M. Prakash Institute **17 April 2022**

Solution - Entrance Test for 3 year foundation Program **7 - 8 pm**

1. Sum of two Natural numbers m and n is 5760 and difference between them is $\frac{1}{3}$ of the larger number. Find larger number.

   Solution: Suppose m is the larger number. So, smaller number is $5760 - m$. So we get $m - (5760 - m) = \frac{m}{3} \Rightarrow \frac{5}{3}m = 5760 \Rightarrow m = 3456$.

2. Find $\dfrac{26}{5} \times \dfrac{35}{13} \times \dfrac{337}{7} \times \dfrac{198}{66} =$.

   Solution: Simple question. All the numbers in the denominator get cancelled and we get 337×6 in the numerator. Answer is 2022.

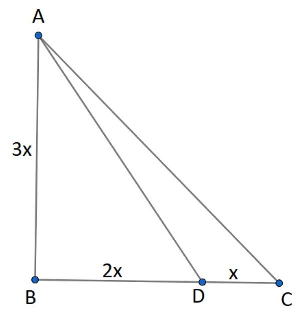
3. If $a : b = 7 : 3$, and $(a^2)(b^2) = 7056$, then $a - b = $?

   Solution: Let $a = 7k,\ b = 3k$, so $a^2b^2 = 49 \times 9 \times k^4 \Rightarrow 49 \times 9 \times k^4 = 7056$ $\Rightarrow k = 2 \Rightarrow a = 14,\ b = 6 \Rightarrow a - b = 8$.

4. $\sqrt{150}$ lies between natural numbers $m-1$ and m. $\sqrt{250}$ lies between natural numbers $n - 1$ and n. $\sqrt{600}$ lies between natural numbers $p - 1$ and p. Find $m + n + p$.

   Solution: We know $144 < 150 < 169$, so $m = 13$. Also, $225 < 250 < 256$, so $n = 16$. Similarly $576 < 600 < 625$, so $p = 25$. So, $m + n + p = 13 + 16 + 25 = 54$.

5. $\triangle ABC$ is right angled triangle as shown. $DC = x, DB = 2x, AB = 3x$, if $AC = 3\sqrt{26}$ find AD.



   Solution: Using Pythagoras theorem in $\triangle ABC$, we get $AC^2 = (3x)^2 + (3x)^2$, i.e. $234 = 18x^2 \Rightarrow x^2 = 13$.
   Using Pythagoras theorem in $\triangle ABD$, we get $AD^2 = (3x)^2 + (2x)^2 = 13x^2 = 169$, so $AD = 13$.

6. Let $A = 75\%$ of 60% of 40 and $B = 40\%$ of 120% of 50. Find $A + B$.

   Solution: $A = 0.75 \times 0.6 \times 40 = \frac{3}{4} \times \frac{3}{5} \times 40 = 18$. Similarly $B = 0.4 \times 1.20 \times 50 = 24$. So, $A + B = 18 + 24 = 42$

7. Let $\dfrac{m}{n} = 4$, Find $\dfrac{2m^2 + 8n^2}{m^2 - 6n^2}$.

   Solution: Let $m = 4n \Rightarrow \dfrac{2m^2 + 8n^2}{m^2 - 6n^2} = \dfrac{2(4n)^2 + 8n^2}{(4n)^2 - 6n^2} = \dfrac{32n^2 + 8n^2}{16n^2 - 6n^2} = \dfrac{40n^2}{10n^2} = 4.$

8. Find $\dfrac{\sqrt{5.29}+\sqrt{13.69}}{\sqrt{0.0001}\times\sqrt{0.36}}$.

Solution: $\dfrac{\sqrt{5.29}+\sqrt{13.69}}{\sqrt{0.0001}\times\sqrt{0.36}} = \dfrac{2.3+3.7}{0.01\times 0.6} = \dfrac{6}{0.006} = 1000.$

9. A number consists of 2 digits.The digit at unit's place is 3 times that in 10's place. If the digits are interchanged a new 2 digited number if formed. Let K be this new number. Also $K-15$ is equal to 2 times the original number. Find the original number.

Solution: Let the digit at ten's place be x. So, the digit at unit place is $3x$. So, the number is $10x+3x=13x$. So, new number looks like $(3x)(x)$, i.e. it is $10(3x)+x=31x$. So, we have $31x-15=2(13x) \Rightarrow x=3$, so the original number is 39.

10. On real number line distance between points with coordinates $\dfrac{13}{7}$ and $\dfrac{5}{3}$ is D_1 and distance between points with coordinates $-\dfrac{97}{7}$ and $-\dfrac{11}{21}$ is D_2. Find $\dfrac{D_2}{D_1}$.

Solution: $D_1 = \dfrac{13}{7}-\dfrac{5}{3} = \dfrac{(13)(3)-(5)(7)}{(7)(3)} = \dfrac{4}{21}$.
$D2 = -\dfrac{11}{21}-\left(-\dfrac{97}{7}\right) = \dfrac{(97)(3)-11}{21} = \dfrac{280}{21}$
$\Rightarrow \dfrac{D_2}{D_1} = \dfrac{280}{21} \div \dfrac{4}{21} = 70.$

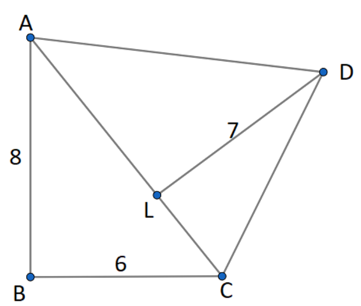
11. B has money equal to $\dfrac{3}{7}^{th}$ of A and C has money equal to $\dfrac{11}{3}^{th}$ of B's. In all, they have 2022 Rs. How much money does A have?

Solution: $B=\dfrac{3}{7}A$ and $C=\dfrac{11}{3}B=\dfrac{11}{3}\times\dfrac{3}{7}A=\dfrac{11}{7}A$
$\Rightarrow A+B+C=\left(\dfrac{3}{7}+\dfrac{11}{7}+1\right)A=3A$. So, we get $3A=2022 \Rightarrow A=674$.

12. Sum of 7 consecutive odd numbers is 133. If we ignore first and last, what is the sum of remaining five?

Solution: Let the first number be n. So, we get $(n)+(n+2)+(n+4)+(n+6)+(n+8)+(n+10)+(n+12)=133 \Rightarrow 7n+42=133 \Rightarrow n=13$. Sum of first and last is $13+(13+12)=38$. So, answer is $133-38=95$

13. $\square ABCD$ is such that $\angle ABC = 90^\circ$ and $\overline{DL} \perp \overline{AC}$ If $AB=8, BC=6$ and $DL=7$ then find the area of the $\square ABCD$.



Solution: By Pythagoras theorem, $AC^2 = AB^2 + BC^2 = 64 + 36 = 100$ $\Rightarrow AC = 10$. So, area of $\square ABCD$ = area of $\triangle ABC$ + area of $\triangle ACD$ = $\frac{1}{2}(AB)(BC) + \frac{1}{2}(AC)(LD) = \frac{48+70}{2} = 59$

14. Which of the fraction is largest among $\frac{2}{5}, \frac{5}{11}, \frac{8}{17}$?
    Report 10 if answer is $\frac{2}{5}$, 20 if answer is $\frac{5}{11}$, 30 if answer is $\frac{8}{17}$.

    Solution: Let's make the denominator same for all.
    $\frac{2}{5} = \frac{2(11)(17)}{(5)(11)(17)} = \frac{374}{935}$, $\frac{5}{11} = \frac{5(5)(17)}{(5)(11)(17)} = \frac{425}{935}$, $\frac{8}{17} = \frac{8(5)(11)}{(5)(11)(17)} = \frac{440}{935}$.
    Since the largest numerator is 440, $\frac{8}{17}$ is the largest. Answer 30.

15. If $a + b + c = 0$ then $\left(\dfrac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\dfrac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$ equals.

    Solution: Since $a + b + c = 0$, we get $a + b = -c, b + c = -a, c + a = -b$, etc. So, $\left(\dfrac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\dfrac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right) = (-1 + -1 + -1)(-1 + -1 + -1) = 9.$

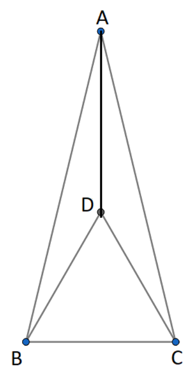
16. 15 workers make 30 machines in 8 days. Find the number of days needed by 30 workers to make 15 machines.

    Solution: Since 15 workers make 30 machines in 8 days, they will make 15 machines in 4 days. So, 30 workers will need only two days to make 15 machines. Answer is 2.

17. If A's score is 25% more than B's score, by what percent is B's score less than A ?

    Solution: Suppose B has 100 score. So, A's score is 125.
    So, B's score is $\frac{100}{125} \times 100 = 80$ percent of A. So, it is 20 percent less.

18. As shown in the figure, △DBC is an equilateral triangle and $\triangle ABC$ is an isosceles triangle, such that $m\angle A : m\angle D = 1 : 3$. Find $m\angle ADC$.



    Solution: Since the figure is symmetric about the line AD, we can claim that $\angle ADB = \angle ADC$. Also, $\angle ADB + \angle ADC + \angle BDC = 360°$. But $\angle BDC = 60°$, so we get $\angle ADC = \frac{360-60}{2} = 150°$.

19. Find the difference in the sums of all two - digit odd numbers and two- digit even numbers.

    Solution: Two digit even numbers are $10, 12, 14, \ldots, 96, 98$. Two digit odd numbers are $11, 13, 15, \ldots, 97, 99$. So, for every odd number, there is an even number, which is one less than the odd number.
    $(11-10)+(13-12)+(15-14)+\cdots+(97-96)+(99-98) = 1+1+1+\cdots+1+1.$
    Totally there are 45 even and 45 odd numbers. So, the answer is 45.

20. Meaning of a^b is a multiplied to a, b times. For example $a^4 = a \times a \times a \times a$. If $775 = 5^x + 5^y + 5^z$ where x, y, z are natural numbers, find $x + y + z$.

Solution: We know that $775 = 625 + 125 + 25 = 5^4 + 5^3 + 5^2$. So answer is $4 + 3 + 2 = 9$.

Answers:

1	2	3	4	5	6	7	8	9	10
3456	2022	8	54	13	42	4	1000	39	70
11	12	13	14	15	16	17	18	19	20
674	95	59	30	9	2	20	150	45	9