M Prakash Institute **24 November 2024** **Solution - XI Entrance Test 1**
Each question carries five marks **10 am to 1 pm** **Paper Type AD**

Chemistry

Scientific data:
Atomic Number: $H = 1, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, Na = 11, Mg = 12,$
$Al = 13, S = 16, Cl = 17, K = 19, Ca = 20, Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25,$
$Fe = 26, Cu = 29, Ga = 31, Ge = 32, In = 49, Cs = 55, Ba = 56, Tl = 81$

Atomic Mass : $H = 1, Li = 7, Be = 9, B = 11, C = 12, N = 14, O = 16, \mathrm{Na} = 23, \mathrm{Mg} = 24,$
$\mathrm{Al} = 27, K = 39, Ca = 40,\ \mathrm{S} = 32, Cl = 35.5,\ \mathrm{K} = 39, \mathrm{Sc} = 45, Ti = 48,\ \mathrm{V} = 51, \mathrm{Cr} = 52,$
$\mathrm{Mn} = 55, \mathrm{Fe} = 56, Cu = 63.5, \mathrm{Ga} = 70, \mathrm{Ge} = 72, \mathrm{In} = 115, Cs = 133, Ba = 137, \mathrm{Tl} = 204$
Avogadro Number $= 6 \times 10^{23}$ per mole

Q.1 The total numbers of atoms present in 40 grams of CH_4 is ———$\times 10^{23}$.

Solution: 1 mole = 16 grams $= 5 \times 6 \times 10^{23}$ atoms in all CH_4 molecules. Then,
40 grams = 2.5 mole $= 5 \times 6 \times 2.5 \times 10^{23} = 75 \times 10^{23}$ atoms in all CH_4 molecules. **Ans. 75.**

Q.2 How many of the followings are equal in their number of moles?
(i) 22 grams CO_2 gas
(ii) 14 grams CO gas
(iii) 1.5×10^{23} O_3 molecules
(iv) 10 grams $CaCO_3$
(v) 19.6 grams H_2SO_4
(vi) 30 grams $C_3H_6O_3$
(vii) 15 grams C_2H_6
(viii) 54 grams $C_2H_2O_4$
(ix) 53 grams Na_2CO_3
(x) 49 grams H_3PO_4
(xi) 1 grams H_2 gas
(xii) 56 grams NH_4NO_3

Solution: i) 22 grams $CO_2 = 0.5$ mole
ii) 14 grams $CO = 0.5$ mole
iii) $1.5 \times 10^{23} O_3 = 0.25$ mole
iv) 10 grams $CaCO_3 = 0.1$ mole
v) 19.6 grams $H_2SO_4 = 0.2$ mole
vi) 30 grams $C_3H_6O_3 = 1/3$mole
vii) 15 grams $C_2H_6 = 0.5$ mole
viii) 54 grams $C_2H_2O_6 = 0.6$ mole
ix) 53 grams $Na_2CO_3 = 0.5$ moles
x) 4 grams $H_3PO_4 = 0.5$ mole
xi) 2 gram $H_2 = 0.5$ mole
xii) 56 grams $NH_4NO_3 = 0.7$ mole
Molecules equal in number of mole = 6 **Ans. 6.**

Q.3 The maximum amount of oxygen gas produced on complete electrolysis of 2.7 kg of acidified water is —— mol of oxygen gas.

Solution: Electrolysis of acidified water = $2\ H_2O_{liquid} \rightarrow \quad O_2 \quad + 4H^+ \ + \ 4e^-$
. 2×18 grams 16 grams
. 2700 grams x grams
x = 1200 grams of O_2 = 75 mole of O_2. **Ans. 75.**

Q.4 The amount of ammonium nitrate is required to prepare 4 lit aqueous solution having molarity 0.05 molar is —— grams.

Solution: molarity $= 0.05 = \frac{mole\ of\ NH_4NO_3}{volume\ of\ NH_4NO_3\ in\ litre} = \frac{mole\ of\ NH_4NO_3}{4}$
$\Rightarrow$ mole of $NH_4NO_3 = 0.2$ mole
$\Rightarrow$ Grams of $NH_4NO_3 = 0.2$ mole $\times\ 80 = 16$ grams. **Ans. 16.**

Q.5 An element having atomic number 95 is present in group number 'X' and period number 'Y' in the modern periodic table. The value of $(X + Y)$ is ——

Solution: The element belongs to the period number 7 and group number 3. **Ans. 10.**

Q.6 Write the sum of number of protons present in the nucleus of largest and smallest elements from the following list:
Na, K, B, Ca, Mg, O, Li, Ba, Cs, C

Solution: largest atom = Cs ;Smallest atom = O $\Rightarrow$ Sum of protons in Cs and O = 55 + 8 = 63. **Ans. 63.**

Q.7 When some amount of sugar is heated with the help of Bunsen burner in an evaporating dish, then 16 g of a black residue is formed. The amount of pure sugar is required for the given experiment is ———- grams.

Solution: $C_{12}H_{22}O_{11} + Heat \rightarrow 12C + 11(H_2O)$
342 grams 144 grams
x grams 16 grams
Hence $x = 38$. **Ans. 38.**

Q.8 In Alumino Thermite process, the amount of iron oxide reacts with 27 g of aluminium is ——- grams

Solution: Alumino-Thermite Reaction : $2Al + Fe_2O_3 \rightarrow 2Fe + Al_2O_3$
. 54 grams 160 grams
. 27 grams x grams
Hence x = 80 grams. **Ans. 80.**

Q.9 Minimum molecular mass of an open chain alkene showing structural isomerism is ——

Solution: The alkene is C_4H_8 and possible isomers are but-1-ene, but-2-ene and 2-methylpropene. Molar Mass of $C_4H_8 = 56$. **Ans. 56.**

Q.10 The difference in the molar mass of diethyl ketone and acetone is ———-.

Solution: Diethyl Ketone = $(C_2H_5)_2CO$; Acetone = $(CH_3)_2CO$
Difference in the molar masses = 28. **Ans. 28.**

Physics

Use $g = 10\ m/s^2$ wherever required.

Q.11 Object A is thrown vertically upwards with speed of $\frac{7v}{6}\ m/sec$ at time $t = 0\ sec$. Object B is thrown vertically upwards with speed of $v\ m/sec$ at time $t = 1\ sec$. Both objects start from the same ground level. At $t = 6\ sec$, both objects are travelling upwards and the distance between them is 65 m. Calculate v.

Solution: At $t = 6$, $s_A = \left(\frac{7v}{6}\right)(6) - \frac{1}{2}(10)(36) = 7v - 180$ and
$s_B = (v)(5) - \frac{1}{2}(10)(25) = 5v - 125 \Rightarrow 7v - 180 - (5v - 125) = 65 \Rightarrow v = 60$. **Ans. 60.**

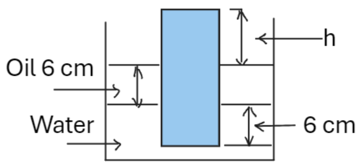
Q.12 Object A is thrown vertically upwards at $2v\ m/sec$ speed at $t = 0\ sec$. Object B is thrown vertically upwards at speed $v\ m/sec$ at $t = 10\ sec$. Both objects start from the same ground level. Object A is vertically above object B at a distance of 250 meters at $t = 15\ sec$. Object A is coming down and object B is going up. Calculate v.

Solution: $s_A = (2v)(15) - \frac{1}{2}(10)(15)^2$ and $s_B = v(5) - \frac{1}{2}(10)(5)^2$.
$s_A - s_B = 250 \Rightarrow (2v)(15) - 1125 - (5v - 125) = 250 \Rightarrow v = 50$. **Ans. 50.**

Q.13 Consider a circular horizontal track of length 200 meters. (i.e. the circumferrence of the circular track is 200 meters.) Ajay starts running at a constant acceleration of $a\ m/sec^2$ from the northmost point of the track in the anticlockwise direction, i.e. north-west-south-east at $t = 0$. Akshay and Mohan start running at constant speed of $u\ m/sec$ at $t = 0$ from the southmost point. Akshay starts running in clokcwise direction. Mohan starts running in anticlockwise direction. Ajay crosses Akshay at $t = 10\ sec$. Ajay catches Mohan at $t = 20\ sec$. Calculate Ajay's acceleration a and mark $15a$ as your answer.

Solution: Linear distance travelled by Ajay in time t is $s_{Ajay} = \frac{1}{2}at^2$
Linear distance travelled by Akshay in time t is $s_{Akshay} = ut$. Since they cross each other at $t = 10$, sum of their linear distances travelled is 100 at $t = 10$. This gives
$\frac{1}{2}a(100) + u(10) = 100 \Rightarrow 5a + u = 10$.
Linear distance travelled by Mohan in time t is $s_{Mohan} = ut$
Since Ajay catches Mohan at $t = 20$, difference in their linear distances travelled is 100 at $t = 20$. This gives
$\frac{1}{2}a(20)^2 - 20u = 100 \Rightarrow 10a - u = 5$. Adding the two equations, we get $15a = 15$. **Ans. 15.**

Q.14 Refer to the diagram.
There is a container. There is a layer of water (density 1 gm/cc) at the bottom. There is a layer of oil of density 0.9 gm/cc of height 6 cm above water. A cylindrical solid of material of density 0.6 gm/cc is floating as shown. The height of the portion of the solid in water is 6 cm. The height of the portion of the solid in air is h cm. Calculate h and write $10h$ as your answer.



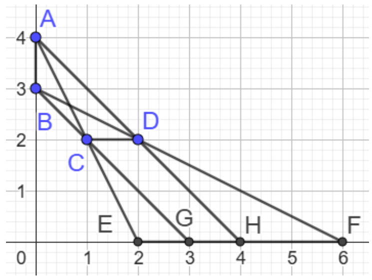
Solution: Consider two points in the container, at the same horizontal level which is the bottom of the cylindrical solid. One point below, i.e. at the base of the solid and the other point in water. Since pressure at these two points is same, we equate the pressure. We get
$\rho_{water}(6) + \rho_{oil}(6) = \rho_{solid}(h + 12)$
$\Rightarrow 6 + 6(0.9) = (0.6)(h + 12) \Rightarrow h = 7$. **Ans. 70.**

Q.15 There are four points A, B, C, D on a straight line in this order from left to right. Various distances are $AB = 3\ cm,\ BC = 1\ cm, CD = x\ cm$. There are static point charges at A, B, C. Charge at A is $+Q_1$ coulombs. Charge at B is $+Q_2$ coulombs. Charge at C is $+Q_3$ coulombs. The net static force on $+Q_2$ due to $+Q_1$ and $+Q_3$ is zero. Now, the charge at A is replaced by a point static charge of $-Q_1$ coulombs. Charge at B is moved to point D. The net static force on $+Q_2$ due to $-Q_1$ and $+Q_3$ is zero. Calculate x and mark $10x$ as your answer.

Solution: Using Coulomb's law and equating net force on $+Q_2$ to zero in both cases, we get
$K\dfrac{Q_1Q_2}{9} = K\dfrac{Q_1Q_3}{1} \Rightarrow Q_1 = 9Q_3$ and
$K\dfrac{Q_1Q_2}{(4+x)^2} = K\dfrac{Q_2Q_3}{x^2} \Rightarrow 9x^2 = (x+4)^2 \Rightarrow 8x^2 - 8x - 16 = 0 \Rightarrow x = -1,\ 2$. **Ans. 20.**

Q.16 Refer to the diagram.

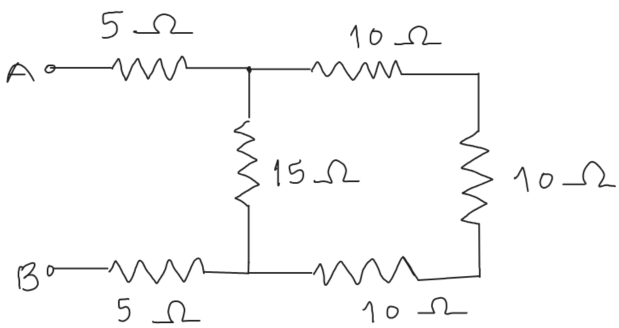
AB is a light source. B is at a height of 3 meters from ground. $AB = 1$ meter. CD is a horizontal opaque object. Horizontal distance of C from the vertcal wall is 1 meter. $CD = 1$ meter. Height of CD from ground is 2 meters. A penumbra and an umbra is formed on the horizontal ground. The ratio of lengths of penumbra and umbra is k. Write $10k$ as your answer.

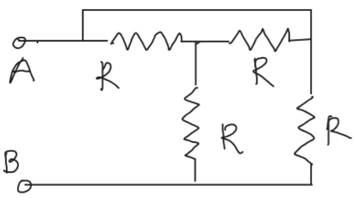


Solution:
The diagram is self-explanatory. $k = \dfrac{EF}{GH} = 4$. **Ans. 40.**

Q.17 Refer to the diagram. The equivalent resistances of the two circuits between points A and B are same. Calculate R -and mark $\frac{3R}{2}$ as your answer.

Solution: Using standard series-parallel formulae, effective resistance between A and B in the first diagram is 20 Ω and it is $\dfrac{3R}{5}$ Ω in the second diagram. So, $R = \dfrac{100}{3}$. **Ans. 50.**





Q.18 A solid block of metal of mass x gm (specific heat 0.3 $cal/gm°C$) at temperature $460°C$, 250 gm of ice at $0°C$ and 50 gm of steam at $100°C$ are kept in an insulated container. After some time, equilibrium temperature of $60°$ is reached. Calculate x. Assume the following values: Latent heat of fusion of water: 80 cal/gm, latent heat of vaporisation of water = 540 cal/gm, specific heat of water= 1 $cal/gm°C$.

Solution: Metal mass gives away heat enerey when it cools from $460°C$ to $60°C$ which is $x(400)(0.3)$ calories.
Steam gives away heat when (i) it gets converted to water at $100°C$ which is $(50)(540)$ calories and (ii) water at $100°C$ cools to water at $60°C$ which is $(50)(40) = 20000$ calories.
Ice absorbs heat when (i) it gets converted to water at $0°C$ which is $(250)(80) = 2000$ calories and (ii) when water at $0°C$ heats up to water at $60°C$ which is $(250)(60) = 15000$ calories.
Equating heat given away to heat absorbed, we get
$120x + 27000 + 2000 = 20000 + 15000 \Rightarrow x = 50$. **Ans. 50.**

Q.19 A bullet of mass 30 gm is fired vertically upwards with initial speed of 400 m/sec from point A on the ground at $t = 0$. At the same time, a wooden ball of mass 370 gm is dropped from a cliff which is 400 meters directly above point A. (Initial speed of the ball is zero.) The bullet hits the wooden ball and gets stuck in the ball. What will be the height of the bullet and ball from ground at $t = 12$ sec?

Solution: If collision occurs at t sec, we get $400t - \frac{1}{2}(10)t^2 + \frac{1}{2}(10)t^2 = 400 \Rightarrow t = 1$.
Observe that the bullet has reached $s = 400(1) - \frac{1}{2}(10)(1) = 395$ meters above ground at the time of collision.
At $t = 1$, velocity of bullet (upwards) $= 400 - (10)(1) = 390$ m/sec.
At $t = 1$, velocity of ball (downwards) $= (10)(1) = 10$ m/sec. Using Law of conservation of linear momentum, we get

$(30)(390) - (370)(10) = (400)v \Rightarrow v = 20\ m/sec$, since v is positive, the bullet and the ball continue moving upwards at $t = 1$ with velocity $20\ m/sec$.
In further 11 seconds, the ball and the bullet travel $20(11) - \frac{1}{2}(10)(11^2) = -385$ meters, so the height of the ball and bullet from the ground at $t = 12$ is $395 - 385 = 10$ meters. **Ans. 10.**

Q.20 An object is kept in front of a screen. A converging lens is kept between them so that a sharp image of the object is obtained on the screen. Distance between the object and the lens is 30 cm. Focal length of the lens is 15 cm. Now, the screen is moved away from the lens by 20 cm. By what distance should the lens be shifted from its original position so that a sharp reduced image is obtained on the screen?

Solution: In the first scenario, we have $\frac{1}{v} - (-\frac{1}{30}) = \frac{1}{15} \Rightarrow v = 30\ cm$.
Let the lens be shifted towards the screen by $x\ cm$. So, $v = 50 - x$, $u = -(30 + x) \Rightarrow \frac{1}{50-x} - (-\frac{1}{30+x}) = \frac{1}{15} \Rightarrow 1200 = 1500 + 20x - x^2 \Rightarrow x^2 - 20x - 300 = 0 \Rightarrow x = 30, -10$. With $x = 30$, we get reduced image and with $x = -10$, we get enlarged image so answer is 30 cm. **Ans. 30.**

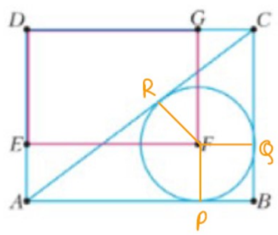
Maths

Q.21 Let $ABCD$ be a quadrilateral with coordinates $(2, 2), (10, 2), (12, 0)$, and $(0, 0)$. This quadrilateral is inscribed in a circle whose area is $K\pi$. Find K

Solution: Let the center of circle be (h, k) Distance between center and each vertex, must be equal.
$(h - 0)^2 + (k - 0)^2 = (h - 2)^2 + (k - 2)^2$
$\Rightarrow h^2 + k^2 = h^2 - 4h + 4 + k^2 - 4k + 4$
$\Rightarrow h + k = 2$,Similarly
$(h - 0)^2 + (k - 0)^2 = (h - 12)^2 + (k - 0)^2$
$\Rightarrow h^2 + k^2 = h^2 - 24h + 144 + k^2$
$\Rightarrow 24h = 144 \Rightarrow h = 6$, Hence $k = -4$
$\Rightarrow$ Radius $R = \sqrt{h^2 + k^2} = \sqrt{36 + 16} = \sqrt{52}$
Hence area of circle is 52π **Ans. 52.**

Q.22 Let $ABCD$ and $DEFG$ be two rectangles so that the point E lies on the side AD, the point G lies on the side CD and the point F is the incenter of $\triangle ABC$. What is the ratio of the area of $ABCD$ and the area of $DEFG$?

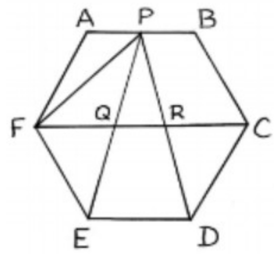
Solution:



Let $AB = DC = L$, Let $AD = BC = W$.
As FP, FQ and FR are inradius Let $FP = FQ = FR = r$
Note $\square FPBQ$ is square.
$\Rightarrow EF = EQ - FQ = L - r, \Rightarrow GF = GP - FP = W - r$
$[ABCD] = LW$ and $[DEFG] = (L - r)(W - r) = r^2 - r[L + W] + LW$
$CQ = CR = W - r$ and $AP = AR = L - r$ Also $AC = AR + RC$
Applying Pythagoras to $\triangle ABC$ we get
$(AR + RC)^2 = L^2 + W^2$

$\Rightarrow (L+W-2r)^2 = L^2 + W^2$
$\Rightarrow 4r^2 - 4r(L+W) + 2LW = 0$
$\Rightarrow 4\left(r^2 - r(L+W) + LW\right) = 2LW$
$\Rightarrow \frac{[ABCD]}{[DEFG]} = \frac{LW}{r^2 - r[LW] + LW} = 2$. **Ans. 2.**

Q.23 In the figure, $ABCDEF$ is a regular hexagon and P is the midpoint of AB. Find the ratio $\frac{Area(DEQR)}{Area(FPQ)}$.



Solution: Let length of the side of the hexagon $= a$
Observe that $QR = \frac{1}{2}ED = \frac{a}{2}$, so by symmetry, $FQ = RC = \frac{1}{2}(2a - \frac{a}{2}) = \frac{3a}{4}$
Since height of $\triangle FPQ$ and distance between parallel sides of the trapezium $\square DEQR$ is same, we have $\dfrac{\text{area}(\square DEQR)}{\text{area}(\triangle FPQ)} = \dfrac{QR + ED}{FQ} = \dfrac{\frac{a}{2} + a}{\frac{3a}{4}} = 2$ **Ans. 2.**

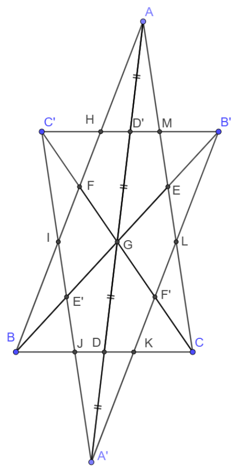
Q.24 Suppose that a, b, c and d are positive integers that satisfy the equations

$$ab + cd = 38, \quad ac + bd = 34, \quad ad + bc = 43.$$

What is the value of $a + b + c + d$?

Solution: Note $(ab + cd) - (ac + bd) = 4 \Rightarrow \quad (a-d)(b-c) = 4 \cdots (\mathbf{4})$
Similarly $(a-c)(d-b) = 5 \cdots (\mathbf{5})$
and $(a-b)(d-c) = 9 \cdots (\mathbf{6})$
As a, b, c, d are positive integers and sum of them is asked we can assume all the brackets positive. If one is positive and the other is negative, RHS will be negative.
By observation we can see that $a > d > b > c \cdots (\mathbf{7})$
Note $a - b \neq 9$ else $d - c = 1$ and (7) is false
$\Rightarrow a - b = 3$ and $d - c = 3 \cdots (\mathbf{8})$
Using (5) we can try $a - c = 5$ and $d - b = 1 \ldots (\mathbf{9})$
$\Rightarrow a = c + 5$ and by (8) $d = c + 3$ and by (9) $d = b + 1 \Rightarrow b = c + 2$
Trial 1: $c = 1 \Rightarrow b = 3, d = 4; a = 6$; given is false
Trial 2: $c = 2 \Rightarrow b = 4; d = 5; a = 7$ works
$\Rightarrow a + b + c + d = 18$. **Ans. 18.**

Q.25 Scalene triangle ABC is reflected through its own centroid G, the image being triangle $A'B'C'$. If $AB = 2BC$ and the area of triangle $A'B'C'$ is 72 , compute the area of the hexagonal region common to both triangle ABC and triangle $A'B'C'$. (Note: the centroid of a triangle is the intersection of its medians.)



Solution: $\frac{AD'}{AD} = \frac{1}{3} \Rightarrow \frac{[AHM]}{[ABC]} = \frac{1}{9} = \frac{[A'\text{KJ}]}{[A'B'C']} \Rightarrow \quad \therefore [A'\text{KJ}] = \frac{72}{9} = 8$

Similarly,

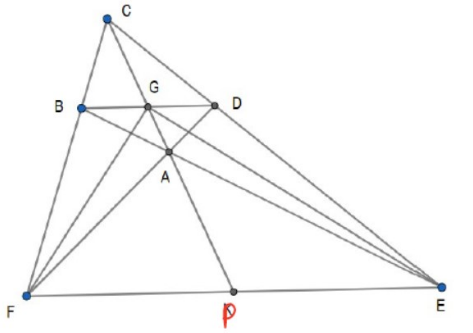
$[AHM] = 8 = [CLK] = [BJI] = [A'JK] = [B'LM] = [C'HI]$

$\therefore$ Area of required hexagon $= 72 - [AHM] - [CLK] - [BJI] = 72 - 24 = 58.$

Ans. 58.

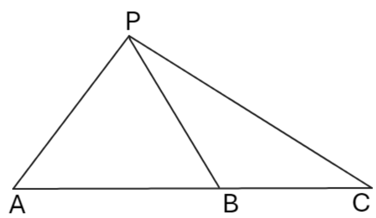
Q.26 Consider a convex quadrilateral $ABCD$. Let rays BA and CD intersect at E, rays DA and CB intersect at F, and the diagonals AC and BD intersect at G. It is given that the triangles DBF and DBE have the same area. Given that the area of triangle ABD is 4 and the area of triangle CBD is 6 , If the area of triangle EFG is α. Report $\frac{\alpha}{10}$

Solution:



Note, $\triangle CDB \sim \triangle CEF$. Let $\frac{DB}{EF} = K = \frac{CG}{CP} = \frac{DG}{EP}$.
Also $\triangle ADB \sim \triangle AFE \therefore \frac{DB}{FE} = K = \frac{AG}{AP} = \frac{DG}{FP}$

Hence P and G are mid points of $EF \& DB$.
Note $\frac{CG}{CP} = \frac{AG}{AP} \Rightarrow \frac{AG}{CG} = \frac{AP}{CP} = \frac{2}{3}. \therefore$ If $AG = 2t, CG = 3t \Rightarrow \quad AP = 10t.$
$\Rightarrow \frac{AG}{AP} = \frac{1}{5}$
Note $[DBE] = [DBF] \Rightarrow DB \| EF$
Note If two triangles have collinear bases and the same height the ratio of areas of triangles is ratio of corresponding bases.



$$\frac{[ABP]}{[ACP]} = \frac{AB}{AC}$$

$\Rightarrow$ Given $\frac{[AQB]}{[CDB]} = \frac{2}{3} = \frac{[AGD]}{[CGD]} = \frac{AG}{CG} = \frac{2}{3}$ Note when two triangles are similar, ratio of their areas is square of ratio of bases. Note $\triangle AGD \sim \triangle APF$ and $\triangle AGB \sim \triangle APE$

$$\Rightarrow \frac{[AGD]}{[APF]} = \left(\frac{AG}{AP}\right)^2 \quad \text{and} \quad \frac{[AGB]}{[APE]} = \left(\frac{AG}{AP}\right)^2$$

$$\text{But [ADG]} = [ABG] \text{ and } [AEP] = [AFP]$$

$$\Rightarrow \frac{[ADB]}{[AFE]} = \left(\frac{AG}{AP}\right)^2 \Rightarrow \frac{4}{[AFG]} = \frac{1}{25} \Rightarrow [AFE] = 100.$$

$$\Rightarrow \frac{[EGF]}{[AEF]} = \frac{GP}{AP} \Rightarrow [EGF] = \frac{12}{10} \times 100 = 120.$$

Ans. 12.

Q.27 The first term of an arithmetic progression is 2, and the common difference is d. If the sum of the squares of the first 5 terms is 5610 , find positive value of d.

Solution:
$a = 2$, common Difference $= d$

$\therefore (2)^2 + (2+d)^2 + (2+2d)^2 + (2+3d)^2 + (2+4d)^2 = 5610.$

$\therefore 5(2)^2 + (1+4+9+16)d^2 + (4+8+12+16)d = 5610$

$\therefore 20 + 30d^2 + 40d = 5610$

$\therefore 3d^2 + 4d - 559 = 0 \quad \text{Now } d = \dfrac{-4 \pm \sqrt{16 - 4(3)(-559)}}{2 \times 3} = 13$

Ans. 13.

Q.28 Find k if $x^6 + 3x^5 + 12x^4 + 19x^3 + 36x^2 + 27x + k$ is perfect cube of a polynomial.

Solution: $P(x) = x^6 + 3x^5 + 12x^4 + 19x^3 + 36x^2 + 27x + k$ is a perfect cube of a polynomial $x^2 + ax + b$

$$\begin{aligned}
&\therefore \left(x^2+ax+b\right)^3 = \left(x^2+ax+b\right)\left(x^2+ax+b\right)^2 \\
&= \left(x^2+ax+b\right)\left(x^4+a^2x^2+b^2+2ax^3+2bx^2+2abx\right) \\
&= \left(x^2+ax+b\right)\left(x^4+2ax^3+\left(a^2+2b\right)x^2+2abx+b^2\right) \\
&= x^6+2ax^5+\left(a^2+2b\right)x^4+(2ab)x^3+\left(b^2\right)x^2+ax^5 \\
&+\left(2a^2\right)x^4+(a^3+2ab)x^3+(2a^2b)x^2+ab^2x+(b)x^4+(2ab)x^3\left(a^2b+2b^2\right)x^2+2ab^2x+b^3 \\
&= x^6+(3a)x^5+\left(3a^2+3b\right)x^4+\left(a^3+6ab\right)x^3+(3a^2b+3b^2)x^2+3ab^x+b^3 \\
&= x^6+3x^5+12x^4+19x^3+36x^2+27x+k
\end{aligned}$$

comparing coefficient, we get,

$$\begin{aligned}
&3a = 3 \Rightarrow a = 1 \quad \text{and} \quad 3ab^2 = 27 \\
&3a^2 + 3b = 12 \\
&\because 3 + 3b = 12 \qquad \therefore b = 3 \Rightarrow k = b^3 = 27
\end{aligned}$$

Ans. 27.

Q.29 The expression $n\,!$ denotes the product $1 \cdot 2 \cdot 3 \cdots n$ and is read as n factorial. For example, $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$. The product $(2!)(3!)(4!)(5!)(6!)(7!)(8!)(9!)(10!)(11!)(12!)$ can be written in the form $M^2(N!)$, where M, N are positive integers. Find smallest suitable value of N.

Solution: This is a long problem.

$$
\begin{aligned}
2 &= 2^1 & &\Rightarrow 2! = 2^1 \\
3 &= 3^1 & &\Rightarrow 3! = 3 \times 2 = 2^1 3^1 \\
4 &= 2^2 & &\Rightarrow 4! = 2^1 3^1 \times 4 = 2^3 3^1 \\
5 &= 5^1 & &\Rightarrow 5! = 2^3 3^1 5^1 \\
6 &= 2^1 3^1 & &\Rightarrow 6! = 2^4 3^2 5^1 \\
7 &= 7^1 & &\Rightarrow 7! = 2^4 3^2 5^1 7^1 \\
8 &= 2^3 & &\Rightarrow 8! = 2^7 3^2 5^1 7^1 \\
9 &= 3^2 & &\Rightarrow 9! = 2^7 3^4 5^1 7^1 \\
10 &= 2^1 5^1 & &\Rightarrow 10! = 2^8 3^4 5^2 71 \\
11 &= 1^1 & &\Rightarrow 11! = 2^8 3^4 5^2 7^1 11^1 \\
12 &= 2^2 3^1 & &\Rightarrow 12! = 2^{10} 3^5 5^2 7^1 11^1
\end{aligned}
$$

$$
\begin{aligned}
&\text{The product is} \\
&= 2^{56} 3^{26} 5^{11} 7^6 11^2 \\
&= \left(2^{52} 3^{24} 5^{10} 7^6 11^2\right)(6!) \\
&= M^2 \cdot N! \\
&\Rightarrow M = 2^{27} 3^{12} 5^5 7^3 11 \\
N &= 6
\end{aligned}
$$

Ans. 6.

Q.30 Find the positive integer n such that

$$\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} + \frac{1}{24} + \frac{1}{42} + \frac{1}{n} = 1$$

Solution: Note that

$$\frac{1}{6}, \quad \frac{1}{7} + \frac{1}{42}, \quad \frac{1}{8} + \frac{1}{24}, \quad \frac{1}{9} + \frac{1}{18}, \quad \text{and} \quad \frac{1}{10} + \frac{1}{15}$$

each equals $\frac{1}{6}$. Thus, $1 = 5 \cdot \frac{1}{6} + \frac{1}{12} + \frac{1}{n} = \frac{11}{12} + \frac{1}{n}$.

Therefore, $n = 12$.

Alternatively, the least common denominator of the fractions in the sum is $2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$. The sum is equivalent to

$$
\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} + \frac{1}{24} + \frac{1}{42} + \frac{1}{n} \\
&= \frac{420 + 360 + 315 + 280 + 252 + 210 + 168 + 140 + 105 + 60}{2520} + \frac{1}{n} \\
&= \frac{2310}{2520} + \frac{1}{n} = \frac{11}{12} + \frac{1}{n},
\end{aligned}
$$

as above. **Ans. 12.**