}{3}, \frac{5}{7}$

$\frac{2}{3} \equiv \frac{14}{21}, \frac{5}{7} \equiv \frac{15}{21}; \frac{14}{21} < \frac{15}{21}$, So $\frac{2}{3} < \frac{5}{7}$

(ii) Cross multiplying $Ns \times Ds$. N with smaller product indicates smaller fraction.

For e.g. Compare: $\frac{2}{3}, \frac{5}{7}$

$2 \times 7(14) < 3 \times 5(15)$ So, $\frac{2}{3} < \frac{5}{7}$

Method 3: 'Less than half-Greater than half'

If one of the fractions in the set is less than half & the rest are more than half, the one that is less than half is the smallest one

Eg: ln $\frac{3}{2}, \frac{7}{5}, \frac{3}{7}, \frac{11}{7}$, only in $\frac{3}{7}$ $D > 2\ N$, hence the smallest fraction

Method 4: Constant difference in N and D

If the difference in Ns and Ds for all the fractions in a set are constant,

• For proper fractions- smaller pair indicates the smaller fraction

• For improper fractions, larger pair indicates the smaller fraction

For e.g. $\frac{3}{5} < \frac{13}{15}$ but $\frac{5}{3} > \frac{15}{13}$

Fraction to Decimals

• All fractions can be converted into decimals by dividing the numerator by the denominator.

Eg: $\frac{4}{5} = 0.8; \frac{3}{4} = 0.75$

Decimals to Fractions

• A decimal can be converted to a fraction by placing the decimal number over its place value. For example, in 0.6, the six is in the tenths place, so we place 6 over 10 to create the equivalent fraction, 6/10.

Eg: $1.5 = \frac{15}{10}; 0.02 = \frac{2}{100}$

Practice Work:

1. Find the second last fraction in the ascending order of the following fractions: $\frac{10}{14}, \frac{12}{28}, \frac{4}{7}, \frac{5}{14}, \frac{11}{14}$.

2. Which of the following fraction/s is greater than $\frac{33}{32}$?

(a) $\frac{3}{2}$ (b) $\frac{8}{11}$ (c) $\frac{15}{16}$ (d) $\frac{23}{22}$.

3. Out of the fractions $\frac{5}{7}, \frac{4}{9}, \frac{6}{11}, \frac{2}{5}$ and $\frac{3}{4}$, find the difference between the largest fraction and the smallest fraction.

4. On real number line distance between points with coordinates $\frac{13}{7}$ and $\frac{5}{3}$ is D_1 and distance between points with coordinates $\frac{97}{-7}$ and $\frac{-11}{21}$ is D_2. Find $\frac{D_2}{D_1}$.

5. Find the value of the following

(a) $\frac{2.223}{0.039} = ?$ (b) $3.75 \div 0.375 + 17.2 \div 1.72$

(c) $\dfrac{0.36 \times 0.27 \times 0.001}{0.06 \times 0.03 \times 0.1 \times 0.2}$ (d) $\dfrac{7.2 \times 7.2 \times 6.8 \times 3.2}{1.7 \times 3.6 \times .32 \times .6 \times .6}$

6. If $\dfrac{2}{39} = \frac{A}{195} = \frac{50}{B}$, then find the value of $A\&B$.

7. If $\dfrac{1}{4.2569} = 0.2349$, then find the value of $\dfrac{1}{0.0042569}$

8. The sum of three fractions is $\dfrac{59}{24}$. When the largest fraction is divided by the smallest, the fraction thus obtained is $\dfrac{7}{6}$ which is $\dfrac{1}{3}$ more than the middle one. Find the fractions?

(a) $\dfrac{3}{5}, \frac{4}{7}, \frac{2}{3}$ (b) $\dfrac{7}{8}, \frac{5}{6}, \frac{3}{4}$ (c) $\dfrac{7}{9}, \frac{2}{3}, \frac{3}{5}$ (d) None of these.

9. The fifth, fifteenth, and twenty-fifth parts of a number together make up 23: find the number.

10. Find the value of $\dfrac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \cdots + \frac{1}{9 \times 10}$

Answer Key

Practice Work :

Que no.	**Answer**
1	$\dfrac{10}{14}$
2	(a) & (d)
3	$\dfrac{7}{20}$
4	70
5	(a) 57 (b) 20 (c) 2.7 (d) 1600
6	10;975
7	234.9
8	2
9	75
10	$\dfrac{9}{10}$

Topic 6

ALGEBRAIC FRACTIONS

• What are Algebraic fractions?

Algebraic fractions are fractions that contain at least one variable. Such fractions involve variables and/or constants, either by themselves or in a combination together.

e.g. $\frac{x}{2}; \frac{x-5}{3}; \frac{3a+b}{a-b}$

VARIABLE

A number which can take various numerical values is known as a **variable.**

Example: x, y, z, etc.

A number which is the product of a constant and a variable is also a **variable.**

Example: $5x^6, 6x^2 - 3x$, etc.

A combination of two or more variables separated by a "+" sign or a "-" sign is also a **variable.**

Example: $x^2 + y + z^5, x^3 - y^3 - z^2$, etc.

CONSTANT

A number having a fixed numerical value is called a constant.

Example: $5, \frac{3}{4}, 3, 6, 2, \overline{3}$, etc.

• Algebraic fractions are subject to the same laws as arithmetic fractions.

• Operations on Algebraic Fractions

Addition & Subtraction

• For fractions with same Ds : Add/Subtract their Ns and keep the D as it is

e.g. $\frac{5x}{3} - \frac{4x}{3} = \frac{x}{3}; \quad \frac{x-5}{3} + \frac{x+2}{3} = \frac{2x-3}{3}$

• For fractions with different Ds:

N of the answer: Sum /Difference of cross multiplications of Ns $\times$ Ds

D of the answer: Product of Ds of the two fractions.

e.g. $\frac{x}{3} - \frac{3x}{10} = \frac{x}{30}; \quad \frac{x-5}{3} + \frac{x+2}{5} = \frac{8x-19}{15}$

• Sometimes the denominators are multiples / factors of one another or some common number. In such cases, we find the LCM of these denominators for simplifying the calculations. In case of Algebraic fractions, this LCM is also called as **LCD (Lowest common Denominator).**

e.g., $\frac{7x}{12} + \frac{5x}{4} = \frac{7x+15x}{12} = \frac{22x}{12} = \frac{11x}{6}; \quad \frac{3a}{10} - \frac{2}{15} = \frac{9a-4}{30}$

Multiplication of Algebraic Fractions

• Multiply the Ns. of the two fractions to get the numerator of the product and the Ds. to get the denominator of the product.

e.g. $\frac{3x}{5} \times \frac{7x}{4} = \frac{21x^2}{20}$; $\frac{x}{3} \times \frac{x+2}{5} = \frac{x^2+2x}{15}$

Division of Algebraic Fractions

• Write the reciprocal of second fraction and multiply with the first fraction.

e.g. $\frac{3x}{5} \div \frac{9x^2}{7} = \frac{3x}{5} \times \frac{7}{9x^2} = \frac{7}{15x}$; $\frac{x+2}{5} \div \frac{3}{25} = \frac{x+2}{5} \times \frac{25}{3} = \frac{5x+10}{3}$

Practice Work:

1. Write down the lowest common denominator for these pairs of fractions.

(a) $\frac{a}{3}, \frac{7a}{4}$ (b) $\frac{x}{2}, \frac{4xy}{6}$ (c) $\frac{3xy}{7}, \frac{-3x}{14}$ (d) $\frac{2}{x}, \frac{3}{2x}$

2. Simplify by cancelling common factors.

(a) $\frac{10x}{2}$ (b) $\frac{24x}{6}$ (c) $\frac{5a}{20}$ (d) $\frac{7}{21a}$

(e) $\frac{-48x^2}{16xy}$ (f) $\frac{120ab^2}{140ab}$

3. Simplify by cancelling common factors.

(a) $\frac{4x+8}{4}$ (b) $\frac{6a-30}{6}$ (c) $\frac{6x-18}{2}$ (d) $\frac{5-15y}{5}$

(e) $\frac{x^2+2x}{x}$ (f) $\frac{6x-4x^2}{2x}$

4. Simplify the following.

(a) $\frac{3}{x} \times \frac{x-1}{6}$ (b) $\frac{x+4}{10} \times \frac{2}{x}$ (c) $\frac{6-18x}{2} \times \frac{5}{1-3x}$ (d) $\frac{b-1}{10} \times \frac{5}{1-b}$

5. Simplify the following.

(a) $\frac{x}{5} \div \frac{x}{15}$ (b) $\frac{x+4}{2} \div \frac{x+4}{6}$ (c) $\frac{6x-12}{5} \div \frac{x-2}{3}$

(d) $\frac{5}{3a+4} \div \frac{15}{-15a-20}$ (e) $\frac{2x-6}{5x-20} \div \frac{x-3}{x-4}$

6. Simplify the following.

(a) $\frac{2}{3} + \frac{a}{7}$ (b) $\frac{3}{8} + \frac{a}{2}$ (c) $\frac{3}{10} - \frac{3b}{2}$ (d) $\frac{2}{5} + \frac{4x}{15}$

(e) $\frac{-4}{x} - \frac{2}{3}$ (f) $\frac{-9}{2x} - \frac{1}{3}$

7. Simplify the following algebraic expressions.

(a) $\frac{x+3}{4} + \frac{x+2}{5}$ (b) $\frac{x+2}{3} + \frac{x+1}{4}$ (c) $\frac{x-3}{4} - \frac{x+2}{2}$

(d) $\frac{x-2}{8} + \frac{2x+4}{12}$ (e) $\frac{5x+3}{10} + \frac{2x-2}{4}$

8. Simplify the following algebraic expressions.

(a) $\frac{5}{x+1} + \frac{2}{x+4}$ (b) $\frac{4}{x-7} + \frac{3}{x+2}$ (c) $\frac{1}{x-3} + \frac{2}{x+5}$

(d) $\frac{2}{x-3} - \frac{3}{3x+4}$ (e) $\frac{8}{3x-2} - \frac{3}{1-x}$

Answer Key

Practice Work :

Que no.	Answer
1	(a) 12 (b) 6 (c) 14 (d) $2x$
2	(a) $5x$ (b) $4x$ (c) $\frac{a}{4}$ (d) $\frac{1}{3a}$ (e) $\frac{-3x}{y}$ (f) $\frac{6b}{7}$
3	(a) $x+2$ (b) $a-5$ (c) $3x-9$ (d) $1-3y$ (e) $x+2$ (f) $3-2x$
4	(a) $\frac{x-1}{2x}$ (b) $\frac{x+4}{5x}$ (c) 15 (d) $\frac{-1}{2}$
5	(a) 3 (b) 3 (c) $\frac{18}{5}$ (d) $\frac{-5}{3}$ (e) $\frac{2}{5}$
6	(a) $\frac{14+3a}{21}$ (b) $\frac{3+4a}{8}$ (c) $\frac{3-15b}{10}$ (d) $\frac{6+4x}{15}$ (e) $\frac{-12-2x}{3x}$ (f) $\frac{-27-2x}{6x}$
7	(a) $\frac{9x+23}{20}$ (b) $\frac{7x+11}{12}$ (c) $\frac{-x-7}{4}$ (d) $\frac{7x+2}{24}$ (e) $\frac{5x-1}{5}$
8	(a) $\frac{7x+22}{(x+1)(x+4)}$ (b) $\frac{7x-13}{(x-7)(x+2)}$ (c) $\frac{3x-1}{(x-3)(x+5)}$ (d) $\frac{3x+17}{(x-3)(3x+4)}$ (e) $\frac{14-17x}{(3x-2)(1-x)}$

Topic 7

RATIO AND PROPORTION

What is Ratio?

Ratio is the comparison between two quantities in terms of their magnitudes. The ratio of two quantities is equivalent to a fraction that one quantity is of the other.

Remember:

• In the ratio of two quantities the two quantities must be of the same kind and in same unit.
• The ratio is a pure number, i.e., without any unit of measurement.
• The ratio would stay unaltered even if both the numerator and the denominator are multiplied or divided by the same number.
• A ratio $a : b$ is said to be in lowest form or simplest form if the HCF of a and b is 1.

Properties of Ratios

(i) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \ldots = \frac{a_1+a_2+a_3+\ldots}{b_1+b_2+b_3+\ldots}$

This means that if two or more ratios are equal, then the ratio whose numerator is the sum of the numerators of all the ratios and denominator is the sum of the denominators of all the ratios is equal to each of the given ratio.

Since $\frac{35}{50} = \frac{7}{10} \therefore \frac{35}{50} = \frac{7}{10} = \frac{35+7}{50+10} = \frac{42}{60}$

(ii) If a quantity n is divided into two parts in the ratio $a : b$ then first part $= \frac{a}{a+b} \times n$ and second part $= \frac{b}{a+b} \times n$.

If a quantity n is divided into three parts in the ratio $a : b : c$ then first part $= \frac{a}{a+b+c} \times n$, second part $= \frac{b}{a+b+c} \times n$ and third part $= \frac{c}{a+b+c} \times n$.

Comparison of Ratios

Method - I : Cross Multiplication Method

$\frac{a}{b} > \frac{c}{d}$, if $ad > bc$ & $\frac{a}{b} < \frac{c}{d}$, if $ad < bc$

For example:

$\frac{6}{7} > \frac{3}{5}$ because $6 \times 5 > 7 \times 3$ and $\frac{4}{5} < \frac{7}{8}$ because $4 \times 8 < 5 \times 7$

Method - II : Denominator Equating Method

By making the denominator of each ratio equal to the LCM of the denominators of both ratios, we can compare the two ratios by checking their numerators. If two ratios have same denominator then the ratio with greater numerator is greater.

Types of Ratios

(i) Duplicate ratio

Duplicate ratio of $x : y$ is $x^2 : y^2$.

(ii) Triplicate ratio

Triplicate ratio of $x : y$ is $x^3 : y^3$.

(iii) Subduplicate ratio

Subduplicate ratio of $x : y$ is $\sqrt{x} : \sqrt{y}$.

(iv) Subtriplicate ratio

Subtriplicate ratio of $x : y$ is $\sqrt[3]{x} : \sqrt[3]{y}$.

(v) Reciprocal or Inverse ratio

Reciprocal ratio of $a : b$ is $\dfrac{1}{a} : \frac{1}{b}$ or $b : a$.

(vi) Compounded ratio

Compounded ratio of two or more ratios say

$\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \ldots\ldots$ etc. is $\dfrac{ace\ldots\ldots}{bdf\ldots\ldots}$ i.e. $\dfrac{\text{Product of numerator}}{\text{Product of denominator}}$

Practice Work:

1. Find the ratio of the following:

(a) Speed of cycle 15 Km/Hr. to speed of scooter 30 Km/Hr.
(b) 5 m to 10 km
(c) 50 paise to Rs. 5

2. The ratio of marks of Shanta and Narmada is 5 : 6. The sum of their marks is 132. Find the marks of Narmada.

3. The present ages of Raman and Pavan are in a ratio 4 : 5. The product of their ages numerically is equal to 980. Find the difference between their ages.

4. The ratio of present ages of two brothers is 4 : 5. After 10 years, the ratio of their ages would be 6 : 7. Find the age of elder brother after 15 years.

5. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the tens place, what is the number?

6. Divide Rs 16250 among A, B and C in the ratio 5 : 7 : 13

7. An alloy contains copper and zinc in the ratio 7 : 3. If it contains 12.6 gm of copper, how much does this alloy weigh?

8. Rama invested Rs, 6000 and started a business. After 7 months Govinda invested some amount in the same business. At the end of the year Govinda got half of the total profit. What is Govinda's investment in the business?

9. A bag contains coins of Re. 1, 50 paise and 25 paise in the ratio $5 : 9 : 4$. What is the worth of the bag, if the total number of coins in the bag is 72?

10. There is four years difference in the present ages of Prachi and Shachi. Five years ago, the ratio of their ages was 5:3. Find the present age of Shachi.

Answer Key:

Practice Work:

Que no.	**Answer**
1	(a) 1:2 (b) 1:2000 (c) 1:10
2	72
3	7
4	40
5	36
6	3250, 4550, 8450
7	18
8	14400
9	42
10	11

Topic 8

RATIO AND PROPORTION

What is Proportion?

When two ratios are equal, the four quantities composing them are said to be proportional. Hence, if $\frac{a}{b} = \frac{c}{d}$, then a, b, c, d are proportional and are written as $a : b :: c : d$. The terms a and d are also called **extremes** while the terms b and c are called the **means.**

$a : b : c : d \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

Hence **Product of extremes = Product of means**

Continued Proportion

(i) If $\frac{a}{b} = \frac{b}{c}$, then a, b, c, are said to be in continued proportion and vice-versa.

If $A : B$ and $B : C$, is given, how to find $A : B : C$?

Eg: If $A : B = 2 : 3$, and $B : C$ is $7 : 9$, find $A : B : C$

A	:	B	:	C
2	:	3		
		7	:	9
2×7	:	3×7	:	3×9
14	:	21	:	27

Properties of Proportion

(i) Invertendo: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$

(ii) Alternando: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

(iii) Componendo: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$

(iv) Dividendo: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$

(v) Componendo and Dividendo: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Practice Work:

1. Find the third proportional to 36, 12

2. Check whether the following numbers are in proportion

(a) $20, 18, 5, 6$ (b) $3.6, 0.4, 4.5, 0.5$ (c) $\frac{1}{5}, \frac{1}{8}, \frac{1}{4}, \frac{1}{10}$

3. If $a : b = 3 : 2$ and $b : c$ is $4 : 5$, find $a : c$

4. The car that I own can go 150 Km with 25 litres of petrol. How far can it go with 30 litres of petrol?

5. If $a : b = c : d$, prove that $\dfrac{(5a + 7b)}{(5a - 7b)} = \frac{(5c+7d)}{(5c-7d)}$

6. If $\dfrac{(4x + 3y)}{(4x - 3y)} = \frac{7}{4}$, use properties of proportion to find $\dfrac{x}{y}$

7. Do the two ratios $100 : 225$ and $32 : 56$ form a proportion?

8. What least number should be subtracted from $7, 17, 47$ so that the remainders are in continued proportion?

Answer Key:

Practice Work:

Que no.	**Answer**
1	4
2	(b) In proportion
3	6:5
4	180 km
5	
6	11:4
7	Not in proportion
8	2

Topic 9

PERCENTAGE

- Percent means 'for every hundred' or a percent is a fraction whose denominator is 100.
- It is denoted by the symbol %
- Percentage is another way of comparing quantities.
- We can convert fractions, decimals, and ratios to percentages and vice versa.
- To increase a number by 30%, we multiply it by 130/100.
- To decrease a number by 30%, we multiply it by 70/100

To convert a given fraction or decimal into percentage, multiply it by 100 and put the sign %. e.g., $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$

If A 's income is $x\%$ more than that of B. Then B 's income is less than that of A by $\left\{\frac{x}{(100+x)} \times 100\right\}\%$

If A 's income is $x\%$ less than that of B. Then B 's income is more than that of A by $\left\{\frac{x}{(100-x)} \times 100\right\}\%$

Profit and Loss

- The calculation for profit and loss in a transaction is an application of percentage.
- Commonly used terms:

1. Cost price (CP) : The price at which a commodity is bought
2. Selling price (SP) : The price at which a commodity is sold
3. Profit (P): If $SP > CP; P = SP - CP$
4. Loss (L) : If $CP > SP; L = CP - SP$
5. Profit % = Gain % = $\frac{\text{Gain} \times 100}{\text{C.P.}}$,

Loss % = $\frac{\text{Loss} \times 100}{\text{C.P.}}$

- S.P. = $\frac{100 + \text{gain}\%}{100} \times$ C.P.
- S.P. = $\frac{100 - \text{loss}\%}{100} \times$ C.P.

Note:

1. Profit and loss are always calculated on the cost price and not the selling price.
2. If the CP is not given, assume it to be 'X' or '100' and calculate.

DISCOUNT

Discount means reduction in the price. This reduction is always given on the marked price (M.P.) or list price or advertised price.

Important Formulae

• When discount is offered on an article, then we calculate the selling price (S.P.) as:
S.P. = Marked price - Discount.

• Discount = M.P. - S.P. = Marked price - Selling price

• Discount $\% = \dfrac{\text{Discount}}{\text{M.P.}} \times 100.$

• S.P. = M.P. - Discount

. $= \text{M.P.} - \dfrac{\text{Discount}\% \times \text{M.P.}}{100}$

Practice Work:

1. Find the percent value of the following :

(a) 10% of 20% of 100 (b) 420 gm of 6 Kg (c) 0.25 of x

(d) 125 paise of Rs. 25 (e) 150 m of 5 Km (f) $\dfrac{3}{5}$ th of x

2. If 20% of x + 40% of $\dfrac{x}{2}$ + 20% of $3x = 100$, then find the value of x.

3. 'A' has scored 16 marks out of 20 and 'B' has scored 35 marks out of 50. Whose score is higher?

4. If 15% of 40 is greater than 25% of an unknown number by 2. Find the unknown number.

5. 'A' bought an article for Rs. 10. He sold it to 'B' with 10% profit. 'B' sold it to 'C' with 10% profit on his cost price. Find the price at which 'C ' bought the article.

6. A watch is sold at 10% profit. If it is sold at Rs. 30 less, there is a loss of 5%, Find the cost price of the watch.

7. The S.P. of 6 toys is equal to the C.P. of 5 toys. What will be the percent profit or loss in the transaction?

8. Meenu bought two fans for Rs. 1200 each. She sold one at a loss of 5% and the other at a profit of 10%. Find the selling price of each and then find out the total profit or loss.

9. 40% of 400 is calculated and again 40% of that number is calculated. What is the ratio of the number obtained to the original number (400) ?

10. A medical student has to secure 40% marks to pass. He scores 40 marks but fails by 40 marks. Find the maximum marks of the question paper.

Answer Key:

Practice Work:

Que no.	**Answer**
1	(a) 2 (b) 7 (c) 25 (d) 5 (e) 3 (f) 60
2	100
3	A
4	16
5	12.10
6	200
7	100/6 or 16.67
8	1140, 1320, 60
9	4/25
10	200

Topic 10

PERCENTAGE

Discount

Discount means reduction in the price. This reduction is always given on the marked price (M.P.) or list price or advertised price.

Important Formulae

• When discount is offered on an article, then we calculate the selling price (S.P.) as: S.P. = Marked price - Discount.

• Discount = M.P. - S.P. = Marked price - Selling price

• Discount $\% = \dfrac{\text{Discount}}{\text{M.P.}} \times 100.$

• S.P. = M.P. - Discount

. $= \text{M.P.} - \dfrac{\text{Discount}\% \times \text{M.P.}}{100}$

Simple Interest and Compound Interest

• The calculation of interest is an application of percentage

• Commonly used terms:

1. Principal: The money borrowed for a certain period is called the principal (P).

2. Interest: The additional money paid by the borrower for having used lenders money is called the Interest.

3. Amount: The sum of the principal and the interest on it, together is called the Amount (A).

4. Rate: The interest on Rs.100 for one year is called rate percent per annum (R)

5. Time: The number of days, months, or years, for which the borrowed money is used is called **Time** (T)

$$\textbf{Simple interest} = \textbf{Principal} \times \textbf{Time} \times \frac{\textbf{Rate}}{\textbf{100}}$$

$$\textbf{SI} = \textbf{P} \times \textbf{T} \times \frac{\textbf{R}}{\textbf{100}}$$

$$\textbf{Amount} = \textbf{Principal} + \textbf{Interest}$$

Compound Interest

If the interest earned for a specific period is added to the principal for calculating the interest for the next period and so on, then such calculated interest is called the compound interest (C.I.).

Important Formulae

• If A is the amount, P is the principal, $R\%$ is the rate of interest compounded annually and n is the number of years, then $\mathbf{A} = \mathbf{P}\left(\mathbf{1} + \dfrac{\mathbf{R}}{\mathbf{100}}\right)^{\mathbf{n}}$

If V_0 is the value of an article at a certain time and $R\%$ per annum is the rate of depreciation, then the value V_n at the end of n years is given by

$$\mathbf{V_n = V_0\left(1 - \frac{R}{100}\right)^n}$$

Important Formulae for calculating Population

• Let P be the population of a city or town at the beginning of a certain year and the population grows at a constant rate of $R\%$ per annum, then

Population after n years $= P\left(1 + \dfrac{R}{100}\right)^n$.

Population n years ago $= \dfrac{P}{\left(1 + \dfrac{R}{100}\right)^n}$

Practice Work:

1. A sum of Rs 400 is lent at the rate of 5% per annum. Find the interest at the end of 2 years.

2. A person deposits Rs 25000 in a firm, which pays an interest at the rate of 20% per annum. Calculate the income he gets from it annually.

3. A man borrowed Rs 8000 from a bank at 8% per annum. Find the amount he has to pay after $4\frac{1}{2}$ years.

4. Rakesh lent out Rs 8000 for 5 years at 15% per annum and borrowed Rs 6000 for 3 years at 12% per annum. How much did he gain or lose?

5. Find the amount on a sum of Rs 5000 for 2 years at the rate of 8% p.a. compounded annually.

6. Find the compound interest on a sum of Rs 8000 for 3 years at a rate of 5% p.a.

7. Find the sum of money which amounts to Rs 13310 in 3 years at the rate of 10% per annum compounded annually.

8. A cycle shop allows a discount of 25% on the marked price and earns a profit of 20% on the cost price. Its marked price, on which the shop earns Rs.40 is .....

9. The population of a city increases by 10% every year. Its present population is $3,02,500$. What was its population 2 years ago?

10. Mr. Mittal purchased a car for Rs $3,00,000$ and a bike for his son for Rs $1,00,000$, He sold the car at a profit of 10% and the bike at a loss of 20%. What is the net gain or loss?

Answer Key:

Practice Work:

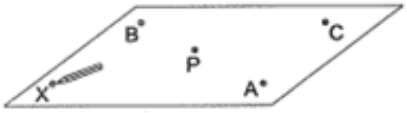
Que no.	Answer
1	40
2	5000
3	10880
4	3840
5	5832
6	1261
7	10000
8	320
9	250000
10	Gain 10000

Topic 11

BASIC GEOMETRY - 1

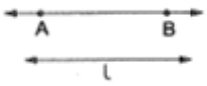
Point

A point is a basic geometrical object. It has some position but neither any length nor any breadth. To represent a point on paper, we make a dot using a sharp pencil. We denote points by capital letters, such as A, B, C, P and X.



Line

A straight line (simply called a line) is formed by a collection of points. A line has some position, shape and length but neither any breadth nor any thickness.



It can be extended on either side to any length; so it has no end points. In the given figure, this is shown by arrowheads at either end of the line. A line is denoted by any two points on it; for example, $\overleftrightarrow{AB}$, or line AB. It may also be denoted by a small letter (e.g. l, m, n, etc.).

Ray

A ray is a part of a straight line. It extends indefinitely in one direction from a fixed point. In the given figure, this is represented by an arrowhead in one direction.

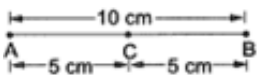
A ray has one end point-the fixed point from which it starts is called the initial point. It is denoted by the initial point and any other point on it, such as $\overrightarrow{AB}$, or simply ray AB.

Line segment

The part of a straight line between two given points is called a line segment.

A line segment has two end points and a finite length. The line segment between the points A and B is denoted as $\overline{AB}$, and its length is the distance between its end points A and B.

If C is a point on the line segment AB such that

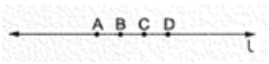


$AC = CB = \frac{1}{2}AB$ then C is called the middle point (or midpoint) of AB.

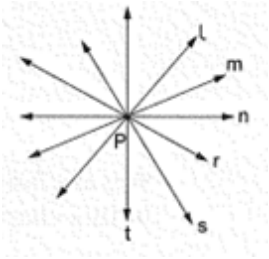
Properties of Lines

The following properties of lines are taken to be universally true.

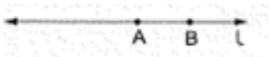
Property 1: There are an infinite number of points on a straight line.



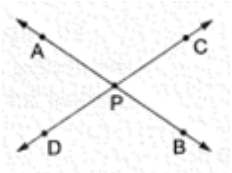
Property 2: An infinite number of straight lines can be drawn through a given point.



Property 3: One and only one straight line passes through two different given points (or two distinct points determine a straight line).

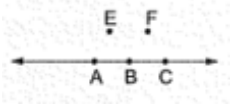


Property 4: Two different straight lines cannot have more than one point in common.



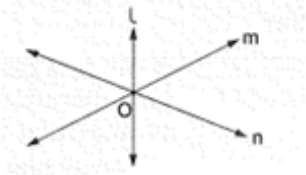
Collinear points

If three or more points lie on a straight line, we say that these points are collinear. In the figure, A, B and C are collinear points, while A, E and F are noncollinear points.



Concurrent lines

If three or more straight lines have one point in common then these lines are called concurrent lines. In the figure, l, m and n are three concurrent lines and O is the point of concurrency.

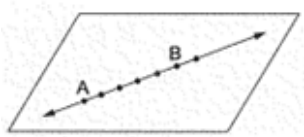


Plane

A plane is a flat surface that has some length and breadth but no thickness. The surface of a sheet of paper and a tabletop are examples of a plane. In fact, they are parts of planes, because a plane is supposed to extend infinitely. It is made up of an infinite number of lines and hence an infinite number of points.

An important property of a plane

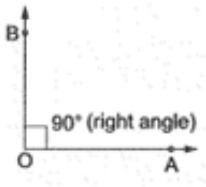
If two points on a line lie in a plane then every point on the line lies in the plane. This follows from the fact that two points are enough to determine a straight line.



Types of angles:

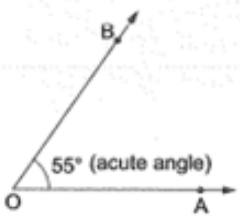
Right angle

An angle of measure 90° is called a right angle. In the given figure, $\angle AOB$ is a right angle. Note that the symbol '□' at the vertex of the angle is used to denote a right angle.



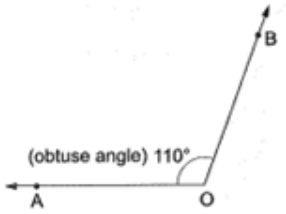
Acute angle

An angle that measures less than 90° is called an acute angle. In the given figure, $\angle AOB$ is an acute angle because $\angle AOB = 55° < 90°$.



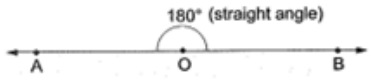
Obtuse angle

An angle of magnitude greater than 90° but less than 180° is called an obtuse angle. Thus, angles measuring $91°, 110°, 120°$, 135° and 179° are obtuse angles.



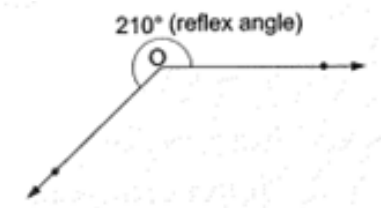
Straight angle

An angle of magnitude 180° is called a straight angle. The two arms of a straight angle together form a straight line.



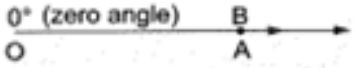
Reflex angle

A reflex angle is an angle that measures more than 180° but less than 360°. Thus, angles measuring $181°, 210°, 250°, 310°$ and 354° are reflex angles.



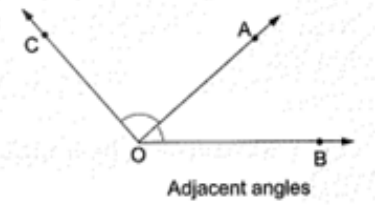
Zero angle

If the two arms of an angle coincide then the angle measures 0°. Such an angle is called a zero angle. In the given figure, $\angle AOB$ is a zero angle because its arms OA and OB represent the same ray.



Adjacent angles

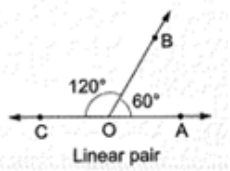
Two angles are called adjacent angles if
(i) they have the same vertex,
(ii) they have a common arm, and
(iii) their other arms are on different sides of the common arm.



In the given figure, $\angle AOB$ and $\angle AOC$ are adjacent angles. However, $\angle AOB$ and $\angle COB$ are not adjacent angles because their other arms are on the same side of the common arm OB.

Property - If the sum of two adjacent angles is 180°, the outer arms of the angles are on the same line.

Let $\angle AOB$ and $\angle BOC$ be two adjacent angles, the sum of which is 180°.
Let $\angle AOB = 60°$ and $\angle BOC = 120°$.
Then, $\angle AOC = \angle AOB + \angle BOC = 60° + 120° = 180°$.
So, OA and OC are in a straight line.

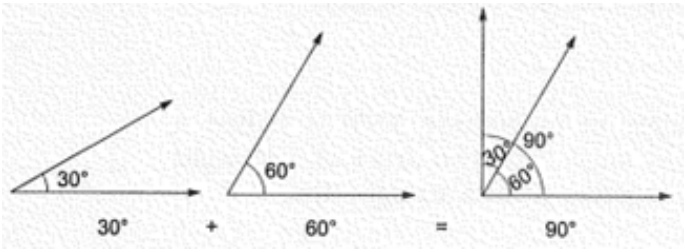


Linear pair

Two adjacent angles are said to form a linear pair if the sum of their measures is 180°.

Complementary angles

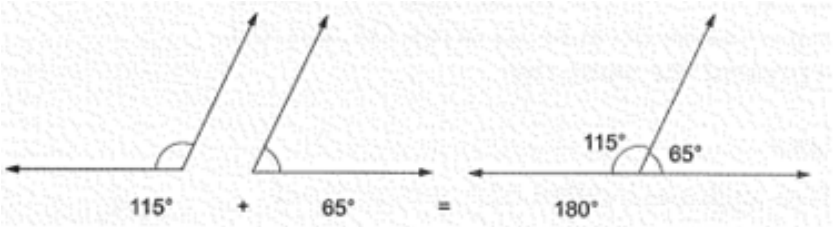
If the sum of the magnitudes of two angles is equal to 90° then the angles are called complementary angles, and each angle is said to be complementary to the other. Example $30° + 60° = 90°$. So, 30° and 60° are complementary to each other.



Complementary angle of an angle = 90°− (measure of the given angle).
Example The complementary angle of 22° is $90° - 22° = 68°$.

Supplementary angles

If the sum of the measures of two angles is equal to 180° then they are called supplementary angles, and each angle is said to be supplementary to the other. Example 115° and 65° are supplementary to each other, since $115° + 65° = 180°$.



Supplementary angle of an angle = 180°− (measure of the given angle) .
Example The supplementary angle of 72° is $180° - 72° = 108°$.

THE 'CLOCK' GAME

We know that
The hour hand (HH) of a clock moves through 30° in one clock hour whereas the minute hand (MH) moves through 360° in one clock hour.

Using this try to find out:

(i) In one cycle of 12 hours, how many times and at exactly what times will
(a) the HH and MH be at right angles (90°) ?
(b) the HH and MH be in 180° (straight line)?
(c) The MH crosses over the HH ?

(ii) The method to find the exact angle (in degrees) in the HH and MH at any given time. Eg: At 5.20, the angle between the HH and MH is exactly 40°

Play a lot.... Have fun!!

Example: Find the measure of the angle between the hour hand and the minute hand of a clock at 6 hours 50 minute.
Minute Hand in 60 mins completes 360°

$\therefore$ in 1 min completes $\dfrac{360}{60} = 6$ degree.

Hour Hand in 12Hrs i.e. in $12 \times 60 = 720$ mins
$\Rightarrow$ 720 mins completes 360°

$\therefore$ in 1 min completes $\dfrac{360}{720} = \frac{1}{2}$ degree.

In given problem with respect to 12 noon position.

Hour hand in 6 hrs50 min = 410 mins. is at $410 \times \dfrac{1}{2} = 205$ degree.

Minute Hand is at $50 \times 6 = 300$ degree

$\therefore$ Angle between minute & Hour hand $= 300 - 205 = 95$ degree.

Practice Work:

1. If $\angle A \& \angle B$ form linear pair & $m\angle A = 3m\angle B$. Find $m\angle B$.

2. If $m\angle P = 57°$ & $m\angle Q = 43°$ are they complementary?

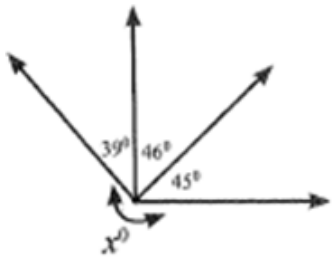
3. Find the measure between the hour hand and minute hand of a clock at 20 minutes past 3.

4. What is the supplementary angle of $(120 - x)°$?

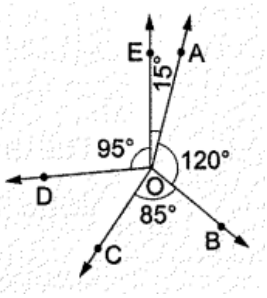
5. The sum of the complementary and supplementary angles of an angle is twice the difference of the supplementary and complementary angles of the same angle. Find the angle.

6. What is the angle between East and North-West directions?

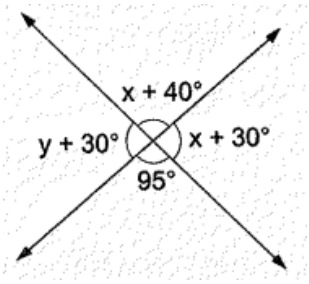
7. Find the value of x.



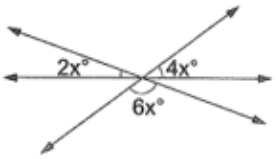
8. In the adjoining figure, $\angle AOB = 120°, \angle BOC = 85°, \angle DOE = 95°$ and $\angle EOA = 15°$. Find $\angle COD$.



9. Find the values of x and y from the adjoining figure.



10. Find the values of x in the following figures.



Answer Key:

Practice Work:

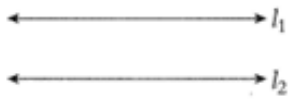
Que no.	**Answer**
1	45
2	NO
3	20
4	$60 + x$
5	45
6	135
7	230
8	45
9	55, 55
10	15

Topic 12

BASIC GEOMETRY - 2

Parallel Lines

Two coplanar lines that do not meet are called **parallel lines.**



In the given figure, l_1 and l_2 are parallel lines. Symbolically we write $l_1 \| l_2$ and read as l_1 is parallel to l_2.

Properties of Parallel Lines

1. For two given parallel lines, the perpendicular distance between the lines is the same everywhere.

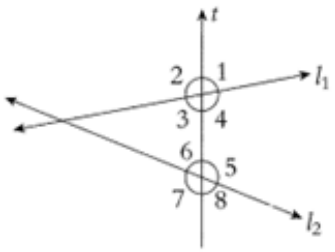
2. If two lines lie in the same plane and are perpendicular to the same line, then they are parallel to each other.

3. If two lines are parallel to the same line, then they are parallel to each other.

4. One and only one parallel line can be drawn to a given line through a given point which is not on the given line.

Transversal:

A straight line intersecting a pair of lines in two distinct points is called a **transversal** for the two given lines.
Let l_1 and l_2 be a pair of lines and t be a transversal.
As shown in the figure, a total of eight angles are formed.



1. $\angle 1, \angle 2, \angle 7$, and $\angle 8$ are exterior angles and $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles.

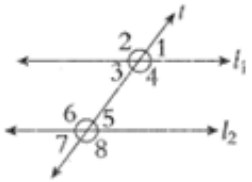
2. ($\angle 1$ and $\angle 5$), ($\angle 2$ and $\angle 6$), ($\angle 3$ and $\angle 7$), and ($\angle 4$ and $\angle 8$) are pairs of corresponding angles.

3. ($\angle 1$ and $\angle 3$), ($\angle 2$ and $\angle 4$), ($\angle 5$ and $\angle 7$), and ($\angle 6$ and $\angle 8$) are pairs of vertically opposite angles.

4. ($\angle 4$ and $\angle 6$) and ($\angle 3$ and $\angle 5$) are pairs of alternate interior angles.

5. ($\angle 1$ and $\angle 7$) and ($\angle 2$ and $\angle 8$) are pairs of alternate exterior angles.

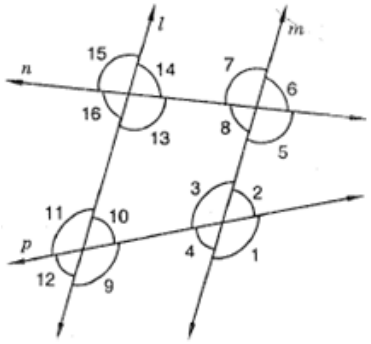
If l_1 and l_2 are parallel, and t is a transversal, then



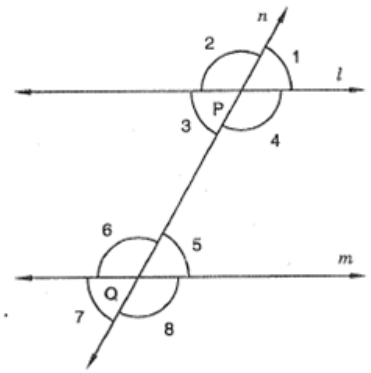
1. Corresponding angles are equal, i.e., $\angle 1 = \angle 5,\ \angle 2 = \angle 6,\ \angle 3 = \angle 7$, and $\angle 4 = \angle 8$.
2. Alternate interior angles are equal, i.e., $\angle 4 = \angle 6$ and $\angle 3 = \angle 5$.
3. Alternative exterior angles are equal, i.e., $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$.
4. Exterior angles on the same side of the transversal are supplementary, i.e., $\angle 1 + \angle 8 = 180°$ and $\angle 2 + \angle 7 = 180°$.
5. Interior angles on the same side of the transversal are supplementary, i.e., $\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$.

Practice Work:

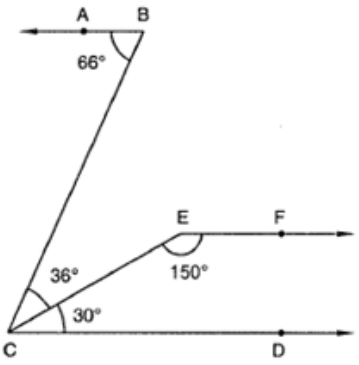
1. The line n is transversal to line / and m in figure. Identify the angle alternate to 13 and angle corresponding to $\angle 15$.



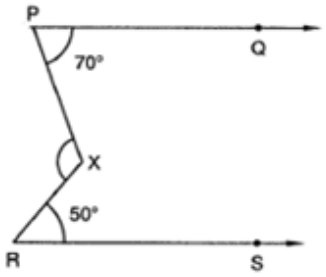
2. In the given figure, line $l \parallel$ line m, n is transversal and $\angle 1 = 40°$. Find all the other angles marked in the figure.



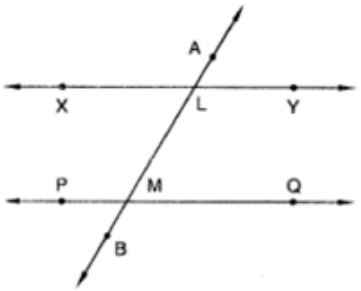
3. Show that $\overrightarrow{BA} \parallel \overrightarrow{EF}$.



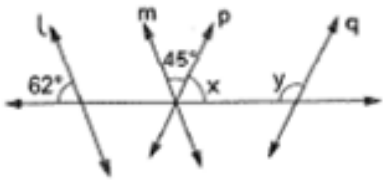
4. Find out $\angle PXR$, if $\overrightarrow{PQ} \parallel \overrightarrow{RS}$.



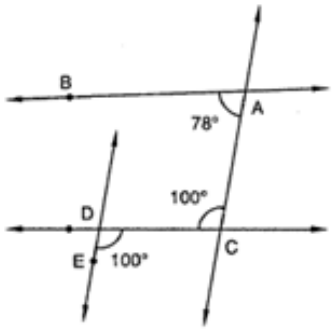
5. In the given figure, $\overline{XY} \parallel \overline{PQ}$ and $\overline{AB}$ is a transversal.
If, (i) $\angle MLY = 2\angle LMQ$, find $\angle LMQ$.
(ii) $\angle XLM = (2x - 10)^\circ$ and $\angle LMQ = x + 30^\circ$, find x.
(iii) $\angle XLM = \angle PML$, find $\angle ALY$
(iv) $\angle ALY = (2x - 15)^\circ, \angle LMQ = (x + 40)^\circ$, find x.



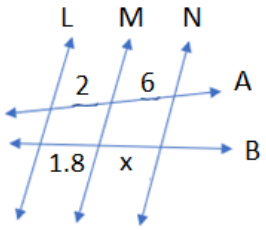
6. If $l \parallel m$ and $p \parallel q$, find x and y.



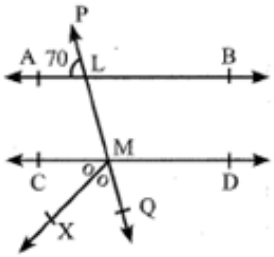
7. State which lines are parallel and why?



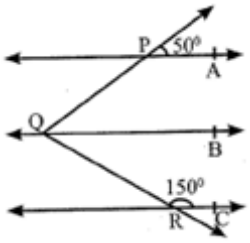
8. In the adjoining figure, line $L \parallel$ line $M \parallel$ line N. If line A and line B are their transversals then find the value of x.



9. In the adjoining figure line $AB \parallel$ line CD and line PQ is their transversal. Then from the given information $m\angle QMX = ?$



10. In the adjoining figure line $PA \parallel$ line $QB \parallel$ line RC. Line PQ and line QR are their transversals. From the given information $m\angle PQR = ?$



Answer Key:

Practice Work:

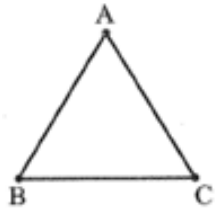
Que no.	Answer
1	$\angle 7,\ \angle 7$
2	140, 40, 140 40, 140, 40, 140
3	-
4	120
5	60, 40, 90, 55
6	73, 107
7	DE$\|\|$AC
8	5.4
9	55
10	80

Topic 13

BASIC GEOMETRY - 3

Triangle properties

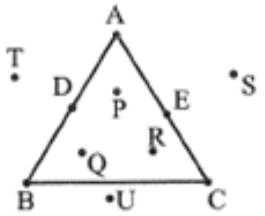
Triangle : A closed figure formed by three angles, three sides and three vertices is called a triangle. The sides and angles are called parts of the triangle
In the adjacent figure,



Sides of triangle : seg AB, seg BC, seg AC
Angles of triangle: $\angle A, \angle B, \angle C$
Vertices of triangle : A, B, C

Interior and exterior of a triangle :

Due to a triangle drawn in a plane, a plane is divided into three parts.

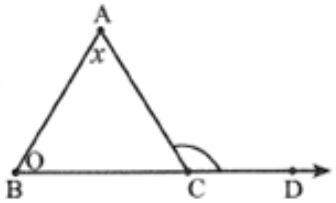


1) Exterior of a triangle 2) Interior of a triangle 3) The triangle itself
Points in the exterior : points S, T, U
Points in the interior : points P, Q, R
Points on the triangle points A, B, C, E, D

Triangular region : Triangular region is formed by the interior of a triangle and the triangle itself.

Exterior angle of a triangle :

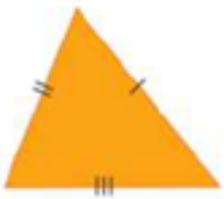
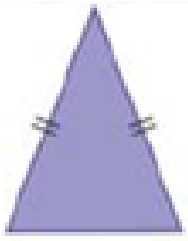
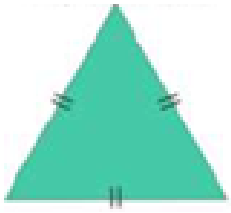
The angle forming a linear pair with an interior angle of a triangle is called an exterior angle.



In the adjoining figure, $\angle ACD$ is an exterior angle of $\triangle$ABC. A triangle has 6 exterior angles with 2 angles at each vertex. $\angle$A and $\angle$B are remote interior angles of $\angle$ACD.

Types of Triangles: Triangles may be classified based on the lengths of sides or measures of angles.

Triangles Based on Sides

Scalene	Isosceles	Equilateral
		
Length of all sides are different	Length of two sides are equal	Length of all sides are equal

Triangles Based on Angles

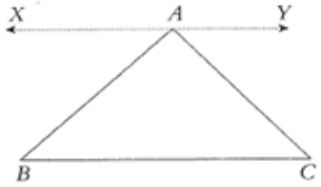
Acute	Right	Obtuse
Each angle is $< 90°$	One angle is $= 90°$	One angle is $> 90°$

Think and decide whether:

a. A scalene triangle can be acute, right or obtuse.
b. An isosceles triangle can be acute, right or obtuse.
c. An equilateral triangle can be acute, right or obtuse.

Properties of a triangle:

Theorem 1: The sum of the three angles of a triangle is 180°.



Given: ABC is a triangle.

To prove: $\angle A + \angle B + \angle C = 180°$

Construction: Draw a line XY through A and parallel to $\overline{BC}$.

Proof: $\angle XAB + \angle BAC + \angle CAY = 180°$ ($\because$ straight line) ...(1)

$\overline{XY}$ and $\overline{BC}$ are parallel and $\overline{AB}$ is a transversal.

$\therefore \angle XAB = \angle ABC$ ($\because$ alternate angles) ...(2)

$\overline{XY}$ and $\overline{BC}$ are parallel and $\overline{AC}$ is a transversal.

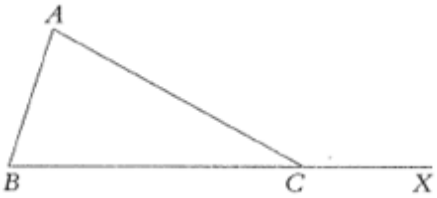
$\angle YAC = \angle ACB$ ($\because$ alternate angles) ...(3)

From Eqs (1), (2), and (3), we have

$\angle ABC + \angle BAC + \angle ACB = 180°$

Hence, proved.

Theorem 2: The exterior angle of a triangle is equal to the sum of the interior angles opposite to it.



Given: ABC is a triangle; BC is produced to the point X.

To Prove: $\angle ACX = \angle A + \angle B$

Proof: $\angle A + \angle B + \angle BCA = 180°$($\because$ Angles of $\triangle ABC$)

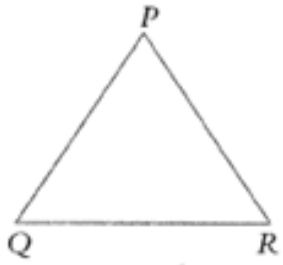
$\angle BCA + \angle ACX = 180°$($\because$ linear pair)

$\therefore \angle A + \angle B + \angle BCA = \angle BCA + \angle ACX$

$\Rightarrow \angle ACX = \angle A + \angle B$

Hence, proved.

Given below are the statements of some of the properties of triangles:



1. The sum of any two sides of a triangle is greater than the third side.
In $\triangle PQR, PQ + QR > PR$, $QR + RP > PQ$, and $RP + PQ > QR$.

2. Difference between any two sides is less than the third side.
$PQ - QR < PR, QR - RP < PQ$, and $RP - PQ < QR$

3. Angles opposite to equal sides are equal and vice versa.
If $\angle P = \angle Q$, then $QR = PR$.
If $QR = PR$, then $\angle P = \angle Q$.

4. If the angles are in increasing or decreasing order, then the sides opposite to them also will be in the same order. If $\angle P > \angle Q > \angle R$, then $QR > PR > PQ$.

Practice Work:

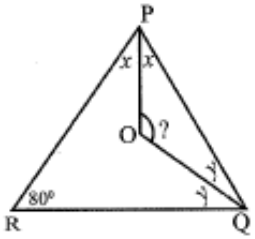
1. The angles of a triangle are $M°, (M+8)°$ and $72°$. Find the smallest angle of the triangle.

2. Determine which of the following sets cannot represent three sides of a triangle

a) $3, 4, 5$ b) $2, 3, 4$ c) $2, 3, 5$ d) $3, 5, 7$

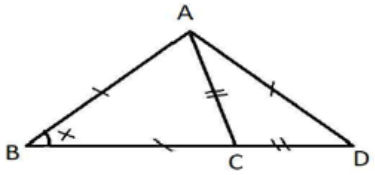
3. In $\triangle PQR, S$ is mid-point of segment QR. Given $SP = SQ = SR$. $\dfrac{m\angle Q}{m\angle R} = \frac{3}{2}$, then $m\angle R$ equals .....

4. Find $m\angle POQ$



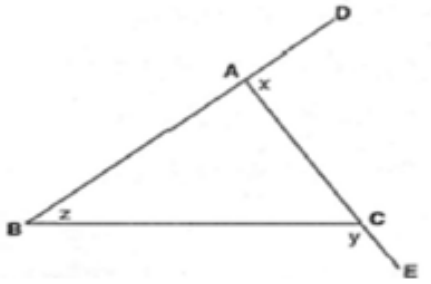
.

5. In this diagram, $AB = BC, BC = AD$ and $AC = CD$. Find measure of $\angle ABC$.



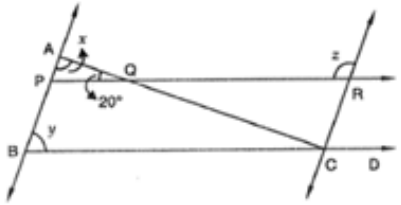
.

6. As shown in the figure $m\angle DAC = x°, m\angle BCE = y°$ and $\angle ABC = z°$.
Find $\dfrac{1}{12}(x + y - z)$.

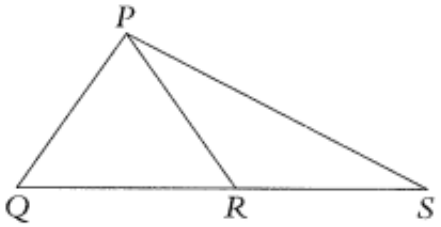


.

7. If $\overline{CA} \perp \overleftrightarrow{AB}$, $\overleftrightarrow{AB} \| \overleftrightarrow{CR}$ and $y = 70$, find z. Prove that $\overline{PR} \| \overline{BC}$.

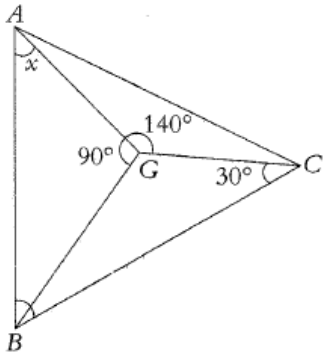


8. In $\triangle PQS, m\angle RPS + m\angle PSR = 60°$. $m\angle QPR = 2\ m\angle RPS$.
Prove that $m\angle PQR = 2\ m\angle PSR$.



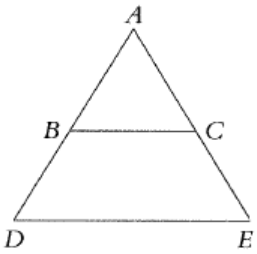
.

9. In the figure below, $\angle ABC = 60°$. Find x.



.

10. In the given figure, $\overline{BC}||\overline{DE}$ and $\angle ABC = \angle CED$. $\angle A = \angle ACB - 30°$. Find $\angle A$.



.

Answer Key:

Practice Work:

Que no.	Answer
1	50
2	c)
3	36
4	130
5	36
6	15
7	110
8	-
9	50
10	40

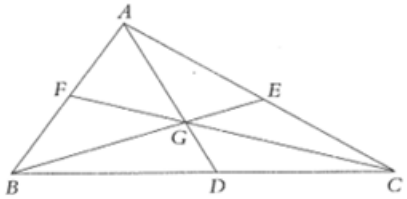
Topic 14

BASIC GEOMETRY - 4

Components of Triangle

Median:

A line segment joining the mid-point of a side and its opposite vertex is a **median** of the triangle. In the adjacent triangle, D is the mid-point of side BC and AD is the median.



For each side of the triangle, there is a corresponding median.
$\overline{AD}$ is the median to $\overline{BC}$, $\overline{BE}$ is the median to $\overline{AC}$, and $\overline{CF}$ is the median to $\overline{AB}$.

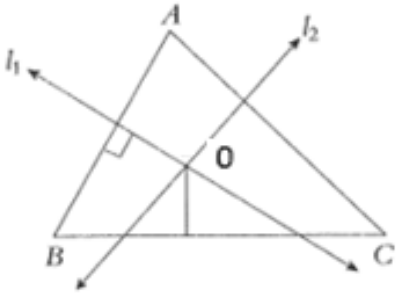
• The medians of the triangle are concurrent and the point of concurrence of the medians is called the **centroid**. It is denoted by G.

• The centroid divides each median in the ratio $2:1$. The smaller part is closer to the side to which the median is drawn.
$\therefore AG : GD = 2 : 1$

• A median divides the triangle into two triangles of equal areas.
The three medians divide the triangle into six triangles of equal area.

Perpendicular bisector:

The perpendicular bisector of a side bisects it and is perpendicular to it. The perpendicular bisector of a side of a triangle need not pass through the opposite vertex.



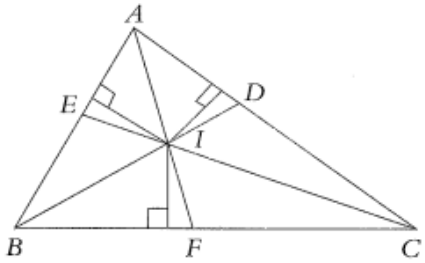
The point of concurrence of perpendicular bisectors is called the **circumcentre**. It is denoted by O. Since OA = OB = OC, taking O as the centre and OA or OB or OC as the radius, a circle can be formed passing through A, B and C. This circle is called the circumcircle of the triangle. O is the circumcentre and OA (or OB or OC) is the circumradius of the triangle.

Position of circumcentre:

1) Circumcentre of an acute angled triangle - In the interior of triangle.
2) Circumcentre of a right angled triangle - midpoint of the hypotenuse.
3) Circumcentre of an obtuse angled triangle - In the exterior of the triangle.

Angle bisector:

The angle bisector of an angle bisects that angle.
For each angle of a triangle, the bisector can be drawn.

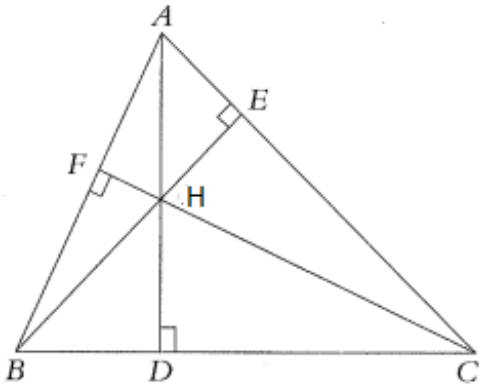


$\overline{BD}, \overline{CE}$, and $\overline{AF}$ are the bisectors of $\angle ABC, \angle ACB$, and $\angle BAC$, respectively.
The bisectors of the interior angles of a triangle are concurrent. The point of concurrence of the bisectors of the interior angles of triangle is the **incentre** of the triangle.

The incentre is denoted by I. This Incentre is equidistant from every side of the triangle. The perpendicular drawn from the incentre (l) to any side is the radius of the incircle, which is inscribed inside the given triangle.

Altitude:

The altitude to a side of a triangle is the perpendicular drawn from its opposite vertex.



In the given triangle, $\overline{AD}$ is the altitude to side BC.
BE and CF are the altitudes to $\overline{AC}$ and $\overline{AB}$, respectively.
The altitudes of a triangle are concurrent. The point of concurrence of the altitudes is the Orthocentre.

The orthocentre is denoted by H.

Position of point of concurrence of altitudes of a triangle.

1) Acute angled triangle - In the interior of the triangle.
2) Right angled triangle - On the vertex of the right angle.
3) Obtuse-angled triangle - In the exterior of the triangle.

Congruency of altitudes of a triangle :

1) Equilateral triangle - All three altitudes are congruent.
2) Isosceles triangle - Altitudes on congruent sides are congruent.
3) Scalene triangle - All three altitudes are of different lengths.

- PERIMETER OF A TRIANGLE:

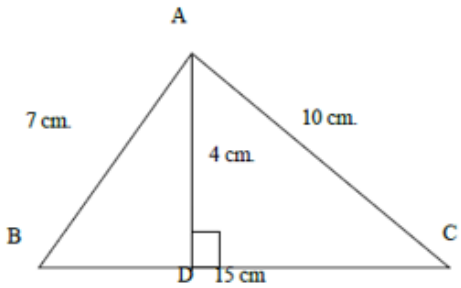
• The sum of the lengths of all the three sides of a triangle is called as the perimeter of the triangle.

• Unit of perimeter is called as the linear unit. It is the same as length; like mm, cm, m, etc.

- AREA OF A TRIANGLE:

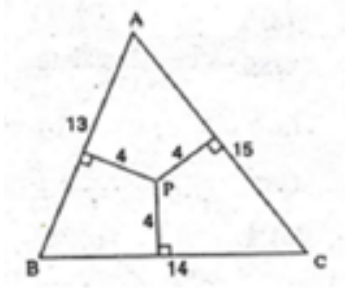
• The area of a triangle is the measurement of the space enclosed in-side the triangle.
• One of the formulae to find the area of any triangle = $1/2\times$ base $\times$ corresponding height of the Δ
• Unit of area is a square unit, like 'square cm$'$ or cm^2

Example: In $\triangle$ABC : I (AB) 7 cm., I(AB)15 cm, I(AC) = 10 cm.I(AD) = 4 cm (Height of the Δ). Find the perimeter and area of the triangle.

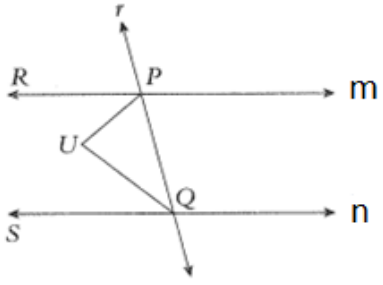


Practice Work:

1. Find the area of the adjoining figure.

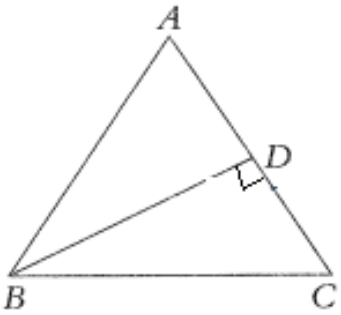


2. In the given figure PU and QU are angle bisectors of $\angle RPQ$ and $\angle SQP$ respectively. m$\angle$PUQ = 90°. Prove that line m is parallel to line n.



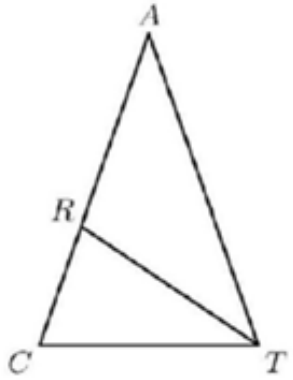
3. The area of the triangle is 162 sq. cm. If the base and the height are equal in length, find the length of the base.

4. In $\triangle ABC$, BD is an altitude. If $m\angle ABD = m\angle CBD$, prove that $\triangle ABC$ is an isosceles triangle.

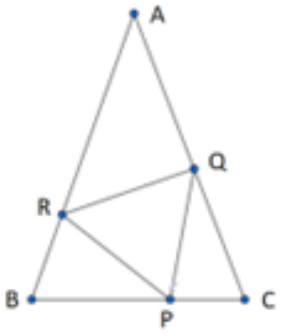


5. In $\triangle ABC$, if $BC = 10, AC = 12$ and altitude $AD = 6$ then the length of altitude BE is

6. In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36°$. $\overline{TR}$ bisects $\angle ATC$, if $CT = 29$ then find AR.



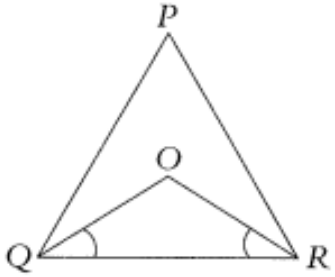
7. In the given figure, $\triangle ABC$ is isoceles and $\triangle PQR$ is equilateral $\triangle$. If $\angle ARQ = 47°$ and $\angle PQC = 33°$, find $\angle RPB$.



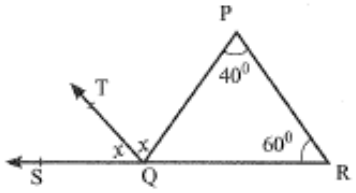
8. In a triangle TOP, its orchocentre lies at O. Then, the circumradius of $\triangle TOP$ is ..... .

(a) $TO/2$ (b) $OP/2$ (c) $TP/2$ (d) $TO/4$

9. In the figure below, $2\angle P = \angle QOR$. OQ and OR are bisectors of $\angle Q$ and $\angle R$ respectively. Find $\angle P$.



10. In the adjoining figure, seg QT is bisector of $\angle PQS$, $m\angle P = 40°, m\angle R = 60°$ then $m\angle TQS = ?$



.

Answer Key:

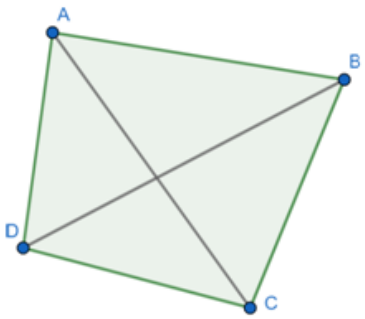
Practice Work:

Que no.	Answer
1	84
2	-
3	18
4	-
5	5
6	29
7	40
8	TP/2
9	60
10	50

Topic 15

BASIC GEOMETRY - 5

Quadrilaterals



• A convex closed figure bounded by four line segments is called a **Quadrilateral.**
E.g.- □ ABCD as shown in the figure.

Parts of a Quadrilateral

• **Sides :** The line segments forming the quadrilateral.
E.g. AB, BC, CD, DA

• **Vertex :** The meeting point of two such adjacent line segments.
E.g. A, B, C, D

• **Angles :** The angle formed at each vertex by two adjacent line segments.
E.g. $\angle ABC, \angle BCD, \angle CDA, \angle DAB$

i) **Adjacent Angles :** Angles with a common side.
E.g. $\angle ABC$ & $\angle BCD, \angle BCD$ & $\angle CDA, \angle CDA$ & $\angle DAB, \angle DAB$ & $\angle ABC$

ii) **Opposite Angles :** Angles with no common side.
E.g. $\angle ABC$ & $\angle CDA, \angle BCD$ & $\angle DAB$

• **Diagonals:** Line segments joining the vertices of opposite angles.
E.g. AC & BD

How to name a Quadrilateral?

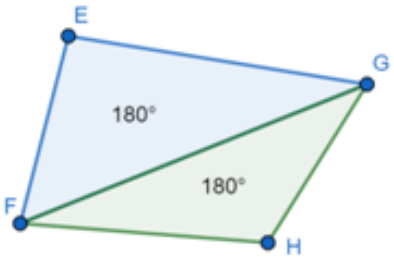
• A Quadrilateral can be named by starting at any vertex and going serially either **clockwise** or **anti-clockwise** around the figure.

E.g.: □ ABCD, □ BCDA, □ CDAB, □ DABC are names of the same quadrilateral shown above.

• While writing the name of a quadrilateral, □ this symbol is used instead of the word quadrilateral for ease of writing.

The sum of all angles of a quadrilateral is always 360°.

• A quadrilateral consists of two triangles.



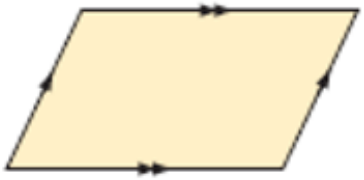
• Sum of all angles of these triangles is 180°.

• Therefore, the **total sum of all angles of the quadrilateral is 360°.**

Types of Quadrilaterals

There are six types of special quadrilaterals.

1. A **parallelogram** is a quadrilateral which has opposites sides parallel.



Properties:

• opposite sides are equal in length.

• opposite angles are equal in size.

• diagonals bisect each other.

2. A **rectangle** is a parallelogram with four equal angles of 90°.

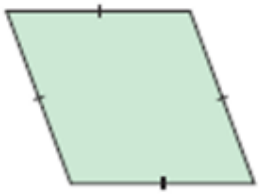
Properties:

• opposite sides are parallel and equal.

• diagonals bisect each other.

• diagonals are equal in length.

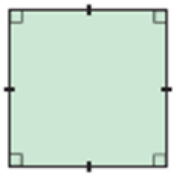
3. A **rhombus** is a parallelogram in which all sides are equal in length.



Properties:

- opposite sides are parallel.
- opposite angles are equal in size.
- diagonals bisect each other at right angles.
- diagonals bisect the angles at each vertex.

4. A **square** is a rhombus with four equal angles of 90°.

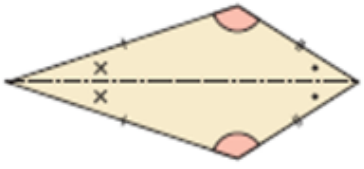


Properties:

- opposite sides are parallel.
- all sides are equal in length.
- diagonals bisect each other at right angles.
- diagonals bisect the angles at each vertex.
- diagonals are equal in length.

5. A **trapezium** is a quadrilateral which has a pair of parallel opposite sides.

6. A **kite** is a quadrilateral which has two pairs of adjacent sides equal in length.



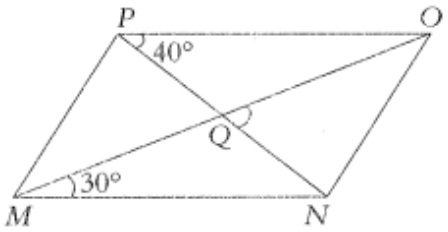
Properties:

- Longer diagonal is an axis of symmetry.
- One pair of opposite angles are equal.
- diagonals cut each other at right angles.

- Longer diagonal bisects one pair of angles at the vertices.
- The longer diagonal bisects the shorter diagonal.

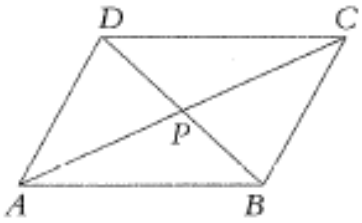
Practice Work:

1. In the given figure, $MNOP$ is a parallelogram, diagonals MO and PN intersect at Q, $\angle OPQ = 40°$ and $\angle OMN = 30°$. Find $\angle OQN$.



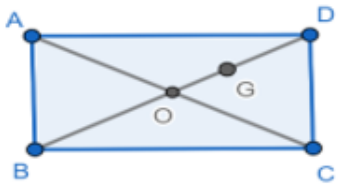
2. $ABCD$ is a parallelogram. If $\angle A + \angle C = 120°$, then $\angle B + \angle D =$ ..... .

3. In the given figure, $ABCD$ is a parallelogram, $AC = 14$ cm and $BD = 10$ cm, then $AP + BP =$ ..... cm.

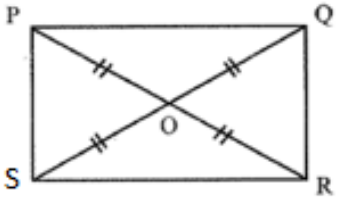


4. Measures of consecutive angles of a quadrilateral are in the ratio $3 : 4 : 5 : 6$. Find the largest angle.

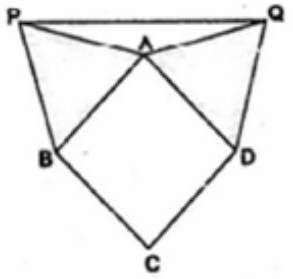
5. In the given figure, $ABCD$ is a rectangle and G is the centroid of $\triangle ADC$. If $BG = 4$ cm, then find the length of AC.



6. In rectangle PQRS, PR and QS intersect each other at O. If m$\angle$PSO $= 55°$ then m$\angle$SOR $=$?



7. As shown in the figure, □ ABCD is a square. △PAB and $\triangle QAD$ are equilateral triangles. Find the measure of $\angle AQP$.



8. One pair of opposite angles of a parallelogram is $(2x - 50°, x + 20°)$.
Then the parallelogram necessarily is ..... .

(a) A rhombus (b) A square (c) A rectangle (d) A trapezium

9. The adjacent angles of a rhombus are $2x - 35°$ and $x + 5°$. Find x.

(a) 70° (b) 40° (c) 35° (d) 45°

10. One angle of a parallelogram is 30° more than twice its adjacent angles. Find the measure of its adjacent angle.

(a) 50° (b) 60° (c) 70° (d) 80°

Answer Key:

Practice Work:

Que no.	Answer
1	70
2	240
3	12
4	120
5	6
6	110
7	15
8	c
9	70
10	50

Topic 16

ANGLE CHASING

Angle Chasing

In the earlier topic of Basic Geometry, we learnt and revised some concepts related to

- Point, Line, Segment, Ray,
- Intersecting Lines and Angles formed by them,
- Parallel Lines and angles formed by them,
- Types of Triangles
- Types of Quadrilaterals
- Types of Polygons

In this topic, we will be applying all these concepts to find the values of angles in various geometric figures. While doing so, we will also prove some theorems already learnt in the previous topic.

Practice Work:

1. List the names of all Polygons with $3 - 10$ sides inclusive.

2. Decide if each of the following is true or false.

(a) The angle sum of a quadrilateral is $300°$.

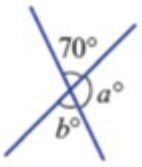
(b) A square has 4 lines of symmetry.

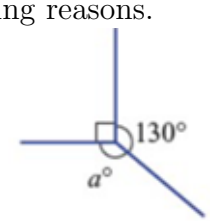
(c) An isosceles triangle has two equal sides.

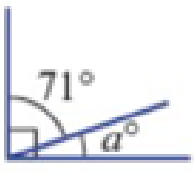
(d) An exterior angle on an equilateral triangle is $120°$.

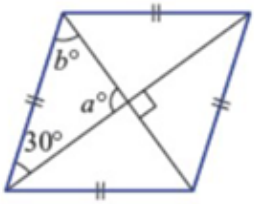
(e) A kite has two pairs of equal opposite angles.

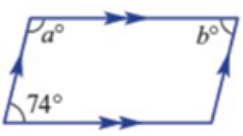
3. Find the values of the pronumerals giving reasons.

a) 

b) 

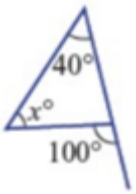
c) 

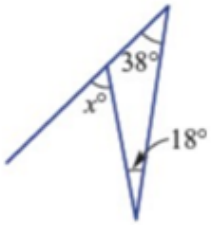
d) 

e) 

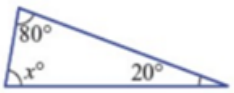
4. Using the exterior angle theorem, find the value of the pronumeral.

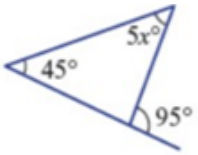
a)

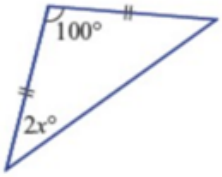
b) 

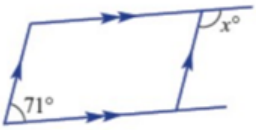
c) 

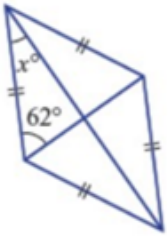
5. Find the value of the pronumeral, giving reasons.

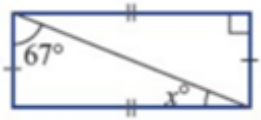
a) 

b) 

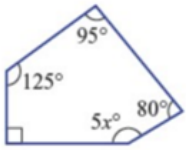
c) 

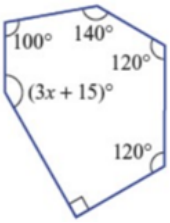
d) 

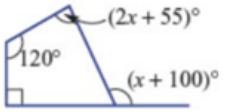
e) 

f) 

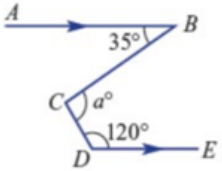
6. Find the value of x in the following, giving reasons.

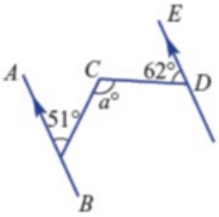
a) 

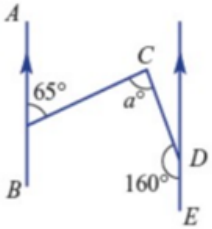
b) 

c) 

7. Find the value of the pronumeral a, giving reasons.

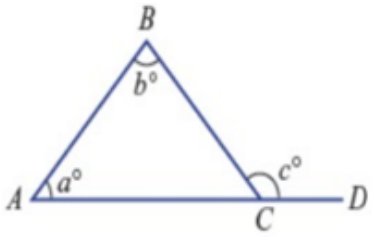
a) 

b) 

c) 

8. Find the value of x in this diagram giving reasons.
Hint: Form isosceles and/or equilateral triangles.

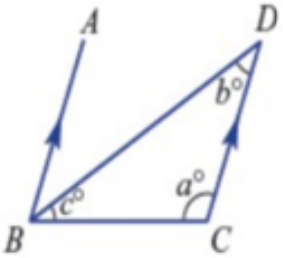
9. Prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles by following these steps.



a) Write $\angle BCA$ in terms of a and b and give a reason.

b) Find c in terms of a and b using $\angle BCA$ and give a reason.

10. a) Explain why in this diagram $\angle ABD$ is equal to $b°$.



b) Using $\angle ABC$ and $\angle BCD$ what can be said about a, b and c ?

c) What does your answer to part b show?

Answer Key:

Practice Work:

Que no.	Answer
1	16
2	3
3	48
4	10
5	65
6	
7	24
8	1440
9	360
10	61

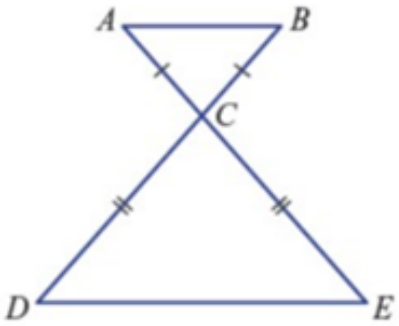
Topic 17

ANGLE CHASING

Angle Chasing

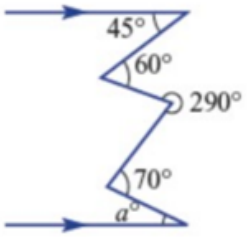
Practice Work:

1. Give reasons why AB and DE in this diagram are parallel, i.e. $AB \| DE$.

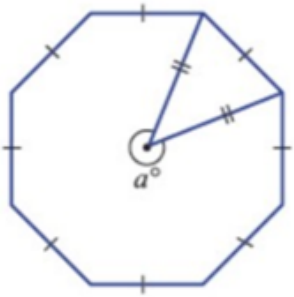


2. Find the value of the pronumerals giving reasons.

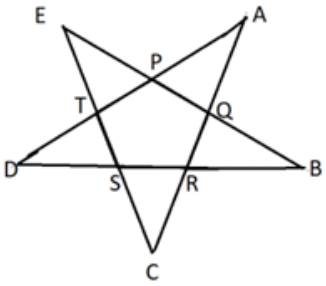
a)



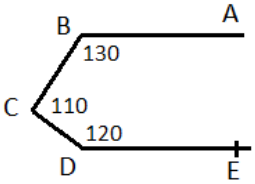
b)



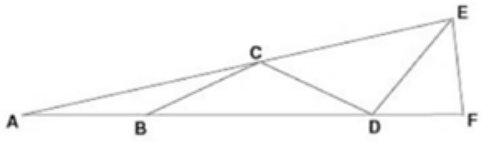
3. PQRST is a regular pentagon. Find $\angle A + \angle B + \angle C + \angle D + \angle E$.



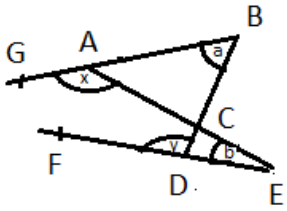
4. Prove that $\overleftrightarrow{AB} \| \overleftrightarrow{DE}$



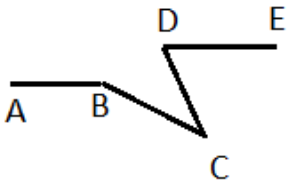
5. In the given diagram, $AB = BC = CD = DEm\angle EAF = 12.5°$. Find $m\angle EDF$



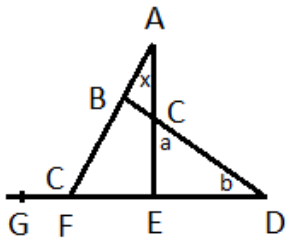
6. Prove that $x - y = a - b$



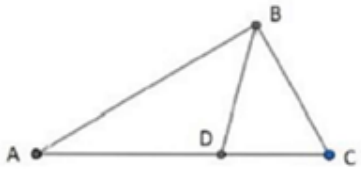
7. If $\overleftrightarrow{AB} || \overleftrightarrow{DE}$. Prove that $m\angle ABC + m\angle CDE = 180 + m\angle BCD$.



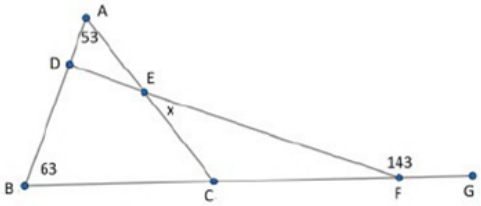
8. Express x interms of a, b, c.



9. In triangle ABC, BD bisects angle B. If $m\angle C = \frac{2}{3}m\angle B$ and $m\angle B = 3m\angle A$ then $m\angle BDC$ is



10. Find x in following figure.



Answer Key:

Practice Work:

Que no.	Answer
1	
2	(a) 15° (b) 315°
3	180°
4	180°
5	50°
6	
7	
8	$x = c - a - b$
9	75°
10	27°

Topic 18

SQUARE AND SQUARE ROOTS

• A number multiplied by itself is a **square** of the number.

Eg: 5×5 is read as 'square of 5' or '5 square'; written as '5^2'.

• The number of which the square is found is called as the **'square root'**. It is read as 'square root of 25'; written as $\sqrt{25}$.

• Note that the product of 2 negative numbers is a positive number.

So, square root can be negative, but square number is always positive.

• The value of the square root of any number is expressed as $\sqrt{25} = \pm 5$ and read as 'Square root of 25 is 'plus or minus' 5'.

• Natural numbers that have natural numbers as square roots are called **'perfect squares'**.

For eg: 25 is a perfect square of $5; 26$ is not a perfect square.

• Squares of numbers from 1 to 10.

Square root	Square	Square root	Square
1	1	6	36
2	4	7	49
3	9	8	64
4	16	9	81
5	25	10	100

• A square number never has $2, 3, 7, 8$ in its unit's place.

• Unit's place of square number, determines the unit's place of square root.

Square ends with	Square root ends with	Square ends with	Square root ends with
1	1,9	6	4,6
4	2,8	25	5
9	3,7	00	0

Examples:

(1) If $11^2 = 121, 110^2 = ?$

(2) Can 100000 be a square number? Why?

(3) Is $15^2 = 10^2 + 5^2$?

Quick methods for finding squares:

Square of number ending with '1':

• The last digit is always 1.

• Multiply the whole number by the digit in the tens' place and add the digit in the tens' place to the product.

• Write this number before 1 to get the square of the number

Eg; $21^2 = (21 \times 2) + 2 = 44$ before $1 = 441$

Square of number ending with '5 ':

• The square of a number ending with 5 always ends with 25.

• Multiply the digit in the tens' place by the next consecutive number.

• Place it before 25, to get the square number.

Eg. $25^2 = (2 \times 3)25 = 6$ before $25 = 625$.

Square of any 2-digit number, say 'ab' :

where 'a' is the digit in the tens' place and 'b' is the digit in the unit's place.

• Step 1 : Find b^2.

- If it is a single digit, write it in the unit's place of the final answer.
- If it has more than one digits, write the unit's place digit of the number in the unit's place of the final answer and carry forward the rest.

• Step 2: Find $a \times b$. Multiply it by 2 . Add the carry over of step 1 (if any) - Write the unit's place digit of this value in the tens' place of the final answer and carry forward the rest.

• Step 3: Find a^2. Add the carry- over of step 2.

• Write it before the tens' place of the final answer.

This is the square of the two- digit number 'ab'

Eg: $(34)^2$

• Step 1: $4^2 = 16$
'6 written in the unit's place of the final answer. '1' carried forward to step 2.

• Step 2: $3 \times 4 = 12 \times 2 = 24 + 1$ (carried forward) $= 25$
'5' written in the tens' place of the final answer. '2' carried forward to step 3

• Step 3 := $3^2 = 9 + 2$ (carried forward) $= 11$
11 written before '56' in the final answer.
Final answer = 1156.

We can easily prove WHY THIS WORKS a little later!!

Examples: Find the squares of :

(1) 35 (2) 66 (3) 31 (4) 7.2 (5) 41/45

Methods to find the square root of a given number

1) Factorisation Method:

• Prime factorise the given number.

• Divide the factors in identical two sets.

• The product of each set is the square root of the given number.

Example: Find $\sqrt{3}6$ by factorization method:

2) Approximation method:

• Identify the squares of two consecutive multiples of 10 between which the given number falls.

• Approximate the square root, using the unit's place digit.

• Confirm by calculation.

Example: Find $\sqrt{1444}$ by approximation method:

Operations on Numbers in Square root:

Addition and Subtraction:

If a and b are two numbers, $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$ and $\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$

For eg:
$\sqrt{4} + \sqrt{9}$
$\sqrt{9} - \sqrt{4}$

Multiplication and Division

If a and b are two numbers, $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

For eg:
$\sqrt{4} \times \sqrt{9}$

Practice Work:

1. Find the square root by factorisation method -

(a) 1089 (b) 1764

2. Find the square root by approximation method -

(a) 3136 (b) 2704

3. Find the length of the side of a square, whose area is 441 m^2.

4. Find the square root of $6\dfrac{43}{361}$

5. Find the square roots of the following decimal numbers.

(a) 2.56 (b) 7.29 (c) 51.84

6. The students of Class VIII donated Rs. 2401 to the Prime Minister's National Relief

Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

7. If $x = 18,\ y = 16$, find the value of $\sqrt{3+\sqrt{2x}} + \sqrt{4y}$

8. Find the smallest number dividing 4375 to give a perfect square.

9. Four times a number multiplied to that number gives 196. Find the number.

10. How many four digit square numbers have 4, 5, 6 in their units place?

Answer Key:

Practice Work:

Que no.	Answer
1	(a) 33 (b) 42
2	(a) 56 (b) 52
3	21
4	$\frac{47}{19}$
5	(a) 1.6 (b) 2.7 (c) 7.2
6	49
7	11
8	7
9	7
10	35 numbers

Topic 19

SQUARE AND SQUARE ROOTS - 2

Converting a non-square number to a square number

• A non-square number can be converted to a square number by adding or removing a factor & making the factors as identical sets.

Example:
(1) Convert 18 into the next possible square number

(2) Convert 72 into the previous possible square number

(3) Convert 500 into the next possible square number

Simplification :

Note that: $\frac{\mathbf{a}}{\sqrt{\mathbf{a}}} = \frac{\sqrt{\mathbf{a}}\times\sqrt{\mathbf{a}}}{\sqrt{\mathbf{a}}} = \sqrt{\mathbf{a}};\ \mathbf{a}\times\sqrt{\mathbf{b}} = \mathbf{a}\sqrt{\mathbf{b}}$

Practice Work:

1. $\sqrt{\frac{x}{169}} = \frac{54}{39}$. Find the value of '$x$'.

2. $\sqrt{176 + \sqrt{2401}}$ =?

3. $\sqrt{98\sqrt{2 + \sqrt{7 - \sqrt{13 - \sqrt{16}}}}}$ =?

4. If the area of a square is $\frac{384}{1014}$ m^2, then find the length of the side of the square.

(a) $\frac{48}{78}m$ (b) $\frac{12}{15}$ m (c) $\frac{7}{13}$ m (d) $\frac{8}{13}$ m

5. Find the value of:

(i) $\frac{\sqrt{80}}{\sqrt{405}}$ (ii) $\frac{\sqrt{441}}{\sqrt{625}}$

6. $\frac{\sqrt{1296}}{x} = \frac{x}{2.25}$; Find the value of x.

7. Find the difference between $\sqrt{25 + 144}$ and $\sqrt{25} + \sqrt{144}$.

8. $\frac{4.9}{(0.7)^2} - \frac{0.25}{(0.5)^2}$ =?

9. The number of whole numbers between $\sqrt{3}$ and $\sqrt{290}$ is .....

10. Evaluate: $\frac{1 + \sqrt{2} + \sqrt{4} + \sqrt{8}}{1 + \sqrt{2}} + \frac{1+\sqrt{3}+\sqrt{9}+\sqrt{27}}{1+\sqrt{3}}$. **Answer Key:**

Practice Work:

Que no.	**Answer**
1	324
2	15
3	14
4	$\frac{8}{13}$
5	(a) $\frac{4}{9}$ (b) $\frac{21}{25}$
6	9
7	4
8	9
9	16
10	7

Topic 20

SURDS - 1

What are Surds ?

• Irrational root of a rational number is called a Surd.

• In general words, a Surd is a **non-terminating & non-recurring decimal.**

• If **a** is a rational number, and **n** is a positive integer, any irrational number of the form $\sqrt[n]{\mathbf{a}}$ will be referred to as a surd.
E.g.- $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{10}$, etc.

• $\sqrt[n]{a}$ = Surd, where '**n**' = Order of the Surd,
. $\sqrt[n]{\ }$ = Radical sign,
. '**a**' = Radicand, which is always a rational number.

Types of Surds:

Surds can be classified based on -

1. Order of the Surd

i) **Quadratic Surd:** Surds of order '2' i.e., Surd with 'n'= 2

ii) **Cubic Surd:** Surds of order '3' i.e., Surd with 'n'= 3

iii) **Biquadratic Surd:** Surds of order '4' i.e., Surd with 'n'= 4

2. Number of Terms

i) **Monomial or Simple Surd:** Surds containing only one term.

ii) **Binomial Surd:** Surds containing two terms in addition / subtraction.

iii) **Trinomial Surd:** Surds containing three terms in addition / subtraction.

3. Type of Terms

i) **Like Surds:** Surds containing the same common Surds factor.

ii) **Unlike Surds:** Surds which are not Like Surds are called Unlike Surds.

iii) **Mixed Surds:** Surds containing a rational factor other than 1 along with the irrational factor.

iv) **Pure Surds:** Surds containing unity or 1 as the rational factor.

Exceptions to Surds

• **All surds are irrational numbers, but all irrational numbers may not be surds.**

e.g., π is an irrational number as it is a non-terminating non-recurring decimal.
$\pi = 3.141592653\ldots$ is an irrational number between 3 and 4.

But according to its definition, a surd is an irrational root of a rational number and expressed as $\sqrt[n]{\mathbf{a}}$, where '**a**' is a rational number.

Hence $\pi \neq \sqrt[n]{\mathbf{a}}$, as π cannot be expressed as a rational number under a root sign.

Therefore π **is not a surd.**

• Similarly, '**e**' also known as **Euler's number** is an irrational number, but not a surd.

• Such irrational numbers, which are not surds are called **Transcendental numbers.**

Laws of Surds:

• Surds can be simplified by using following basic laws-

1. $(\sqrt[n]{a})^{\mathbf{n}} = \mathbf{a}$
2. $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$, **where 'm' is a positive integer.**
5. $\mathbf{b}\sqrt[n]{a} \pm \mathbf{c}\sqrt[n]{a} = (\mathbf{b} \pm \mathbf{c})\sqrt[n]{a}$
6. $\mathbf{p} \times (\mathbf{q}\sqrt[n]{a}) = \mathbf{pq}\sqrt[n]{a}$
7. $(\mathbf{a}\sqrt{\mathbf{b}})^{\mathbf{2}} = \mathbf{a^2} \times (\sqrt{\mathbf{b}})^{\mathbf{2}} = \mathbf{a^2 b}$
8. $\dfrac{\mathbf{b}}{\sqrt{\mathbf{a}}} = \frac{\mathbf{b}}{\sqrt{\mathbf{a}}} \times \frac{\sqrt{\mathbf{a}}}{\sqrt{\mathbf{a}}} = \frac{\mathbf{b}\sqrt{\mathbf{a}}}{\mathbf{a}}$
9. $\dfrac{\mathbf{1}}{\sqrt{\mathbf{a}} + \sqrt{\mathbf{b}}} = \frac{\mathbf{1}}{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}} \times \frac{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}}{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}} = \frac{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}}{(\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}})(\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}})} = \frac{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}}{\mathbf{a}-\mathbf{b}}$,

where $\sqrt{\mathbf{a}} - \sqrt{\mathbf{b}}$ is called conjugate of the given surd.

10. $\dfrac{\mathbf{1}}{\sqrt{\mathbf{a}} - \sqrt{\mathbf{b}}} = \frac{\mathbf{1}}{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}} \times \frac{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}}{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}} = \frac{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}}{(\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}})(\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}})} = \frac{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}}{\mathbf{a}-\mathbf{b}}$,

where $\sqrt{\mathbf{a}} + \sqrt{\mathbf{b}}$ is called conjugate of the given surd.

Simplifying Surds:

• To simplify a surd, first factorise it, find the highest perfect square and get its square root, then rewrite the surd as a product of this square root and the remaining factors. This is the **Lowest form** i.e., $\mathbf{b}\sqrt[n]{a}$ of the given surd.

Reversing the Process:

• To compare the value of two or more surds, we may need to reverse the process and express a surd in the form $\sqrt[n]{a}$ rather than the form $\mathbf{b}\sqrt[n]{a}$.

Exponential form of Surds:

• Surds can be expressed in the exponential form as $\sqrt[n]{a} = a^{\frac{1}{n}}$

Practice Work:

1. Choose the correct word from the words given in bold to make the sentence true.

(a) A number that cannot be expressed as a fraction is a **rational/irrational number.**

(b) A surd is an irrational number that uses a **root/square** symbol.

(c) The decimal representation of a surd is a **terminating/recurring/non-recurring** decimal.

(d) $\sqrt{25}$ is a **surd/rational number**.

2. Decide if these numbers are surds.

(a) $\sqrt{7}$ (b) $2\sqrt{11}$ (c) $2\sqrt{25}$ (d) $-5\sqrt{144}$

3. Write down the highest square factor of these numbers. For example, the highest square factor of 45 is 9.

(a) 20 (b) 18 (c) 125 (d) 24

4. Simplify the following surds.

(a) $\sqrt{12}$ (b) $\sqrt{45}$ (c) $\sqrt{24}$ (d) $\sqrt{48}$

(e) $\sqrt{128}$ (f) $\sqrt{360}$

5. Simplify the following.

(a) $2\sqrt{18}$ (b) $3\sqrt{20}$ (c) $4\sqrt{48}$ (d) $2\sqrt{63}$

(e) $\dfrac{\sqrt{24}}{4}$ (f) $\dfrac{\sqrt{54}}{12}$ (g) $\dfrac{\sqrt{80}}{20}$ (h) $\dfrac{\sqrt{99}}{18}$

(i) $\dfrac{2\sqrt{108}}{18}$ (j) $\dfrac{3\sqrt{147}}{14}$

6. Simplify the following.

(a) $\sqrt{\dfrac{8}{9}}$ (b) $\sqrt{\dfrac{12}{49}}$ (c) $\sqrt{\dfrac{18}{25}}$ (d) $\sqrt{\dfrac{11}{25}}$

(e) $\sqrt{\dfrac{15}{27}}$ (f) $\sqrt{\dfrac{27}{4}}$

7. Express these surds as a square root of a positive integer.

(a) $2\sqrt{3}$ (b) $4\sqrt{2}$ (c) $5\sqrt{2}$ (d) $3\sqrt{3}$

(e) $9\sqrt{10}$ (f) $5\sqrt{5}$

8. (a) List all the factors of 450 that are perfect squares.

(b) Now simplify $\sqrt{450}$ using the highest of these factors.

9. $\sqrt{x} = x^{1/2}; \sqrt{25} = 25^{1/2} = 5; x^{1/2} = 7$. Find x.

Answer Key:

Practice Work:

Que no.	Answer
1	(a) irrational (b) root (c) non-recurring (d) rational
2	(a) Yes (b) Yes (c) No (d) No
3	(a) 4 (b) 9 (c) 25 (d) 4
4	(a) $2\sqrt{3}$ (b) $3\sqrt{5}$ (c) $2\sqrt{6}$ (d) $4\sqrt{3}$ (e) $8\sqrt{2}$ (f) $6\sqrt{10}$
5	(a) $6\sqrt{2}$ (b) $6\sqrt{5}$ (c) $16\sqrt{3}$ (d) $6\sqrt{7}$ (e) $\frac{\sqrt{3}}{\sqrt{2}}$ (f) $\frac{\sqrt{3}}{2\sqrt{2}}$ (g) $\frac{1}{\sqrt{5}}$ (h) $\frac{\sqrt{11}}{6}$ (i) $\frac{2}{\sqrt{3}}$ (j) $\frac{3\sqrt{3}}{2}$
6	(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{2\sqrt{3}}{7}$ (c) $\frac{3\sqrt{2}}{5}$ (d) $\frac{\sqrt{11}}{5}$ (e) $\frac{\sqrt{5}}{3}$ (f) $\frac{3\sqrt{3}}{2}$
7	(a) $\sqrt{12}$ (b) $\sqrt{32}$ (c) $\sqrt{50}$ (d) $\sqrt{27}$ (e) $\sqrt{810}$ (f) $\sqrt{125}$
8	(a) 9, 25, 225 (b) $15\sqrt{2}$
9	49

Topic 21

SURDS - 2

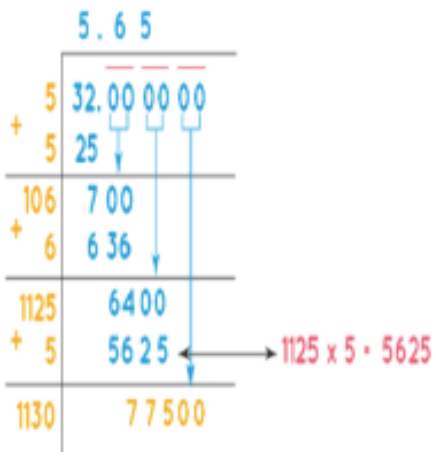
Surds on a Number Line:

Surds are irrational numbers hence can be located on the number line by following two methods -

1) Long Division Method-

Long division is a method for dividingl arge numbers into steps or parts, breaking the division problem into a sequence of easier steps. We can find the exact square root of any given number using this method.

e.g., Let's find square root of 32 by this method -



• Step 1: Place a bar over 32. We also pair the 0s in decimals in pairs of 2 from left to right.

• Step 2: Find a number such that when you multiply it with itself, the product is less than or equal to 32.

We know that 5×5 is 25 and is less than 32. Now let us divide 32 by 5.

• Step 3: Let us place a decimal point and pairs of zeros and continue our division.

Now, add the divisor 5 with itself and the sum becomes the starting digit of our next divisor.

• Step 4: Choose a number in the unit's place for the new divisor such that its product with the same number is less than or equal to 700. If we put a 6 in the one's place, our product is less than 700. We get $106 \times 6 = 636$.

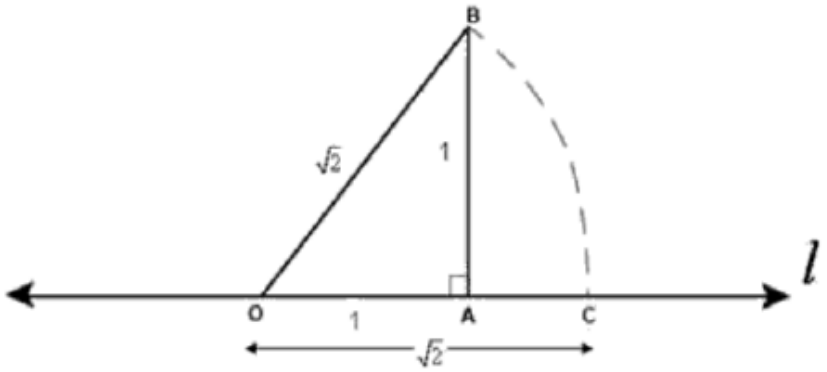
• We continue doing the same steps till we get the number of decimals we require.

2) Using Pythagoras Theorem -

Square roots of surds can be found out by using Pythagoras theorem and then located on the number line.

a) e.g., Let's find square root of 2 by this method -

Let 'l' be the number line with the zero point O as origin. On this line cut off $OA = 1$ unit. At A draw perpendicular to number line 'l' and on it cut off $AB = 1$ unit. Then $\triangle OAB$ is a right triangle, right angled at A.



By Pythagoras theorem, we have:

$OB^2 = OA^2 + AB^2 = 1^2 + 1^2 = 2 \Rightarrow OB = \sqrt{2}$

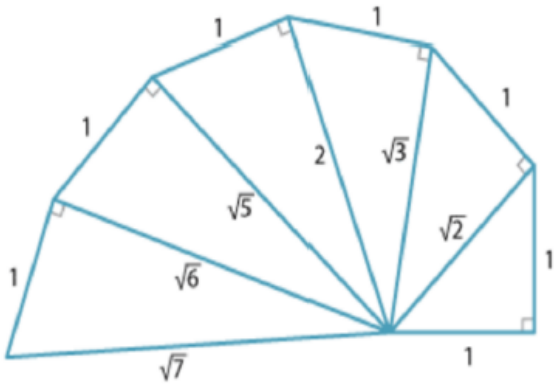
With O as centre and radius OB, draw an arc of the circle meeting the line 'l' at C.

Then $OC = OB$(Radii of circle)

$\Rightarrow OC = \sqrt{2}$

Hence, the point 'C' on the number line corresponds to the irrational number $\sqrt{2}$ i.e. OC = $\sqrt{2}$.

b) e.g., Pythagoras theorem can be used to find square roots of certain surds in a spiral form as shown -



In this method, the hypotenuse obtained for the previous surd ($\sqrt{2}$) is used as a base and the required height is drawn perpendicular to it for the next surd ($\sqrt{3}$), followed by the next hypotenuse that equals to 2 as shown in the figure. Since every new surd obtained thus is used as a base, the operation continues in a spiral fashion hence the name.

Practice Work:

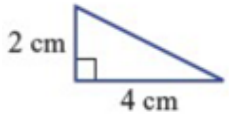
1. Determine the exact side length in simplest form of a square with the given area.

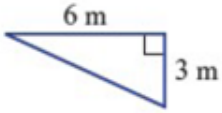
(a) 32 m^2 (b) 120 cm^2 (c) 240 mm^2

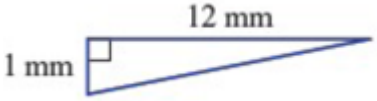
2. Determine the exact radius and diameter of a circle in simplest form with the given area.

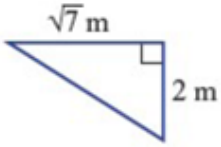
(a) 24π cm^2 (b) 54π m^2 (c) 128π m^2

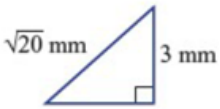
3. Use Pythagoras' theorem to find the unknown length in these triangles in simplest form

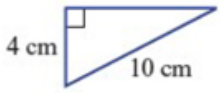
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

4. Express each number as a decimal and decide if it is rational or irrational. Then locate all the numbers on the same number line.

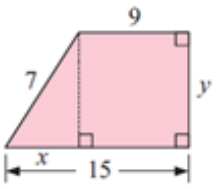
(a) $1\frac{5}{7}$ (b) $-\sqrt{2}$ (c) $2\sqrt{3}$ (d) π

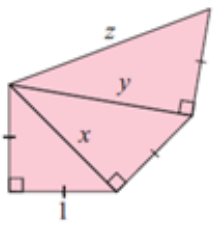
5. Use Pythagoras' theorem to construct a line segment with the given lengths. You can only use a ruler and a set square or compasses. Do not use a calculator.

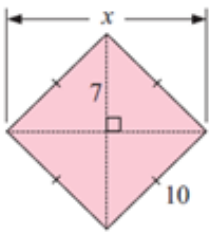
(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{10}$ (d) $\sqrt{22}$

6. Number of whole natural numbers between $\sqrt[3]{7}$ and $\sqrt[3]{344}$ is

7. Find the unknown lengths-

(a) 

(b) 

(c) 

8. $\sqrt{200}$ lies between consecutive natural numbers m and $m+1$. $\sqrt{300}$ lies between consecutive natural numbers n and $n+1$. $\sqrt{500}$ lies between consecutive natural numbers k and $k+1$. Find $m+n+k$. **Answer Key:**

Practice Work:

Que no.	Answer
1	(a) $4\sqrt{2}$ (b) $2\sqrt{30}$ (c) $4\sqrt{15}$
2	(a) $r = 2\sqrt{6}; D = 4\sqrt{6}$ (b) $r = 3\sqrt{6}; D = 6\sqrt{6}$ (c) r = $8\sqrt{2}$; D = $16\sqrt{2}$
3	(a) $2\sqrt{5}$ (b) $3\sqrt{5}$ (c) $\sqrt{145}$ (d) $\sqrt{11}$ (e) $\sqrt{11}$ (f) $2\sqrt{21}$
4	(a) 1.714285 Rational (b) -1.41421 Irrational (c) 3.4640 Irrational (d) 3.142857 Irrational
5	(a) 2.23606 (b) 2.44948 (c) 3.16227 (d) 4.69041
6	6
7	(a) $x = 6; y = \sqrt{13}$ (b) $x = \sqrt{2}; y = \sqrt{3}; z = 2$ (c) $x = 2\sqrt{51}$
8	53

Topic 22

SURDS - 3

Operations on Surds:

Addition & Substraction:

• Unless they are Like Surds, we cannot add or subtract surds.

$\sqrt{a}+\sqrt{a}=2\sqrt{a}$ but $\sqrt{a}+\sqrt{b}\neq\sqrt{a+b}$

• When two or more monomials are added / subtracted, they need to be in their lowest form to determine if they are Like surds or Unlike surds.

• If they are **Like surds**, they **can be added / subtracted**, hence the resulting surd is also a Monomial.

• If they are **Unlike surds**, they **cannot be added / subtracted**, hence the resulting surd is a Binomial or Trinomial or Polynomial.

Surds in Fraction form:

Addition and Subtraction:

• To add / subtract Surds in fraction form, we first need to find the **Lowest Common Denominator (LCD)** of the terms in the given surd. Then we can solve them in the same way as we solve fractions.

Practice Work:

1. Decide if the following pairs of numbers are Like Surds.

(a) $\sqrt{3}, 2\sqrt{3}$ (b) $5, \sqrt{5}$ (c) $-2\sqrt{5}, 3\sqrt{5}$ (d) $-\sqrt{7}, -2\sqrt{7}$

2. Find $\sqrt{\sqrt{16}+\sqrt[3]{125}}$.

3. (a) Simplify the surd $\sqrt{48}$.

(b) Hence simplify the following.

(i) $\sqrt{3}+\sqrt{48}$ (ii) $\sqrt{48}-7\sqrt{3}$ (iii) $5\sqrt{48}-3\sqrt{3}$

4. Simplify the following

(a) $2\sqrt{5}+4\sqrt{5}$ (b) $5\sqrt{3}-2\sqrt{3}$ (c) $7\sqrt{5}+4\sqrt{5}$ (d) $6\sqrt{3}-5\sqrt{3}$
(e) $\sqrt{21}-5\sqrt{21}+2\sqrt{21}$ (f) $3\sqrt{11}-8\sqrt{11}-\sqrt{11}$

5. Simplify the following

(a) $2\sqrt{3}+3\sqrt{2}-\sqrt{3}+2\sqrt{2}$ (b) $5\sqrt{6}+4\sqrt{11}-2\sqrt{6}+3\sqrt{11}$
(c) $2\sqrt{3}+2\sqrt{7}+2\sqrt{3}-2\sqrt{7}$ (d) $5\sqrt{11}+3\sqrt{6}-3\sqrt{6}-5\sqrt{11}$

6. Simplify the following

(a) $\sqrt{8}-\sqrt{2}$ (b) $\sqrt{8}+3\sqrt{2}$ (c) $\sqrt{27}+\sqrt{3}$
(d) $3\sqrt{44}+2\sqrt{11}$ (e) $3\sqrt{8}-\sqrt{18}$ (f) $\sqrt{24}+\sqrt{54}$

7. Simplify the following

(a) $\sqrt{2}+\sqrt{50}+\sqrt{98}$ (b) $\sqrt{6}-2\sqrt{24}+3\sqrt{96}$
(c) $\sqrt{150}-\sqrt{96}-\sqrt{162}+\sqrt{72}$ (d) $\sqrt{12}+\sqrt{125}-\sqrt{50}+\sqrt{180}$
(e) $2\sqrt{200}+3\sqrt{125}+\sqrt{32}-3\sqrt{242}$

8. Simplify these surds that involve fractions. Remember to use the LCD (lowest common denominator).

(a) $\dfrac{\sqrt{3}}{2}+\frac{\sqrt{3}}{3}$ (b) $\dfrac{\sqrt{5}}{4}+\frac{\sqrt{5}}{3}$ (c) $\dfrac{\sqrt{2}}{5}-\frac{\sqrt{2}}{6}$
(d) $\dfrac{-5\sqrt{10}}{6}+\frac{3\sqrt{10}}{8}$ (e) $\sqrt{1\dfrac{9}{16}}-\sqrt{1\dfrac{7}{9}}$

Answer Key:

Practice Work:

Que no.	**Answer**
1	(a) Yes (b) No (c) Yes (d) Yes
2	3
3	(a) $4\sqrt{3}$ (b)(i) $5\sqrt{3}$ (b)(ii) $-3\sqrt{3}$ (b)(iii) $17\sqrt{3}$
4	(a) $6\sqrt{5}$ (b) $3\sqrt{3}$ (c) $11\sqrt{5}$ (d) $\sqrt{3}$ (e) $-2\sqrt{21}$ (f) $-6\sqrt{11}$
5	(a) $\sqrt{3}+5\sqrt{2}$ (b) $3\sqrt{6}+7\sqrt{11}$ (c) $4\sqrt{3}$ (d) 0
6	(a) $\sqrt{2}$ (b) $5\sqrt{2}$ (c) $4\sqrt{3}$ (d) $8\sqrt{11}$ (e) $3\sqrt{2}$ (f) $5\sqrt{6}$
7	(a) $13\sqrt{2}$ (b) $9\sqrt{6}$ (c) $\sqrt{6}-3\sqrt{2}$ (d) $2\sqrt{3}+11\sqrt{5}-5\sqrt{2}$ (e) $15\sqrt{5}-9\sqrt{2}$
8	(a) $\frac{5}{2\sqrt{3}}$ (b) $\frac{7\sqrt{5}}{12}$ (c) $\frac{1}{15\sqrt{2}}$ (d) $\frac{-11\sqrt{5}}{12\sqrt{2}}$ (e) $\frac{-1}{12}$

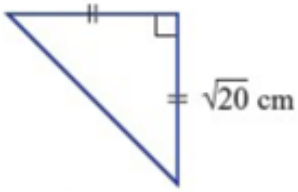
Topic 23

SURDS - 4

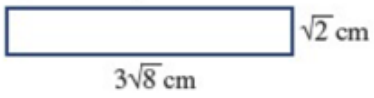
Practice Work:

1. Find the perimeter of these quadrilaterals and triangles in simplest form.

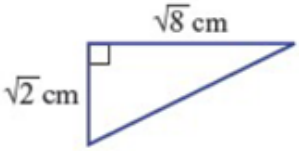
(a)



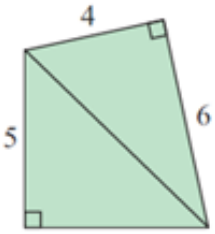
(b)



(c)



(d)



2. Explain why $\sqrt{5}$ and $\sqrt{20}$ can be thought of like surds.

3. Prove that each of the following simplifies to zero by showing all steps.

(a) $5\sqrt{3}-\sqrt{108}+\sqrt{3}$ (b) $\sqrt{6}+\sqrt{24}-3\sqrt{6}$ (c) $3\sqrt{2}-2\sqrt{27}-\sqrt{50}+6\sqrt{3}+\sqrt{8}$

4. Prove that the surds in these expressions cannot be added or subtracted.

(a) $3\sqrt{12}-\sqrt{18}$ (b) $4\sqrt{8}+\sqrt{20}$ (c) $\sqrt{50}-2\sqrt{45}$

5. To simplify the following you will need to simplify surds and combine using a common denominator.

(a) $\dfrac{\sqrt{8}}{3}-\frac{\sqrt{2}}{5}$ (b) $\dfrac{\sqrt{12}}{4}+\frac{\sqrt{3}}{6}$ (c) $\dfrac{3\sqrt{5}}{4}-\frac{\sqrt{20}}{3}$

(d) $\dfrac{5\sqrt{48}}{6}+\frac{2\sqrt{147}}{3}$ (e) $\dfrac{2\sqrt{96}}{5}-\frac{\sqrt{600}}{7}$ (f) $\dfrac{3\sqrt{125}}{14}-\frac{2\sqrt{80}}{21}$

6. Simplify the following surds -

(a) $\sqrt{2\dfrac{14}{25}}-\sqrt{1\dfrac{7}{9}}$ (b) $\dfrac{\sqrt{48}-\sqrt{27}+\sqrt{75}}{\sqrt{98}-\sqrt{8}+\sqrt{2}}$ (c) $\dfrac{\sqrt{10800}+\sqrt{16200}+\sqrt{27000}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

(d) $\dfrac{3\sqrt{150}-3\sqrt{24}+2\sqrt{54}}{\sqrt{90}}$ (e) $\dfrac{\sqrt{72}}{\sqrt{36}}+\frac{3}{\sqrt{2}}=?$

7. An equilateral triangle has an altitude of length 10 cm. Find the length of a side.

8. Find the value of

(a) $\dfrac{\sqrt{3}+\sqrt{27}+\sqrt{75}+\sqrt{147}+\sqrt{243}+\sqrt{363}}{3\sqrt{3}}$ (b) $\dfrac{\sqrt[3]{81}+\sqrt[3]{-192}+\sqrt[3]{375}}{\sqrt[3]{24}}$

Answer Key:

Practice Work:

Que no.	**Answer**
1	(a) $4\sqrt{5}+2\sqrt{10}$ (b) $14\sqrt{2}$ (c) $3\sqrt{2}+\sqrt{10}$ (d) $15+3\sqrt{3}$
2	Same radicand
3	-
4	(a) $6\sqrt{3}-3\sqrt{2}$ (b) $8\sqrt{2}+2\sqrt{5}$ (c) $5\sqrt{2}-6\sqrt{5}$
5	(a) $\dfrac{7\sqrt{2}}{15}$ (b) $\dfrac{2}{\sqrt{3}}$ (c) $\dfrac{\sqrt{5}}{12}$ (d) $8\sqrt{3}$ (e) $\dfrac{6\sqrt{6}}{35}$ (f) $\dfrac{29\sqrt{5}}{42}$
6	(a) $\dfrac{4}{15}$ (b) $\dfrac{\sqrt{3}}{\sqrt{2}}$ (c) $30\sqrt{6}$ (d) $\sqrt{15}$ (e) $\dfrac{5}{\sqrt{2}}$
7	$\dfrac{20}{\sqrt{3}}$
8	(a) 12 (b) 2

Topic 24

SURDS - 5

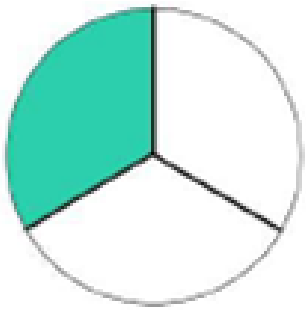
Operations on Surds:

Multiplication and Division:

- Surds can be multiplied or divided using following Laws of Surds -
- $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ (Multiplication)
- $\mathbf{p} \times (\mathbf{q}\sqrt[n]{a}) = \mathbf{pq}\sqrt[n]{a}$ (Multiplication)
- $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ (Division)
- Surds in fraction form can be multiplied / divided in the same way as in fractions.
- If the given surds are monomials, they can be simply multiplied/divided, and the resulting surd will be a monomial.

Distributive Law in Surds

- To find the product of a monomial and a binomial.



Practice Work:

1. Copy and complete.

(a) $\sqrt{15} \div \sqrt{3} = \sqrt{\frac{15}{\_\_\_}} = \sqrt{\_\_\_}$ (b) $\sqrt{42} \div \sqrt{7} = \sqrt{\frac{42}{\_\_\_}} = \sqrt{\_\_\_}$

(c) $\sqrt{6} \times \sqrt{5} = \sqrt{6 \times \_\_} = \sqrt{\_\_}$ (d) $\sqrt{11} \times \sqrt{2} = \sqrt{11 \times \_\_} = \sqrt{\_\_}$

2. Simplify the following.

(a) $\sqrt{3} \times \sqrt{5}$ (b) $\sqrt{11} \times \sqrt{3}$ (c) $\sqrt{2} \times \sqrt{13}$ (d) $-\sqrt{3} \times -\sqrt{2}$

3. Simplify the following.

(a) $\sqrt{20} \div \sqrt{2}$ (b) $\sqrt{18} \div \sqrt{3}$ (c) $\sqrt{33} \div -\sqrt{11}$

(d) $-\dfrac{\sqrt{26}}{\sqrt{2}}$

(e) $-\dfrac{\sqrt{50}}{\sqrt{10}}$

4. Simplify the following.

(a) $\sqrt{7} \times \sqrt{3}$ (b) $\sqrt{2} \times \sqrt{5}$ (c) $\sqrt{10} \times \sqrt{3}$

(d) $\sqrt{14} \times \sqrt{7}$ (e) $\sqrt{2} \times \sqrt{22}$ (f) $\sqrt{3} \times \sqrt{18}$

5. Simplify the following.

(a) $2\sqrt{5} \times \sqrt{15}$ (b) $3\sqrt{7} \times \sqrt{14}$ (c) $4\sqrt{6} \times \sqrt{21}$

(d) $3\sqrt{14} \times 2\sqrt{21}$ (e) $-4\sqrt{6} \times 5\sqrt{15}$ (f) $2\sqrt{10} \times -2\sqrt{25}$

6. Simplify the following.

(a) $\dfrac{6\sqrt{14}}{3\sqrt{7}}$ (b) $\dfrac{15\sqrt{12}}{5\sqrt{2}}$ (c) $\dfrac{4\sqrt{30}}{8\sqrt{6}}$

7. Use the distributive law to expand the following and simplify the surds where necessary.

(a) $\sqrt{3}(\sqrt{2} + \sqrt{5})$ (b) $\sqrt{2}(\sqrt{7} - \sqrt{5})$ (c) $3\sqrt{2}(2\sqrt{13} - \sqrt{11})$

(d) $4\sqrt{5}(\sqrt{5} - \sqrt{10})$ (e) $-2\sqrt{8}(2\sqrt{2} - 3\sqrt{20})$ (f) $2\sqrt{3}(7\sqrt{6} + 5\sqrt{3})$

8. Simplify the following.

(a) $(2\sqrt{7})^2$ (b) $(-3\sqrt{2})^2$ (c) $-(5\sqrt{3})^2$

(d) $\sqrt{24} - 2\sqrt{2}(\sqrt{3} - 4)$ (e) $2\sqrt{3}(\sqrt{6} - \sqrt{3}) - \sqrt{50}$

9. Which among the following is true.

(a) $6 < \sqrt{35} < \sqrt[3]{217}$ (b) $\sqrt[3]{217} < 6 < \sqrt{35}$

(c) $\sqrt{35} < 6 < \sqrt[3]{217}$ (d) $\sqrt{35} < \sqrt[3]{217} < 6$

Answer Key:

Practice Work:

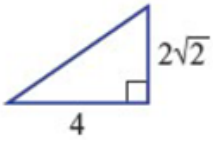
Que no.	**Answer**
1	(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{30}$ (d) $\sqrt{22}$
2	(a) $\sqrt{15}$ (b) $\sqrt{33}$ (c) $\sqrt{26}$ (d) $\sqrt{6}$
3	(a) $\sqrt{10}$ (b) $\sqrt{6}$ (c) $-\sqrt{3}$ (d) $-\sqrt{13}$ (e) $-\sqrt{5}$
4	(a) $\sqrt{21}$ (b) $\sqrt{10}$ (c) $\sqrt{30}$ (d) $7\sqrt{2}$ (e) $2\sqrt{11}$ (f) $3\sqrt{6}$
5	(a) $10\sqrt{3}$ (b) $21\sqrt{2}$ (c) $12\sqrt{14}$ (d) $42\sqrt{6}$ (e) $-60\sqrt{10}$ (f) $-20\sqrt{10}$
6	(a) $2\sqrt{2}$ (b) $3\sqrt{6}$ (c) $\frac{\sqrt{5}}{2}$
7	(a) $\sqrt{6}+\sqrt{15}$ (b) $\sqrt{14}-\sqrt{10}$ (c) $6\sqrt{26}-3\sqrt{22}$ (d) $20-20\sqrt{2}$ (e) $-16+24\sqrt{10}$ (f) $42\sqrt{2}+30$
8	(a) 28 (b) 18 (c) -75 (d) $8\sqrt{2}$ (e) $\sqrt{2}-6$
9	$\sqrt{35} < \sqrt{6} < \sqrt[3]{217}$

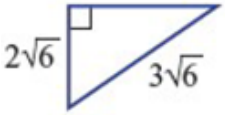
Topic 25

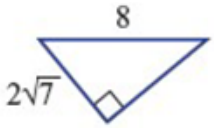
SURDS - 6

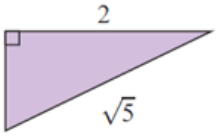
Claaswork:

1. Determine the unknown side of the following right-angled triangles.

(a) 

(b) 

(c) 

(d) 

(e)

2. (a) The perimeter of a square is $2\sqrt{3}$ cm. Find its area.

(b) Find the length of a diagonal of a square if its area is 12 cm^2.

3. $\sqrt{8} \times \sqrt{27}$ could be simplified in two ways as shown.

Method A

$$\begin{aligned}\sqrt{8}\times\sqrt{27} &= \sqrt{4\times2}\times\sqrt{9\times3}\\ &= 2\sqrt{2}\times3\sqrt{3}\\ &= 2\times3\times\sqrt{2\times3}\\ &= 6\sqrt{6}\end{aligned}$$

Method B

$$\begin{aligned}\sqrt{8}\times\sqrt{27} &= \sqrt{8\times27}\\ &= \sqrt{216}\\ &= \sqrt{36\times6}\\ &= 6\sqrt{6}\end{aligned}$$

(a) Describe the first step in method A.

(b) Why is it useful to simplify surds before multiplying, as in method A?

(c) Multiply by first simplifying each surd.

i. $\sqrt{18} \times \sqrt{27}$ ii. $\sqrt{24} \times \sqrt{20}$ iii $\sqrt{98} \times \sqrt{300}$ iv $2\sqrt{72} \times 3\sqrt{80}$

4. $\dfrac{\sqrt{12}}{\sqrt{3}}$ could be simplified in two ways.

Method A

$$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}}$$
$$= \sqrt{4}$$
$$= 2$$

Method B

$$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{4\times 3}{3}}$$
$$= \frac{2\sqrt{3}}{\sqrt{3}}$$
$$= 2$$

Use method B to simplify these surds.

(a) $-\dfrac{2\sqrt{2}}{5\sqrt{8}}$ (b) $\dfrac{2\sqrt{45}}{15\sqrt{5}}$ (c) $\dfrac{5\sqrt{27}}{\sqrt{75}}$

5. Look at this example before simplifying the following.

$(2\sqrt{3})^3 = 2^3(\sqrt{3})^3$
$= 2 \times 2 \times 2 \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}$
$= 8 \times 3 \times \sqrt{3}$
$= 24\sqrt{3}$

(a) $(3\sqrt{2})^3$ (b) $(5\sqrt{3})^3$ (c) $(-\sqrt{3})^4$ (d) $(2\sqrt{2})^5$

(e) $5(2\sqrt{3})^4$ (f) $\dfrac{(2\sqrt{7})^3}{4}$ (g) $\dfrac{(5\sqrt{2})^2}{4} \times \frac{(2\sqrt{3})^3}{3}$ (h) $\dfrac{(2\sqrt{3})^2}{9} \times \frac{(-3\sqrt{2})^4}{3}$

(i) $\dfrac{(2\sqrt{5})^4}{50} \div \frac{(2\sqrt{3})^3}{5}$ (j) $\dfrac{(2\sqrt{2})^3}{9} \div \frac{(2\sqrt{8})^2}{(\sqrt{27})^3}$

6. How many positive integers are there in the set
$A = \{\sqrt{1}, \sqrt{2}, \sqrt{3}, \cdots, \sqrt{2019}, \sqrt{2020}\}$

7. A shed has dimensions, in metres, of height $\sqrt{5}$, width $\sqrt{6}$ and length $\sqrt{10}$ Find the volume of the shed.

8. (a) If $a = \sqrt{2} + 1$ what is the value of $\left(1 + \dfrac{1}{2 + \dfrac{1}{a}}\right)$

(b) If $\dfrac{5}{K} = \dfrac{1}{\sqrt{0.0025}}$ then $K =$

i. 0.5 ii. 0.25 iii. 0.01 iv. 5

Answer Key:

Practice Work:

Que no.	Answer
1	(a) $2\sqrt{6}$ (b) $\sqrt{30}$ (c) 6 (d) 1 (e) 2
2	(a) $\frac{3}{4}$ (b) $2\sqrt{6}$
3	(c) i. $9\sqrt{6}$ (c) ii. $4\sqrt{30}$ (c) iii. $70\sqrt{6}$ (c) iv. $144\sqrt{10}$
4	(a) $\frac{-1}{5}$ (b) $\frac{2}{5}$ (c) 3
5	(a) $54\sqrt{2}$ (b) $375\sqrt{3}$ (c) 9 (d) $128\sqrt{2}$ (e) 720 (f) $14\sqrt{7}$ (g) $100\sqrt{3}$ (h) 144 (i) $\frac{5}{3\sqrt{3}}$ (j) $\frac{9\sqrt{3}}{\sqrt{2}}$
6	44
7	$10\sqrt{3}$
8	(a) $\frac{3\sqrt{2}+4}{2\sqrt{2}+3}$ (b) 0.25

Topic 26

Binomial Product in Surds

Surds of the **same order** can be multiplied according to the following law-
$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

• **To find the product of a monomial and a binomial -**

$$\mathbf{a\,(b+c) = ab + ac}$$

e.g., i) $2\sqrt{3}(4+3\sqrt{3}) = 2\sqrt{3} \times 4 + 2\sqrt{3} \times 3\sqrt{3} = 8\sqrt{3} + 18$

ii) $4\sqrt{5}(9+2\sqrt{3}) = 4\sqrt{5} \times 9 + 4\sqrt{5} \times 2\sqrt{3} = 36\sqrt{5} + 8\sqrt{15}$

• **To find the product of two binomial terms -**

$$\mathbf{(a+b)(c+d) = ac + ad + bc + bd}$$

e.g., $(3\sqrt{2} - 4\sqrt{3})(5\sqrt{3} - \sqrt{2}) = 3\sqrt{2} \times 5\sqrt{3} - 3\sqrt{2} \times \sqrt{2} - 4\sqrt{3} \times 5\sqrt{3} + 4\sqrt{3} \times \sqrt{2}$
$= 15\sqrt{6} - 6 - 60 + 4\sqrt{6} = 19\sqrt{6} - 66$

• **To find the square of the given binomial -**

The surd can be rewritten as a product of two binomials and then simplified using distributive law -

$\mathbf{(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2}$

$\mathbf{(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2}$

e.g., i) $(5\sqrt{2} + 3\sqrt{3})^2 = (5\sqrt{2} + 3\sqrt{3})(5\sqrt{2} + 3\sqrt{3})$

. $= (5\sqrt{2})^2 + 2 \times 5\sqrt{2} \times 3\sqrt{3} + (3\sqrt{3})^2$

. $= 50 + 30\sqrt{6} + 27 = 77 + 30\sqrt{6}$

ii) $(3\sqrt{5} - 4\sqrt{7})^2 = (3\sqrt{5} - 4\sqrt{7})(3\sqrt{5} - 4\sqrt{7})$

. $= (3\sqrt{5})^2 - 2 \times (3\sqrt{5})(4\sqrt{7}) + (4\sqrt{7})^2$

. $= 45 - 24\sqrt{35} + 112 = 157 - 24\sqrt{35}$

• **To find the difference of two squares -**

$\mathbf{(a+b)(a-b) = a^2 - ab + ab + b^2 = a^2 - b^2}$

e.g., i) $(\sqrt{11} + \sqrt{3})(\sqrt{11} - \sqrt{3}) = (\sqrt{11})^2 - (\sqrt{3})^2 = 11 - 3 = 8$

ii) $(7\sqrt{2} + 4\sqrt{5})(7\sqrt{2} - 4\sqrt{5}) = (7\sqrt{2})^2 - (4\sqrt{5})^2 = 98 - 80 = 18$

Practice Work:

1. Simplify the following.

(a) $\sqrt{3} \times \sqrt{7}$ (b) $-\sqrt{2} \times \sqrt{5}$ (c) $2\sqrt{3} \times 3\sqrt{2}$

(d) $(\sqrt{11})^2$ (e) $(\sqrt{13})^2$ (f) $(2\sqrt{3})^2$

(g) $(5\sqrt{5})^2$ (h) $(7\sqrt{3})^2$ (i) $(9\sqrt{2})^2$

2. Expand and simplify.

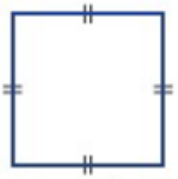
(a) $(2+\sqrt{2})(\sqrt{2}-3)$ (b) $(4+\sqrt{5})(\sqrt{5}-2)$ (c) $(\sqrt{6}+2)(\sqrt{6}-1)$

(d) $(5-\sqrt{3})(2+\sqrt{3})$ (e) $(3+\sqrt{7})(4-\sqrt{7})$ (f) $(\sqrt{2}-5)(3+\sqrt{2})$

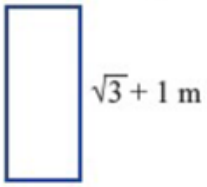
(g) $(4\sqrt{3}-5)(2-3\sqrt{3})$ (h) $(1-5\sqrt{2})(3-4\sqrt{2})$ (i) $(4\sqrt{5}-3)(3-4\sqrt{5})$

3. Find the area of these rectangles and triangles in expanded and simplified form.

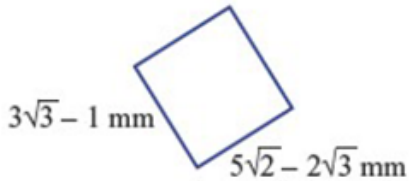
(a)



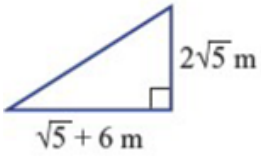
(b)



(c)



(d)



4. Expand and simplify these difference of perfect squares.

(a) $(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})$ (b) $(\sqrt{11}+\sqrt{5})(\sqrt{11}-\sqrt{5})$ (c) $(\sqrt{3}-\sqrt{7})(\sqrt{3}+\sqrt{7})$

5. Fully expand and simplify these surds.

(a) $(2\sqrt{3}-\sqrt{2})^2+(\sqrt{3}+\sqrt{2})^2$ (b) $(\sqrt{5}-\sqrt{3})^2+(\sqrt{5}+\sqrt{3})^2$

(c) $(\sqrt{3}-4\sqrt{5})(\sqrt{3}+4\sqrt{5})-(\sqrt{3}-\sqrt{5})^2$ (d) $-10\sqrt{3}-(2\sqrt{3}-5)^2$

6. Expand and simplify these perfect squares.

(a) $(\sqrt{7}+\sqrt{2})^2$ (b) $(\sqrt{11}-\sqrt{2})^2$ (c) $(\sqrt{10}-\sqrt{3})^2$

(d) $(\sqrt{13}+\sqrt{19})^2$ (e) $(\sqrt{17}+\sqrt{23})^2$ (f) $(\sqrt{31}-\sqrt{29})^2$

7. Let $K=(2\sqrt{3}-\sqrt{6})(2\sqrt{6}+\sqrt{12})$. Find the value of K^2.

Answer Key:

Practice Work:

Que no.	Answer
1	(a) $\sqrt{21}$ (b) $-\sqrt{10}$ (c) $6\sqrt{6}$ (d) 11 (e) 13 (f) 12 (g) 125 (h) 147 (i) 162
2	(a) $-4-\sqrt{2}$ (b) $-3+2\sqrt{5}$ (c) $4+\sqrt{6}$ (d) $7+3\sqrt{3}$ (e) $5+\sqrt{7}$ (f) $-13-2\sqrt{2}$ (g) $-46+23\sqrt{3}$ (h) $43-19\sqrt{2}$ (i) $-89+24\sqrt{5}$
3	(a) $7+4\sqrt{3}$ (b) 2 (c) $15\sqrt{6}-5\sqrt{2}+2\sqrt{3}-18$ (d) $5+6\sqrt{5}$
4	(a) 3 (b) 6 (c) -4
5	(a) $19-2\sqrt{6}$ (b) 16 (c) $-85+2\sqrt{15}$ (d) $-37+10\sqrt{3}$
6	(a) $9+2\sqrt{14}$ (b) $13-2\sqrt{22}$ (c) $13-2\sqrt{30}$ (d) $32+2\sqrt{247}$ (e) $40+2\sqrt{391}$ (f) $60-2\sqrt{899}$
7	72

Topic 27

Rationalising Denominator

The process of removing the surd/ irrational number from the denominator of a fraction is traditionally called **Rationalising the Denominator.** This means converting the surd/ irrational number in the denominator to a rational number.

Rationalising is done mainly for fractions with surds, which have a single term or two terms in the denominator.

Denominator with a single term –

• Multiply the numerator and the denominator by the same surd / irrational number. The denominator obtained is a rational number.

• $\frac{\mathbf{b}}{\sqrt{\mathbf{a}}} = \frac{\mathbf{b}}{\sqrt{\mathbf{a}}} \times \frac{\sqrt{\mathbf{a}}}{\sqrt{\mathbf{a}}} = \frac{\mathbf{b}\sqrt{\mathbf{a}}}{\mathbf{a}}$ - (Refer 8th Law of Surds)

e.g., i) $\frac{\sqrt{48}}{\sqrt{5}} = \frac{\sqrt{16\times3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{15}}{5}$ ii) $\frac{5\sqrt{3}}{2\sqrt{2}} = \frac{5\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{6}}{4}$

Denominator with two terms -

• Multiply both the numerator and the denominator by the **conjugate** of the denominator. The denominator eventually obtained is a rational number.

• $\frac{\mathbf{1}}{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}} = \frac{\mathbf{1}}{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}} \times \frac{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}}{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}} = \frac{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}}{(\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}})(\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}})} = \frac{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}}{\mathbf{a}-\mathbf{b}}$,

where $\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}$ is called **conjugate** of the given surd - (9th Law of Surds)

• $\frac{\mathbf{1}}{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}} = \frac{\mathbf{1}}{\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}}} \times \frac{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}}{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}} = \frac{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}}{(\sqrt{\mathbf{a}}-\sqrt{\mathbf{b}})(\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}})} = \frac{\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}}{\mathbf{a}-\mathbf{b}}$,

where $\sqrt{\mathbf{a}}+\sqrt{\mathbf{b}}$ is called **conjugate** of the given surd - (10 th Law of Surds)

e.g.,

$\frac{3\sqrt{2}}{3+\sqrt{5}} = \frac{3\sqrt{2}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3\sqrt{2}\times3-3\sqrt{2\times5}}{(3)^2-(\sqrt{5})^2} = \frac{9\sqrt{2}-3\sqrt{10}}{9-5} = \frac{9\sqrt{2}-3\sqrt{10}}{4}$

Practice Work:

1. Simplify the following-

(a) $\frac{\sqrt{6}}{\sqrt{6}}$ (b) $\frac{\sqrt{11}}{\sqrt{11}}$ (c) $\frac{\sqrt{72}}{\sqrt{2}}$ (d) $-\frac{3\sqrt{45}}{9\sqrt{5}}$

2. Rationalise the denominators-

(a) $\frac{5}{\sqrt{3}}$ (b) $\frac{8}{\sqrt{2}}$ (c) $\frac{\sqrt{5}}{\sqrt{3}}$ (d) $\frac{\sqrt{2}}{\sqrt{7}}$

(e) $\frac{3\sqrt{6}}{\sqrt{7}}$ (f) $\frac{7\sqrt{3}}{\sqrt{10}}$ (g) $\frac{2\sqrt{7}}{\sqrt{15}}$

3. Explain why multiplying a number by $\frac{\sqrt{x}}{\sqrt{x}}$ does not change its value.

4. Rewrite in the form $\frac{\sqrt{a}}{\sqrt{b}}$ and then rationalise the denominators.

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{5}{7}}$ (c) $\sqrt{\frac{6}{11}}$ (d) $\sqrt{\frac{2}{5}}$

5. Rationalise the denominators-

(a) $\frac{2\sqrt{7}}{3\sqrt{35}}$ (b) $\frac{5\sqrt{12}}{3\sqrt{27}}$ (c) $\frac{9\sqrt{6}}{2\sqrt{3}}$ (d) $\frac{7\sqrt{90}}{2\sqrt{70}}$

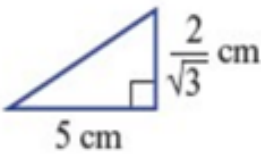
6. Rationalise the denominators-

(a) $\frac{1+\sqrt{2}}{\sqrt{3}}$ (b) $\frac{3+\sqrt{5}}{\sqrt{7}}$ (c) $\frac{2-\sqrt{3}}{\sqrt{5}}$ (d) $\frac{\sqrt{3}-\sqrt{5}}{\sqrt{2}}$

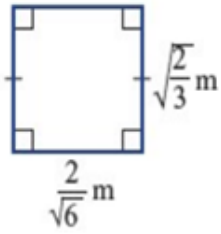
(e) $\frac{3\sqrt{5}+5\sqrt{2}}{\sqrt{10}}$ (f) $\frac{3\sqrt{10}+5\sqrt{3}}{\sqrt{2}}$

7. Determine the exact value of the area of the following shapes. Express your answers using a rational denominator.

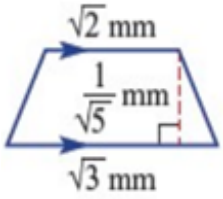
(a)



(b)



(c)



8. Simplify the following by first rationalising denominators and then using a common denominator.

(a) $\frac{3}{\sqrt{5}} + \frac{1}{\sqrt{2}}$ (b) $\frac{5}{2\sqrt{3}} - \frac{2}{3\sqrt{2}}$ (c) $\frac{10\sqrt{6}}{3\sqrt{5}} + \frac{4\sqrt{2}}{3\sqrt{3}}$ (d) $\frac{5\sqrt{2}}{3\sqrt{5}} - \frac{4\sqrt{7}}{3\sqrt{6}}$

Answer Key:

Practice Work:

Que no.	Answer
1	(a) 1 (b) 1 (c) 6 (d) -1
2	(a) $\frac{5\sqrt{3}}{3}$ (b) $4\sqrt{2}$ (c) $\frac{\sqrt{15}}{3}$ (d) $\frac{\sqrt{14}}{7}$ (e) $\frac{3\sqrt{42}}{7}$ (f) $\frac{7\sqrt{30}}{10}$ (g) $\frac{2\sqrt{105}}{15}$
3	-
4	(a) $\frac{\sqrt{6}}{3}$ (b) $\frac{\sqrt{35}}{7}$ (c) $\frac{\sqrt{66}}{11}$ (d) $\frac{\sqrt{10}}{5}$
5	(a) $\frac{2\sqrt{5}}{15}$ (b) $\frac{10}{9}$ (c) $\frac{9\sqrt{2}}{2}$ (d) $\frac{3\sqrt{7}}{2}$
6	(a) $\frac{\sqrt{3}+\sqrt{6}}{3}$ (b) $\frac{3\sqrt{7}+\sqrt{35}}{7}$ (c) $\frac{2\sqrt{5}-\sqrt{15}}{5}$ (d) $\frac{\sqrt{6}-\sqrt{10}}{2}$ (e) $\frac{3\sqrt{2}+2\sqrt{5}}{2}$ (f) $\frac{6\sqrt{5}+5\sqrt{6}}{2}$
7	(a) $\frac{5\sqrt{3}}{3}$ (b) $\frac{2}{3}$ (c) $\frac{\sqrt{10}+\sqrt{15}}{10}$
8	(a) $\frac{6\sqrt{5}+5\sqrt{2}}{10}$ (b) $\frac{5\sqrt{3}-2\sqrt{2}}{6}$ (c) $\frac{6\sqrt{30}+4\sqrt{6}}{9}$ (d) $\frac{3\sqrt{10}-2\sqrt{42}}{9}$