

M Prakash Institute

19 January 2025

Solution - XI Entrance Test 2

Each question carries five marks

10 am to 1 pm

Paper Type AD

Student Receipt number:

Student Name:

Chemistry

Scientific data:

Avogadro Number = 6×10^{23} per mole

Atomic Number: H = 1, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, Ne = 10, Na = 11, Mg = 12, Al = 13, P = 15, S = 16, Cl = 17, K = 19, Ca = 20, Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25, Fe = 26, Cu = 29, Zn = 30, Ga = 31, Ge = 32, Cd = 48, In = 49, Cs = 55, Ba = 56, Tl = 81, Po = 84

Atomic Mass: H = 1, Li = 7, Be = 9, B = 11, C = 12, N = 14, O = 16, Ne = 20, Na = 23, Mg = 24, Al = 27, K = 39, Ca = 40, S = 32, P = 31, Cl = 35.5, K = 39, Sc = 45, Ti = 48, V = 51, Cr = 52, Mn = 55, Fe = 56, Cu = 63.5, Zn = 65, Ga = 70, Ge = 72, Cd = 112, In = 115, Cs = 133, Ba = 137, Tl = 204, Po = 209

Q.1 The number of neon atoms present in 2×10^{-24} Kg sample of pure neon is ——— atoms.

Solution: Weight of Neon = 2×10^{-24} Kg = 2×10^{-21} grams.

Number of mole = $\frac{(\text{Mass in grams})}{(\text{Molar Mass})} = \frac{2 \times 10^{-21}}{20} = 10^{-22}$ mole.

Number of Neon atoms = $6 \times 10^{23} \times 10^{-22} = 60$ atoms. **Ans. 60.**

Q.2 Write the molar mass of any one of the chemical species which has molar mass equal to the other chemical species from the given list $\Rightarrow \text{CH}_3\text{COOH}, \text{NH}_4\text{NO}_2, \text{CO}_3^{2-}, \text{SO}_4^{2-}, \text{H}_2\text{CO}_3, \text{SO}_3$

Solution: CH_3COOH and $\text{CO}_3^{2-} = 60$ grams /mol. **Ans. 60.**

Q.3 The difference between the pH values of 0.01 M NaOH and 0.001 M HCl solution is

Solution: pH of NaOH = 12, pH of HCl = 3 $\therefore$ Difference in the pH = 9 **Ans. 9.**

Q.4 What is the total number of moles of oxygen atoms present in 3 moles of aluminium dichromate salt?

Solution: 1 mole of $\text{Al}_2(\text{Cr}_2\text{O}_7)_3 = 21$ oxygen atoms $\therefore$ Three mole has $21 \times 3 = 63$ atoms. **Ans. 63.**

Q.5 An element present in group number six and period number six in the modern periodic table. The atomic number of the element is ———

Solution: The element is Tungsten with the atomic number 74. **Ans. 74.**

Q.6 Last element of second series of transition element say (A) combines with second element of sixteenth group say (B), forming a compound having molecular formula A_xB_y . The value of $\left(\frac{\text{Molar Mass of } A_xB_y}{2} \right)$ is ———

Solution: The compound is CdS; Its molar mass is 144 and $144/2 = 72$. **Ans. 72.**

Q.7 The amount of reactive form of oxygen gas produced when $\frac{1}{6}$ mol of $\text{K}_2\text{Cr}_2\text{O}_7$ (potassium dichromate) is treated with dilute H_2SO_4 is ——— grams.

Solution: $\text{K}_2\text{Cr}_2\text{O}_7 + 4\text{H}_2\text{SO}_4 \rightarrow \text{K}_2\text{SO}_4 + \text{Cr}_2(\text{SO}_4)_3 + \text{H}_2\text{O} + 3[\text{O}]$

1 mole of $\text{K}_2\text{Cr}_2\text{O}_7 \equiv 48$ grams of oxygen atoms Hence, $(1/6)$ mole of $\text{K}_2\text{Cr}_2\text{O}_7$ gives 8 grams of oxygen atoms. **Ans. 8.**

Q.8 $\frac{2}{3}$ mol of copper is obtained by heating ——— grams of Cu_2S ore in air while performing metallurgy of copper.

Solution: $\text{Cu}_2\text{S} + \text{O}_2 \rightarrow 2\text{Cu} + \text{SO}_2$

2 mol Cu requires one mol Cu_2S

$(2/3)$ mol Cu require $(1/3)$ mol Cu_2S

Weight of $(1/3)$ mol $\text{Cu}_2\text{S} = (159/3) = 53$. **Ans. 53.**

Q.9 The minimum molar mass of a saturated open chain alcohol which can show two different structural isomeric alcohols is ———

Solution: $\text{C}_3\text{H}_7\text{OH}$ (Molar Mass = 60) $\Rightarrow$ Possible isomers : Propan-1-ol and Propan-2-ol.
Ans. 60.

Q.10 How many of the following can decolourise bromine water?

- 1) C_2H_4 2) C_4H_{10} 3) $\text{C}_{17}\text{H}_{35}\text{COOH}$
 4) C_2H_2 5) C_3H_4 6) $\text{C}_{15}\text{H}_{31}\text{COOH}$
 7) C_3H_8 8) C_6H_{14} 9) $\text{C}_{17}\text{H}_{31}\text{COOH}$
 10) C_5H_8 11) C_5H_{10} 12) $\text{C}_{17}\text{H}_{33}\text{COOH}$

Solution: Compounds that can decolorize bromine water are : i, iv,,v ix, x, xi, xii Compounds: (i, iv, v, x, xi): Unsaturated hydrocarbons Compound : (ix, xii): Unsaturated carboxylic acids i.e. oils. **Ans. 7.**

Physics

Use $g = 10 \text{ m/s}^2$ wherever required.

Q.11 Object A is thrown vertically upwards from the top of a tower with speed $u \text{ m/sec}$ at $t = 0$. The top of the tower is $12u \text{ m}$ from the ground. At the same time, object B is thrown vertically upwards from ground at speed $ku \text{ m/sec}$. It is observed that object A and object B are at the same height from the ground at $t = 3 \text{ sec}$. Calculate k and **mark 10k as your answer.**

Solution: At $t = 3$ we have $s_A + 12u = s_B \Rightarrow u(3) - \frac{1}{2}(10)(9) + 12u = ku(3) - \frac{1}{2}(10)(9)$
 $\Rightarrow k = 5$. **Ans. 50.**

Q.12 Object A is thrown vertically upwards at $u \text{ m/sec}$ speed at $t = 0 \text{ sec}$. Object B is dropped from the height of 200 meters at $t = 0 \text{ sec}$ with initial speed zero. Object C is dropped from the same height at $t = 2 \text{ sec}$ with initial speed zero. It is observed that object A and object B cross each other at $t = t_1 \text{ sec}$. Object A and object C cross each other at $t = t_1 + 2 \text{ sec}$. Calculate u in m/sec .

Solution: At $t = t_1$, sum of distance travelled by A and B=200.

$$u(t_1) - \frac{1}{2}(10)(t_1^2) + \frac{1}{2}(10)(t_1^2) = 200 \Rightarrow ut_1 = 200$$

At $t = t_1 + 2$, sum of distance travelled by A and C=200.

$$u(t_1+2) - \frac{1}{2}(10)((t_1+2)^2) + \frac{1}{2}(10)(t_1^2) = 200 \Rightarrow 200 + 2u - 5(4t_1 + 4) = 200 \Rightarrow u - 10\left(\frac{200}{u}\right) - 10 = 0$$

$$\Rightarrow u^2 - 10u - 2000 = 0 \Rightarrow u = 50, -40. \text{ Ans. 50.}$$

Q.13 Consider a circular horizontal track of length 200 meters. (i.e. the circumference of the circular track is 200 meters.) Ajay starts running at a constant acceleration of $a \text{ m/sec}^2$ from the northmost point of the track in the anticlockwise direction, i.e. north-west-south-east at $t = 0$. Akshay and Mohan start running at $t = 0$ from the southmost point. Akshay starts running in clockwise direction. Mohan starts running in anticlockwise direction.

Akshay starts running from zero speed at constant acceleration of $ka \text{ m/sec}^2$ for 2 seconds and then decelerates at $ka \text{ m/sec}^2$ for two seconds. He continues running in the same pattern.

Mohan starts running from zero speed at constant acceleration of $\frac{3ka}{2} \text{ m/sec}^2$ for 2 seconds and then decelerates at $\frac{3ka}{2} \text{ m/sec}^2$ for two seconds. He continues running in the same pattern.

Ajay crosses Akshay at $t = 10$ sec. Ajay catches Mohan at $t = 20$ sec. Calculate k and **mark 4k as your answer**.

Solution: Akshay travels $\frac{1}{2}(ka)(4) = 2ka$ (linear distance) meters in every two seconds. So, Akshay travels $10ka$ meters in 10 seconds. Ajay travels $\frac{1}{2}a(100) = 50a$ distance in 10 seconds. Sum of their distances must be 100.

$$10ka + 50a = 100 \Rightarrow a(k + 5) = 10$$

Mohan travels $\frac{1}{2} \left(\frac{3ka}{2} \right) (2)^2 = 3ka$ distance in every two seconds, so he travels $30ka$ distance in 20 seconds. Difference of distance travelled by Ajay and Mohan is 100 in 20 seconds, so

$$\frac{1}{2}a(20)^2 - 30ka = 100 \Rightarrow a(20 - 3k) = 10$$
 Equating LHS of both equations, we get

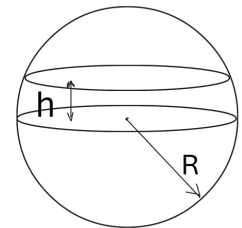
$$k + 5 = 20 - 3k \Rightarrow 4k = 15. \text{ Ans. } 15.$$

Q.14 Refer to the diagram.

The problem is based on the following formula:

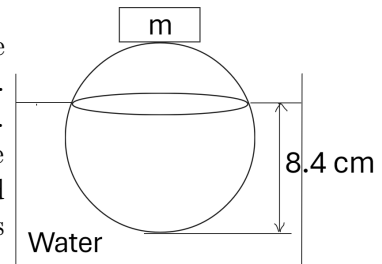
Consider a sphere of radius R . It is cut by two parallel planes. First plane is passing through the centre of the sphere. The second plane is at a distance of h from the first plane. The volume of the cut section of width h is given

$$V = \pi \left(R^2 h - \frac{h^3}{3} \right)$$



Now the problem:

Take the value of π as $\frac{22}{7}$. A spherical ball of radius 6.3 cm made of wood (density 0.5 gm/cc) is floating in water (density 1 gm/cc). A solid of mass m gm is kept on the top of the sphere carefully. It is observed that the depth of the portion of the wooden sphere inside the water is 8.4 cm. Calculate m in grams. Calculate $\frac{m}{10}$ and mark the integer part of $\frac{m}{10}$ as your answer. E.g. if your answer is $m = 123.456$ gm or $m = 128.898$ gm, then mark the answer as 12.



Solution: Using Archimedes principle, weight of the displaced water = weight of wooden ball and the mass above it.

Volume of the submerged part of the sphere is made up of a hemisphere and a slice of sphere of width 2.1 cm.

$$\text{Volume of hemisphere} = \frac{2}{3}\pi R^3 = \frac{2}{3} \frac{22}{7} (6.3)^3 = \frac{(22)(21)^2(54)}{1000}$$

$$\text{Volume of the slice of the sphere} = \pi \left(R^2 h - \frac{h^3}{3} \right) = \frac{22}{7} \left((6.3)^2 (2.1) - \frac{(2.1)^3}{3} \right) = \frac{22}{7} (2.1)^3 \left(9 - \frac{1}{3} \right) = \frac{(22)(21)^2(26)}{1000}$$

So, total volume of the submerged part = $\frac{(22)(21)^2(80)}{1000}$

$$\text{Using Archimedes principle and ignoring } g \text{ on both sides, we get}$$

$$\frac{(22)(21)^2(80)}{1000} = \frac{4}{3} \frac{22}{7} (6.3)^3 (0.5) + m = \frac{(22)(21)^2(54)}{1000} + m \Rightarrow m = \frac{(22)(21)^2(80)}{1000} - \frac{(22)(21)^2(54)}{1000} = \frac{(22)(21)^2(26)}{1000} = 252.252. \text{ Ans. } 25.$$

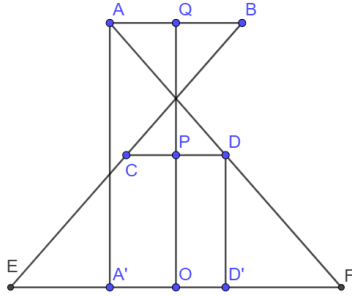
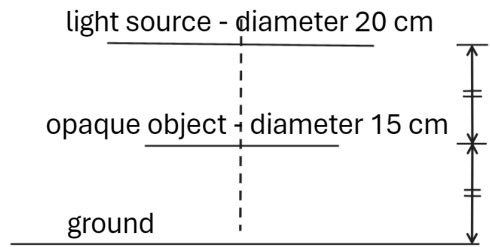
Q.15 There are four points A, B, C, D on a straight line in this order from left to right. Various distances are $AB = \frac{d}{2}$ cm, $BC = \frac{3d}{2}$ cm, $CD = 4d$ cm. There are static point charges at A, C, D . Charge at A is $+Q_1$ coulombs. Charge at C is $+Q_2$ coulombs. Charge at D is $+Q_3$ coulombs. The net static force on $+Q_2$ due to $+Q_1$ and $+Q_3$ is zero. Now, the charge at A is moved to point B . The charge at D is replaced by $+kQ_3$ coulombs. The net static force on $+Q_2$ due to $+Q_1$ and $+kQ_3$ is zero. Calculate k and **mark 9k as your answer**.

$$\text{Solution: Coulomb's law in first situation: } \frac{KQ_1Q_2}{4d^2} = \frac{KQ_3Q_2}{16d^2} \Rightarrow Q_3 = 4Q_1$$

$$\text{Second situation: } \frac{Q_1Q_2}{\left(\frac{3d}{2}\right)^2} = \frac{kQ_3Q_2}{16d^2} \text{ and using } Q_3 = 4Q_1, \text{ we get } 9k = 16. \text{ Ans. } 16.$$

Q.16 Refer to the diagram.

An extended light source in the form of a circular disc of diameter 20 cm is at some distance from ground and horizontal. An opaque object in the form of a circular disc of diameter 15 cm is kept exactly midway between the light source and the ground. The light source and the opaque object are coaxial. The object is horizontal. A penumbra (in the shape of a ring with inner and outer circles) is formed on the ground. Calculate the outer diameter of the penumbra in centimeters.

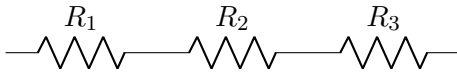


Solution: EF is the outer diameter of the penumbra.

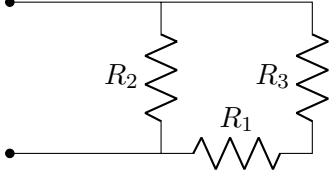
$AQ = A'O = 10$ and $PD = OD' = 7.5$ so, $A'D' = 17.5$. This means $D'F = 17.5$, so $OF = 25$, i.e. $EF = 50$. **Ans. 50.**

Q.17 Refer to the diagram. The equivalent resistance of circuit 4 has two possible values, say R_4 and R_5 where $R_4 > R_5$. $R_4 - R_5 = \frac{p}{q}$ where p, q are prime numbers. Write the product pq as your answer.

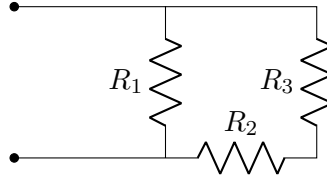
circuit 1 equivalent resistance 11 ohms



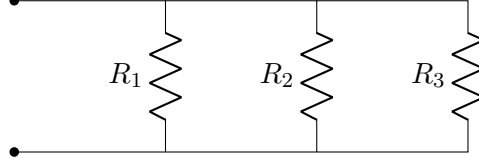
circuit 3 equivalent resistance $\frac{24}{11}$ ohms



circuit 2 equivalent resistance $\frac{18}{11}$ ohms



circuit 4



Solution: Circuit 1: $R_1 + R_2 + R_3 = 11$

$$\text{Circuit 2: } \frac{1}{R_1} + \frac{1}{R_2 + R_3} = \frac{11}{18} \Rightarrow \frac{R_1 + R_2 + R_3}{R_1(R_2 + R_3)} = \frac{11}{18} \Rightarrow R_1(R_2 + R_3) = 18$$

$$\Rightarrow R_1 = 9 \text{ and } R_2 + R_3 = 2 \text{ or } R_1 = 2 \text{ and } R_2 + R_3 = 9$$

$$\text{Circuit 3: } \frac{1}{R_2} + \frac{1}{R_1 + R_3} = \frac{11}{24} \Rightarrow \frac{R_1 + R_2 + R_3}{R_2(R_1 + R_3)} = \frac{11}{26} \Rightarrow R_2(R_1 + R_3) = 24$$

$$\Rightarrow R_2 = 8 \text{ and } R_1 + R_3 = 3 \text{ or } R_2 = 3 \text{ and } R_1 + R_3 = 8$$

Considering all possibilities, we get two possible sets of values for R_1, R_2, R_3

Either 1) $R_1 = 2, R_2 = 8, R_3 = 1$ or 2) $R_1 = 2, R_2 = 3, R_3 = 6$. So, effective resistance of circuit

4 is $\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$, its two possible values are

$$1) \frac{1}{\frac{1}{2} + \frac{1}{8} + \frac{1}{1}} = \frac{8}{13} \text{ or } 2) \frac{1}{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}} = 1, \text{ so } \frac{p}{q} = 1 - \frac{8}{13} = \frac{5}{13}. \text{ Ans. 65.}$$

Q.18 A solid block of metal of mass 50 gm (specific heat $0.3 \text{ cal/gm}^\circ\text{C}$) at temperature 460°C , 250 gm of ice at 0°C and $x \text{ gm}$ of steam at 100°C are kept in an insulated container. After some time, equilibrium temperature of 60° is reached. Calculate x . Assume the following values: Latent heat of fusion of water: 80 cal/gm , latent heat of vaporisation of water = 540 cal/gm , specific heat of water = $1 \text{ cal/gm}^\circ\text{C}$.

Solution: Heat given away by metal mass = $(50)(0.3)(400) = 6000$. Heat given away by steam = $x(540) + x(40) = x(580)$. Heat absorbed by ice = $250(80) + 250(60) = 35000$. So, we get $6000 + x(580) = 35000 \Rightarrow x = 50$. **Ans. 50.**

Q.19 A bullet of mass m gm is fired vertically upwards with initial speed of 400 m/sec from point A on the ground at $t = 0$. At the same time, a wooden ball of mass 370 gm is dropped from a cliff which is 400 meters directly above point A . (Initial speed of the ball is zero.) The bullet hits the wooden ball and gets stuck in the ball. The direction of movement of the bullet and the wooden ball after collision is vertically upwards. The height of the bullet and ball from ground at $t = 12$ sec is 10 meters. Calculate m .

Solution: Suppose the collision occurs at time t . So, sum of distances travelled by bullet and wooden ball is 400 . $400(t) - \frac{1}{2}(10)t^2 + \frac{1}{2}(10)t^2 = 400 \Rightarrow t = 1$.

At $t = 1$, distance travelled by bullet = $400 - \frac{1}{2}(10) = 395$. Suppose, the velocity of the bullet and the ball after collision is v , (considering the level of 395 meters as base level), the displacement after 11 seconds is -385 .

$$-385 = v(11) - \frac{1}{2}(10)121 \Rightarrow v = 20.$$

Velocity of bullet at the time of collision = $400 - (10)(1) = 390$ m/sec

Velocity of ball at the time of collision = $(10)(1) = 10$ m/sec

Using law of conservation of linear momentum, we get $390(m) - 370(10) = (m + 370)(20) \Rightarrow 370m = 370(30) \Rightarrow m = 30$. **Ans. 30.**

Q.20 An object is kept in front of a screen. A converging lens is kept between them so that a sharp image of the object is obtained on the screen. Image is of the same size as that of object. Now, the screen is moved towards the lens by 10 cm. The object is moved away from the lens by 30 cm so that a sharp image is obtained on the screen. Calculate the focal length of the lens in centimeters.

Solution: Initially the object and the image are at a distance of $2f$ from the lens where f is the focal length of the lens.

New object distance = $2f + 30$, new image distance = $2f - 10$, so we get $\frac{1}{2f-10} + \frac{1}{2f+30} = \frac{1}{f} \Rightarrow f(4f + 20) = (2f + 30)(2f - 10) \Rightarrow 20f = 300$. **Ans. 15.**

Maths

Q.21 Let $ABCD$ be a trapezoid with $AB \parallel CD$, $AB = 20$, $CD = 24$, and area 880 . Compute the area of the triangle formed by the midpoints of AB , AC , and BD .

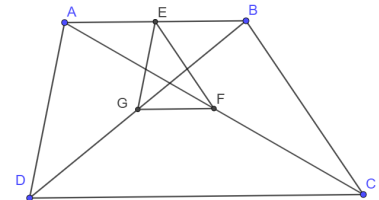
Solution:

$$\text{Distance between the parallel lines} = \frac{2(\text{area}(\square ABCD))}{AB + CD} = 40.$$

So, height of the $\triangle EFG$ with GF as base is 20 .

By standard property of trapezium, $GF = \frac{1}{2}(CD - AB) = 2$.

So, area of $\triangle EFG = \frac{1}{2}(2)(20) = 20$. **Ans. 20.**



Q.22 Points A and B lie on a circle centered at O such that $AB = 14$. The perpendicular bisector of AB intersects $\odot O$ at point C such that O lies in the interior of $\triangle ABC$ and $AC = 35\sqrt{2}$. Lines BO and AC intersect at point D . Let λ be the ratio of the area of $\triangle DOC$ to the area of $\triangle DBC$. Report 98λ .

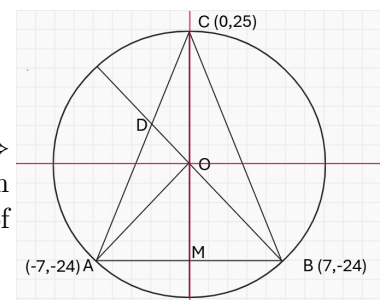
Solution: Let the radius of the circle be R .

$$CM^2 = AC^2 - AM^2 = (35\sqrt{2})^2 - 7^2 = 7^4 \Rightarrow CM = 49$$

$\Rightarrow OM = 49 - R$. Using Pythagoras in $\triangle OAM$,

$$R^2 - (49 - R)^2 = 49 \Rightarrow 98R = 49^2 + 49 = (49)(50) \Rightarrow R = 25 \Rightarrow$$

$OM = 24$. Now, let's establish a frame of reference in which origin is the center of the circle, Y axis along OC . So, coordinates of various points are $O(0, 0)$, $A(-7, -24)$, $B(7, -24)$, $C(0, 25)$.

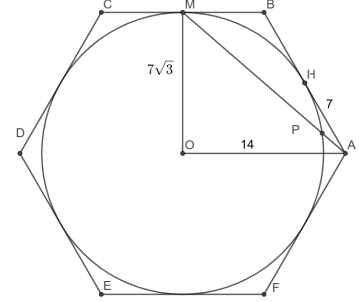


Equation of AC is $y = 7x + 25$ and equation of BO is $24x + 7y = 0$. Solving simultaneously, we get coordinates of D as $(-\frac{175}{73}, \frac{600}{73})$.

Now, $\frac{[\triangle DOC]}{[\triangle DBC]} = \frac{DO}{DB}$ which can be shown to be $\frac{25}{98}$. **Ans. 25.**

Q.23 Let ω be the circle inscribed in regular hexagon $ABCDEF$ with side length 14, and let the midpoint of side BC be M . Segment AM intersects ω at point $P \neq M$. Compute the length of AP . Report AP^2

Solution: $AM^2 = 14^2 + (7\sqrt{3})^2 = (7\sqrt{7})^2 \Rightarrow AM = 7\sqrt{7}$
 $AP \cdot AM = AH^2 \Rightarrow (AP^2)(AM^2) = (AH)^4$
 $\Rightarrow (AP^2)(7\sqrt{7})^2 = 7^4 \Rightarrow AP^2 = 7$. **Ans. 7.**

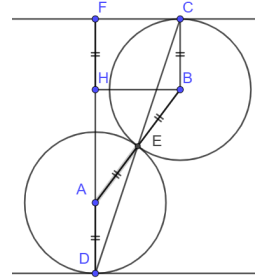


Q.24 The figure below shows two parallel lines, ℓ and m , that are distance 35 apart:

A circle is tangent to line ℓ at point A . Another circle is tangent to line m at point B . The two circles are congruent and tangent to each other as shown. The distance between A and B is 37. Let R be the radius of each circle such that $N < R < N + 1$ where N is natural number. Report the largest value of N .

Solution:

$HB^2 = CF^2 = CD^2 - DF^2 = 37^2 - 35^2 = 144 \Rightarrow HB = 12$
 $AH = 35 - 2R, AB = 2R \Rightarrow (2R)^2 - (35 - 2R)^2 = 144$
 $\Rightarrow 140R = 1369 \Rightarrow R = \frac{1369}{140} \Rightarrow 9 < R < 10$. **Ans. 9.**



Q.25 If a, b, c, d , and e are constants such that every $x > 0$ satisfies

$$\frac{5x^4 - 8x^3 + 2x^2 + 4x + 7}{(x+2)^4} = a + \frac{b}{x+2} + \frac{c}{(x+2)^2} + \frac{d}{(x+2)^3} + \frac{e}{(x+2)^4},$$

then what is the value of $a + b + c + d + e$?

Solution: $5x^4 - 8x^3 + 2x^2 + 4x + 7 \equiv a(x+2)^4 + b(x+2)^3 + c(x+2)^2 + d(x+2) + e$.
 Substituting $x = -1$ in both sides, we get
 $5 + 8 + 2 - 4 + 7 = a + b + c + d + e$. **Ans. 18.**

Q.26 Sum of all real solutions of $\sqrt{x+14-8\sqrt{x-2}} + \sqrt{x+23-10\sqrt{x-2}} = 3$. is

Solution: $\sqrt{x+14-8\sqrt{x-2}} + \sqrt{x+23-10\sqrt{x-2}}$
 $= \sqrt{(x-2) - 8\sqrt{x-2} + 16} + \sqrt{(x-2) - 10\sqrt{x-2} + 25}$
 $= \sqrt{(\sqrt{x-2} - 4)^2} + \sqrt{(\sqrt{x-2} - 5)^2} = |\sqrt{x-2} - 4| + |\sqrt{x-2} - 5|$

This gives either $\sqrt{x-2} = 3$ or $\sqrt{x-2} = 6 \Rightarrow x-2 = 9$ or $x-2 = 36 \Rightarrow x = 11$ or $x = 38$
Ans. 49.

Q.27 If $a = (\sqrt{3} + \sqrt{2})^{-3}$ and $b = (\sqrt{3} - \sqrt{2})^{-3}$, find the value of $(a+1)^{-1} + (b+1)^{-1}$

Solution: Observe that $ab = 1$.

$$(a+1)^{-1} + (b+1)^{-1} = \frac{1}{a+1} + \frac{1}{b+1} = \frac{a+b+2}{ab+a+b+1} = \frac{a+b+2}{a+b+2} = 1. \quad \text{Ans. 1.}$$

Q.28 ABC is a right angled triangle with $\angle B = 90^\circ$. M is the midpoint of AC and $BM = \frac{\sqrt{193}}{2}$. Sum of the lengths of the other two sides AB and BC is 19. Find the area of the triangle.

Solution: In the right angled triangle, $BM = AM = CM$, so $AC^2 = 193 \Rightarrow AB^2 + BC^2 = 193$.
 $AB + BC = 19 \Rightarrow AB^2 + BC^2 + 2(AB)(BC) = 361 \Rightarrow 193 + 2(AB)(BC) = 361$
 $\Rightarrow \frac{(AB)(BC)}{2} = 42. \quad \text{Ans. 42.}$

Q.29 A circle is drawn through points $A(4, 6), B(6, 2), C(-3, 5)$. D is diametrically opposite point of C on the circle. AD is distance between A and D . Find AD^2

Solution: Slope of $AC = \frac{6-5}{4+3}$ so slope of the $\perp$ bisector of AC is -7 . Midpoint of AC is $(\frac{1}{2}, \frac{11}{2})$.
 So, equation of $\perp$ bisector is $(y - \frac{11}{2}) = -7(x - \frac{1}{2}) \Rightarrow 7x + y = 9$
 Similarly, equation of $\perp$ bisector of BC is $3x - y = 1$. They intersect at $O(1, 2)$ which is the center of the circle. The diametrically opposite point to C is therefore $D(5, -1)$, so $AD^2 = (5-4)^2 + (-1-6)^2 = 50. \quad \text{Ans. 50.}$

Q.30 Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then sum of the digits of $S_{15} - S_5$ is :

Solution: With standard notation, $S_{15} - S_5 = \frac{15}{2}(2a + 14d) - \frac{5}{2}(2a + 4d) = 10a + 95d = 5(2a + 19d) = \frac{S_{20}}{2} = 395 \quad \text{Ans. 17.}$