M. Prakash Institute **22 Jan 2023**
Solutions to Entrance Test **10am-12noon**
3 Year foundation program **Paper type: AB**

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1. Sum of seven consecutive odd natural numbers is 651. Find the largest number.

   Solution: Let the largets number be n. So, the other numbers are $n-12, n-10, n-8, n-6, n-4, n-2$. So, we have $7n - 42 = 651 \Rightarrow 7n = 693 \Rightarrow n = 99$. **Ans. 99.**

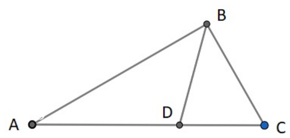
2. What is the smallest natural number with which if we multiply 2023, we get perfect square.

   Solution: $2023 = 7 \times 17 \times 17$. So, we need to multiply it by 7 to get a perfect square. **Ans. 7.**

3. Number of whole natural numbers between $\sqrt[3]{7}$ and $\sqrt[3]{344}$ is

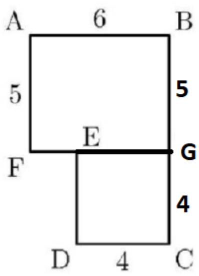
   Solution: Since $1 < 7 < 8$, we have $1 < \sqrt[3]{7} < 2$.
   Also, $343 < 344 < 512 \Rightarrow 7 < \sqrt[3]{344} < 8$. So, whole numbers between $\sqrt[3]{7}$ and $\sqrt[3]{344}$ are $2, 3, 4, 5, 6, 7$. **Ans. 6.**

4. In triangle ABC, BD bisects angle B. If $m\angle C = \frac{2}{3} m\angle B$ and $m\angle B = 3m\angle A$ then $m\angle BDC$ is

   Solution: $m\angle C = \frac{2}{3} m\angle B$ and $m\angle B = 3m\angle A$
   $\Rightarrow m\angle C = 2m\angle A$. Since sum of the angles in a triangle is $180°$, we get $3m\angle A + 2m\angle A + m\angle A = 180$
   $\Rightarrow m\angle A = 30, m\angle B = 90, m\angle C = 60 \Rightarrow m\angle DBC = 45°$
   $\Rightarrow m\angle BDC = 180 - (60 + 45) = 75°$. **Ans. 75.**

5. All angles of the polygon $ABCDEF$ are right angles. Find the area of the polygon $ABCDEF$.

   Solution: If you extend the line FE to intersect BC in G, then the figure gets divided in two parts, the top rectangle $\square ABGF$ of size 6×5 and bottom square $\square GCDE$ of side 4.
   So, total area $= 6 \times 5 + 4 \times 4 = 46$. **Ans. 46.**

6. If $a = -2$, the value of largest number in the set $\left\{-4a, 4a, \frac{24}{a}, a^2, 1\right\}$ is

   Solution: After substituting $a = -2$, we get the set as $\{8, -8, -12, 4, 1\}$ **Ans. 8.**

7. F is fraction halfway between $\frac{1}{5}$ and $\frac{1}{3}$ (on the number line). Find $105F$.

   Solution: $F = \dfrac{\frac{1}{5} + \frac{1}{3}}{2} = \dfrac{4}{15} \Rightarrow 105F = 28$. **Ans. 28.**

8. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are $6.2, 8.3$, and 9.5. The area of the square is

   Solution: Suppose the side of the square is x, so we get $4x = 6.2 + 8.3 + 9.5 = 24 \Rightarrow x = 6$ so, area of the square is $6^2 = 36$. **Ans. 36.**

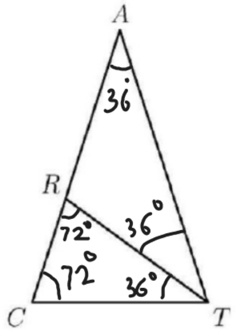
9. Simplify and find $\dfrac{95}{2-\dfrac{5}{12}} =$

Solution: $\dfrac{95}{2-\frac{5}{12}} = \dfrac{95}{\frac{2\times 12-5}{12}} = \dfrac{95\times 12}{19} = 60$. **Ans. 60.**

10. The number 64 has the property that it is divisible by its units digit. How many whole numbers between 10 and 50 have this property?

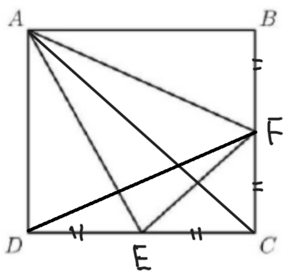
Solution: The numbers are $11, 12, 15, 21, 22, 24, 25, 31, 32, 33, 35, 36, 41, 42, 44, 45, 48$. **Ans. 17.**

11. In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36°$. $\overline{TR}$ bisects $\angle ATC$, If $CT = 29$ then find AR



Solution: Using sum of angles $= 180$, we get $\angle ACT = \angle ATC = 72$, so $\angle CTR = \angle ATR = 36$ which gives $\angle CRT = 72$. So, we get $CT = TR = RA$. **Ans. 29.**

12. The area of rectangle $ABCD$ is 72 . If point A and the midpoints of $\overline{BC}$ and $\overline{CD}$ are joined to form a triangle, the area of that triangle is



Solution: Clearly, area(ΔACD) $=$ area(ΔACB) $= 36$. Since base of ΔADE is DE, which is half of the base of ΔACD and they have the same height AD, area(ΔADE) $= \frac{1}{2}$ area(ΔACD) $= 18$. By the same logic, we can show that area(ΔFEC) $= \frac{1}{2}$ area(ΔDCF) $= 9$. So, area(ΔAEF) $=$ area($\Box ABCD$) $-$ area(ΔAED)$-$area(ΔABF)$-$area(ΔECF) $= 72-18-18-9 = 27$. **Ans. 27.**

13. For any positive integer n, define $\boxed{n}$ (n inside a square box) to be the sum of all positive factors of n. For example, $\boxed{6} = 1+2+3+6 = 12$. $K = \boxed{11}$ Find $\boxed{K}$.

Solution: $\boxed{11} = 12$. Factors of 12 are $1, 2, 3, 4, 6, 12$, so $\boxed{12} = 1+2+3+4+6+12 = 28$. **Ans. 28.**

14. The base of an isosceles $\triangle ABC$ is 24 and its area is 60 . What is the perimeter of $\triangle ABC$?

Solution: So, height of the triangle is $\dfrac{2\times 60}{24} = 5$. Suppose BC is the base. Suppose D is the midpoint of BC. Since the triangle is isoceles, $AD \perp BC$, so using Pythagoras theorem, we get $AC^2 = AD^2 + DC^2 = 5^2 + 12^2 = 169 \Rightarrow AB = AC = 13$, so perimeter$= 13+13+24 = 50$. **Ans. 50.**

15. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of 26640 is

Solution: $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $26640 = \dfrac{26640}{2\times 3\times 4\times 5\times 6} = 37$. **Ans. 37.**

16. If $25^{3-2x} = 5^{-6}$, find x.

Solution: $25 = 5^2$ so $25^{3-2x} = 5^{2(3-2x)} = 5^{6-4x}$ so, we have $6 - 4x = -6 \Rightarrow x = 3$. **Ans. 3.**

17. 50 ml of concentrated Kokam syrup is mixed with water for making a glass of 250 ml tasty Kokam Sharabat. How many liters of water is required to make 70 glasses of Kokam Sharabat.

Solution: Since one glass of 250 ml contains 50 ml of concentrated syrup, it contains 200 ml of water. So, 70 glasses need $70 \times 200 = 14000$ ml $= 14$ liters of water. **Ans. 14.**

18. $\dfrac{\sqrt{200}+\sqrt{300}}{\sqrt{8}+\sqrt{12}} =$

Solution: Observe that $\sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$. Similarly, $\sqrt{300} = 10\sqrt{3}$, $\sqrt{8} = 2\sqrt{2}$, $\sqrt{12} = 2\sqrt{3}$, so we have $\dfrac{\sqrt{200}+\sqrt{300}}{\sqrt{8}+\sqrt{12}} = \dfrac{10(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = 5$. **Ans. 5.**

19. If $\dfrac{3}{7}\left(1-\dfrac{7}{94}k\right)+\dfrac{1}{5}\left(1+\dfrac{7}{94}k\right)+\dfrac{2}{3}\left(1-\dfrac{7}{94}k\right) = 0$, then find the value of $\dfrac{7k}{2}$.

Solution: Let $\frac{7k}{94} = u$. So, we have, after transferring terms of u on one side, $\frac{3}{7}+\frac{1}{5}+\frac{2}{3} = (\frac{3}{7}-\frac{1}{5}+\frac{2}{3})u \Rightarrow u = \frac{136}{94} \Rightarrow \frac{7k}{94} = \frac{136}{94} \Rightarrow 7k = 136 \Rightarrow \frac{7k}{2} = \frac{136}{2} = 68$. **Ans. 68.**

20. R is a rational number. Instead of multiplying R by 3 and then subtracting 7, Rahul divided it by 3 and then added 7. Surprisingly he got the same answer. Report $4R$

Solution: We have $3R - 7 = \frac{R}{3} + 7 \Rightarrow 3R - \frac{R}{3} = 14 \Rightarrow \frac{8R}{3} = 14 \Rightarrow 4R = 21$. **Ans. 21.**