M. Prakash Institute **23 April 2023**
Solutions - Entrance Test **10am-12noon**
3 Year foundation program **Paper type: AB**

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1. Find the value of $20 \times \left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)^{-1}$?

   Solution: $20 \times \left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)^{-1} = 20 \times \left(\frac{5+2+1}{10}\right)^{-1} = 20 \times \left(\frac{10}{8}\right) = 25$
   Ans. 25.

2. Sum of five consecutive natural numbers is 35. Find the sum of next five consecutive natural numbers.

   Solution: Each of the next five numbers is five more than the corresponding number of the first five. For example, if the first five numbers are $7, 8, 9, 10, 11$ then the next five numbers are $12, 13, 14, 15, 16$. Now $12 - 7 = 5, 13 - 8 = 5$, etc. Which means the sum will increase by 5×5. **Ans. 60.**

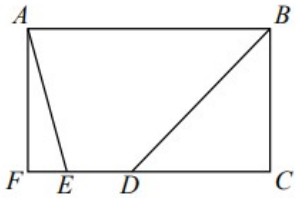
3. Find the smallest natural number to be added to 2023 so that we get a perfect square.

   Solution: Clearly, the nearest square is $2025 = 45^2$. **Ans. 2.**

4. Points A, B, C, and D are on a line in that order. The distance from A to D is 24 . The distance from B to D is 3 times the distance from A to B. Point C is halfway between B and D. What is the distance from A to C ?

   Solution: If distance between A and B is x then $BD = 3x$ and $AD = 24 \Rightarrow 4x = 24 \Rightarrow x = 6$. So, $BD = 18,\ BC = 9 \Rightarrow AC = 6 + 9 = 15$ **Ans. 15.**

5. In the diagram, $ABCF$ is a rectangle with $AB = 30$ and $AF = 14$. Points E and D are on FC so that $FE = 5$ and the area of quadrilateral $ABDE$ is 266. The length of DC is

   Solution: $Area(\square ABCD) = AB \times AF = 420$. $Area(\triangle AFE) = \frac{1}{2}AF \times FE = \frac{1}{2}14 \times 5 = 35$. So, $Area(\triangle BDC) = 420 - 266 - 35 = 129 \Rightarrow \frac{1}{2}14 \times DC = 119 \Rightarrow DC = 17$ **Ans. 17.**

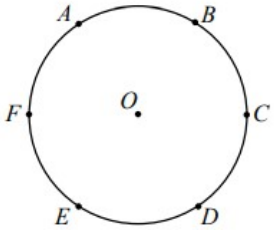
6. The average of a, b and c is 16 . The average of c, d and e is 26 . The average of a, b, c, d, and e is 20 . The value of c is

   Solution: Sum of $a, b, c = 3 \times 16 = 48$, sum of $c, d, e = 3 \times 26 = 78$ and sum of $a, b, c, d, e = 5 \times 20 = 100$ so, $c = 78 + 48 - 100 = 26$ **Ans. 26.**

7. A positive number is increased by 25%. By what percentage should the result be decreased to return to the original value?

   Solution: Suppose the original number is 100, so it becomes 125. Now 125 is decreased by 25 which is $\frac{25}{125} \times 100 = 20\%$ **Ans. 20.**

8. Points A, B, C, D, E, and F are evenly spaced around the circle with centre O, as shown. The measure of $\angle ACO$ is



Solution: Since the circle is divided in six equal parts, we have $m\angle AOC = 120°$. $\triangle AOC$ is isoceles, so its base angle is $\frac{180-120}{2} = 30°$ **Ans. 30.**

9. A rectangle has positive integer side lengths and an area of 24 . The perimeter of the rectangle cannot be
(If your answer is 20, mark 20 as your answer. If your answer is 28, mark 28 as your answer, etc.)
(A) 20 (B) 22 (C) 28 (D) 50 (E) 36

Solution: The possible sides and perimeters are given in the following table:

Side 1	Side 2	Perimeter
1	24	50
2	12	28
3	8	22
4	6	20

Ans. 36.

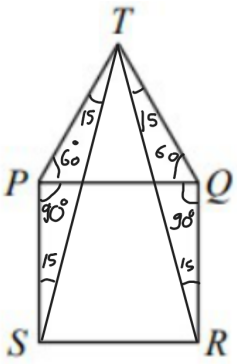
10. The operation $a\nabla b$ is defined by $a\nabla b = \frac{a+b}{a-b}$ for all integers a and b with $a \neq b$. For example, $2\nabla 3 = \frac{2+3}{2-3} = -5$. If $3\nabla b = -4$, what is the value of b ?

Solution: $3\nabla b = -4 \Rightarrow \frac{3+b}{3-b} = -4 \Rightarrow 3 + b = -12 + 4b \Rightarrow 3b = 15 \Rightarrow b = 5$
Ans. 5.

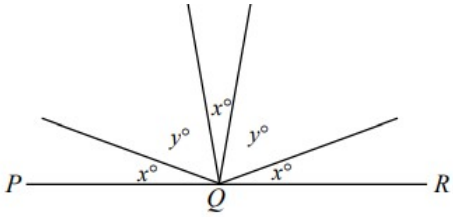
11. If x is 20% of y and x is 50% of z, then what percentage is z of y ?

Solution: $y = 5x$ and $z = 2x \Rightarrow z = \frac{2}{5}y$ **Ans. 40.**

12. In the diagram, pentagon TPSRQ is constructed from equilateral $\triangle PTQ$ and square $PQRS$. The measure of $\angle STR$ is equal to
Solution: In $\triangle TPS, TP = SP$ and $\angle TPS = 90 + 60 = 150°$, so $\angle PTS = \frac{180-150}{2} = 15$. Similarly $\angle QTR = 15$. Because $\triangle PQT$ is equilateral, $\angle PTQ = 60°$, so $\angle STR = 60° - 15 - 15 = 30°$ **Ans. 30.**



13. In the diagram, PQR is a straight line segment. If $x + y = 76$, what is the value of x?
Solution: $x + y + x + y + x = 180 \Rightarrow x + 2(x + y) = 180 \Rightarrow x + 2(76) = 180 \Rightarrow x = 28°$ **Ans. 28.**



14. If $3^x = 5$, the value of 3^{x+2} is

Solution: $3^{x+2} = (3^x)(3^2) = (3^x)(9) = 45$ **Ans. 45.**

15. A group of friends are sharing a bag of candy. On the first day, they eat $\frac{1}{2}$ of the candies in the bag. On the second day, they eat $\frac{2}{3}$ of the remaining candies. On the third day, they eat $\frac{3}{4}$ of the remaining candies. At the end of the third day, there is 1 candy remaining in the bag. How many candies were in the bag before the first day?

Solution: At the end of day 1, $\frac{1}{2}$ of candies remaining. On second day, $\frac{2}{3}$ of these are eaten and $\frac{1}{3}$ remain, i.e. $\frac{1}{3}$ of $\frac{1}{2}$ remain, i.e. $\frac{1}{6}$ remain. On day 3, $\frac{3}{4}$ of $\frac{1}{6}$ eaten, so $\frac{1}{4}$ of $\frac{1}{6}$ remain, i.e. $\frac{1}{24}$ candies remaining. This is one candy. So, total candies $= 24$. **Ans. 24.**

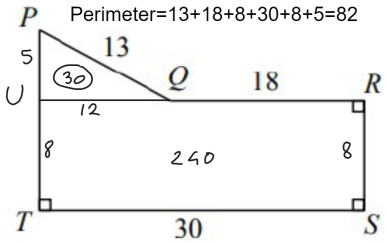
16. The variables a, b, c, d, e, and f represent the numbers $4, 12, 15, 27, 31$, and 39 in some order. Suppose that

$$a + b = c$$
$$b + c = d$$
$$c + e = f$$

The value of $a + c + f$ is

Solution: The pairs required are $12 + 15 = 27, 4 + 27 = 31$ and $27 + 12 = 39$. Of these, the number appearing thrice is c, so, $c = 27$. Number appearing twice is $b = 12$. So,
$a + b = c - - - > 15 + 12 = 27$
$b + c = d - - - > 12 + 27 = 39$
$c + e = f - - - > 27 + 4 = 31 \Rightarrow a = 15, b = 12, c = 27, d = 39, e = 4, f = 31$
Ans. 73.

17. In the diagram, pentagon $PQRST$ has $PQ = 13$, $QR = 18, ST = 30$, and area of the pentagon $PQRST$ is 270.. Also, $\angle QRS = \angle RST = \angle STP = 90°$. Find the perimeter of $PQRST$.



18. Simplify and find $\dfrac{\dfrac{3168}{13}}{3 - \dfrac{7}{13}} =$

Solution: $\dfrac{\dfrac{3168}{13}}{3 - \dfrac{7}{13}} = \dfrac{\dfrac{3168}{13}}{\dfrac{39-7}{13}} = \dfrac{\dfrac{3168}{13}}{\dfrac{32}{13}} = \dfrac{3168}{13} \times \dfrac{13}{32} = \dfrac{3168}{32} = 99$ **Ans. 99.**

19. Find the sum of all natural numbers between $\sqrt{37}$ and $\sqrt{120}$.

Solution: $36 < 37 < 49 \Rightarrow 6 < \sqrt{37} < 7$ and $100 < 120 < 121 \Rightarrow 10 < \sqrt{120} < 11$, so the natural numbers between $\sqrt{37}$ and $\sqrt{120}$ are $7, 8, 9, 10$ **Ans. 34.**

20. $\dfrac{\sqrt{507}+\sqrt{845}+\sqrt{1183}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} =$.

Solution: $\dfrac{\sqrt{507}+\sqrt{845}+\sqrt{1183}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} = \dfrac{\sqrt{169\times 3}+\sqrt{169\times 5}+\sqrt{169\times 7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}}$

$= 13\left(\dfrac{\sqrt{3}+\sqrt{5}+\sqrt{7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}}\right) = 13$ **Ans. 13.**

Key:

1	2	3	4	5	6	7	8	9	10
25	60	2	15	17	26	20	30	36	5
11	12	13	14	15	16	17	18	19	20
40	30	28	45	24	73	82	99	34	13