M Prakash Institute **Solutions of 28 January 2024** **XI Entrance Test 2**
Each question carries five marks **10 am to 1 pm**

Key:

Q.No.	1	2	3	4	5	6	7	8	9	10
Answer	54	28	16	98	33	61	9	11	40	76
Q.No.	11	12	13	14	15	16	17	18	19	20
Answer	90	11	27	11	15	95	60	84	80	25
Q.No.	21	22	23	24	25	26	27	28	29	30
Answer	22	40	84	40	0	13	17	31	4	74

Chemistry

Scientific data:
Atomic Number : $H = 1, C = 6, N = 7, O = 8, Na = 11, Mg = 12, Al = 13, S = 16, Cl = 17, K = 19, Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25, Fe = 26, Cu = 29, Ga = 31, Ge = 32, In = 49, Tl = 81$

Atomic Mass : $H = 1, C = 12, N = 14, O = 16, \mathrm{Na} = 23, \mathrm{Mg} = 24, \mathrm{Al} = 27,\ \mathrm{S} = 32, Cl = 35.5,\ \mathrm{K} = 39, \mathrm{Sc} = 45, Ti = 48,\ \mathrm{V} = 51, \mathrm{Cr} = 52, \mathrm{Mn} = 55, \mathrm{Fe} = 56, Cu = 63.5, \mathrm{Ga} = 70, \mathrm{Ge} = 72, \mathrm{In} = 115, \mathrm{Tl} = 204$
Avogadro Number $= 6.022 \times 10^{23}$ per mole

Q.1 In the year 1866 Newland arranged the elements in an increasing order of their atomic masses. It later on resulted in the Law of Octaves. According to the Newland's law, the sum of number of neutrons present in the elements kept just below the element aluminium and the element silicon = ____.

Solution: $28 + 26 = 54$ $\ {}^{52}_{24}\mathrm{Cr}$ and ${}^{48}_{22}\mathrm{Ti}$ **Ans. 54**.

Q.2 The volume of air inhaled or exhaled by a healthy man per minute is known as tidal volume which is 500 ml. The exhaled air contains about 4% of carbon dioxide gas. Then the volume of exhaled carbon dioxide gas by a healthy man per day $= X \cdot YZ$ Litre. (Here X is natural number from 1 to 99, and Y and Z are single digit numbers. e.g. If the answer is 20.13 then $X = 20, Y = 1, Z = 3$) The value of $X =$ ____.

Solution: Volume of Exhaled CO_2 gas $= \frac{720000}{1000} \times 0.04 = 28800\mathrm{ml} = 28.80$ Litre **Ans. 28**.

Q.3 Molecular mass of fourth member of alkyne series is M_1 and molecular mass of fourth member of cycloalkane is M_2. The value of $(M_2 - M_1) =$ ____.

Solution: $84 - 68 = 16$ **Ans. 16**.

Q.4 A pink coloured alkaline solution of an oxidizing agent having molecular mass M_1 oxidizes ethanol into a carboxylic acid having molecular mass M_2. The value of $(M_1 - M_2) =$ ____.

Solution: $158 - 60 = 98$ (Oxidizing agent is $KMnO_4$ and Carboxylic acid is CH_3COOH) **Ans. 98**.

Q.5 The mass of 0.6 mole of iron atom is $X \cdot YZ \times 10^{-3}$ Kg. (Here X is natural number from 1 to 99, and Y and Z are single digit numbers. e.g. If the answer is 20.13×10^{-3} then $X = 20, Y = 1, Z = 3$)The value of $X =$ ____.

Solution: 0.6 mole iron $= 0.6 \times 6.022 \times 10^{23}$ iron atoms
one iron atom $= 56\mathrm{amu} = 56 \times 1.67 \times 10^{-27}\mathrm{Kg}$
$\therefore$ 0.6 mole ironatoms $= 0.6 \times 6.022 \times 10^{23} \times 56 \times 1.67 \times 10^{-27}\mathrm{Kg}$
0.6 mole iron atoms $= 337.90 \times 10^{-4}\mathrm{Kg}$ **Ans. 33**.

Q.6 Mendeleev kept vacant spaces in the periodic table for elements not discovered till then. Two of these unknown elements were given the name eka-boron and eka-silicon. The number of protons present in the nucleus of eka-boron is 'Z' and number of neutrons present in the nucleus of eka-silicon is 'N'. The value of $(Z+N) =$ ______.

Solution: eka-boron is Scandium $\left({}^{45}_{21}\text{Sc}\right)$ and eka-silicon is Germanium $\left({}^{72}_{32}\text{Ge}\right)$ $21+40=61$ **Ans. 61**.

Q.7 Aqueous solution of 0.01M NaOH has $pH = A$ and aqueous solution of 0.001M HCl has $pH = B$. The value of $(A-B) =$______.

Solution: We know that $pH =$ power of hydrogen ion. The pH scale extends from 0 to 14.
$\because$ OH^- ion concentration for 0.01M NaOH $= 10^{-2}$M
$\therefore$ H^+ ion concentration for 0.01M NaOH $= 10^{-12}$M
hence, pH of 0.01M NaOH $= A = 12$
H^+ ion concentration for 0.001M HCl $= 10^{-3}$M
hence, pH of 0.001M HCl $= B = 3$
$(A-B) = 12-3 = 9$ **Ans. 9**.

Q.8 When an aqueous solution of 0.18M $CuSO_4$ is electrolyzed completely, the maximum amount of copper metal deposited at cathode electrode is $X \cdot YZ$ *grams*. (Here X is natural number from 1 to 99, and Y and Z are single digit numbers. e.g. If the answer is 20.13 then $X = 20, Y = 1, Z = 3$)The value of $X =$______.

Solution: The maximum amount of copper metal deposited at cathode electrode
= The maximum amount of copper present in 0.18M $CuSO_4$ = 11.43 g.**Ans. 11**.

Q.9 When two moles of copper metal is treated with excess of dilute nitric acid, the maximum amount of gas produced =______ *grams*.

Solution: $3Cu_{(s)} + 8HNO_{3(dilute)} \longrightarrow 3Cu\,(NO_3)_{2(aq)} + 2NO_{(g)} + 4H_2O_{(l)}$
$\because$ 3 mol of Cu produces 2 mol of $NO_{(g)}$
$\therefore$ 3 mol of Cu can produce $= \frac{4}{3}$ *mol of* $NO_{(g)} = \frac{4}{3} \times 30 = 40$ *grams* **Ans. 40**.

Q.10 Acidic solution of $KMnO_4$ oxidizes ferrous sulphate into ferric sulphate. Using 0.1 mole of $KMnO_4$, the maximum amount of ferrous sulphate can be oxidized = ______*grams*.

Solution: $2KMnO_4 + 10FeSO_4 + 8H_2SO_4 \longrightarrow K_2SO_4 + MnSO_4 + 5Fe_2\,(SO_4)_3 + 8H_2O$
$\because$ 2 mol of $KMnO_4$ *oxidizes* 10 mol of $FeSO_4$
$\therefore$ 0.1 mol of $KMnO_4$ can oxidizes $=\frac{10\times0.1}{2}$ mol of $FeSO_4$
$\frac{10\times0.1}{2}$ mol of $FeSO_4$ = 0.5 mol of $FeSO_4$
0.5 mol of $FeSO_4$ = $0.5 \times 152 = 76$ g **Ans. 76**.

Physics

Use $g = 10\ m/s^2$ wherever required.

Q.11 An object A is dropped from the top of a cliff at $t = 0$ (with zero initial velocity.) At the same instant an object B and an object C are thrown vertically upwards from the bottom of the cliff, i.e. from ground level, with initial velocities of 9 m/sec and 10 m/sec. B crosses A at $t = t_1$ seconds. C crosses A at $t = t_2$ seconds. If $t_1 - t_2 = 1$ second, then find the height of the cliff in meters.

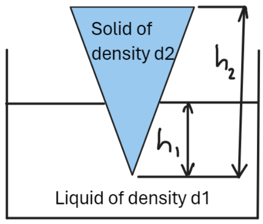
Solution: Let the height of cliff be h meters.
Distance travelled by A in t_1 seconds $= \frac{1}{2} \times 10 \times t_1^2$
Distance travelled by B in t_1 seconds $= 9t_1 - \frac{1}{2} \times 10 \times t_1^2$
$\therefore \frac{1}{2} \times 10t_1^2 + 9t_1 - \frac{1}{2}10 \times t_1^2 = h \therefore 9t_1 = h$
Similarly $10t_2 = h$ but $t_2 + 1 = t_1 \therefore 9\,(t_2+1) = 10t_2 \therefore t_2 = 9 \therefore h = 90$ meters **Ans. 90**.

Q.12 Ram starts running (on a straight track) from rest at constant acceleration of 1 m/sec^2 for some time. He then decelerates at the deceleration of 0.1 m/sec^2 and comes to rest. He has

travelled a total distance of s meters in total t seconds.
Sham starts running from rest (on a straight track) at constant acceleration of $a\ m/sec^2$ and he runs for t seconds. Its found that he has covered a distance of s meters. Find the value of $\frac{1}{a}$.

Solution: Suppose Ram accelerates for t_1 seconds. He reaches velocity v $\therefore v = t_1$. He then decelerates for t_2 seconds to come to rest. $\therefore v = \frac{1}{10}t_2 \quad \therefore \quad t_2 = 10t_1 \quad \therefore t_1 = \frac{1}{11}t \quad t_2 = \frac{10}{11}t$
$\therefore$ distance travelled $= \frac{1}{2}\left(\left(\frac{t}{11}\right)^2 + \left(\frac{1}{10}\right)\left(\frac{10}{11}t\right)^2\right) = \frac{1}{22}t^2$
For Sham, distance travelled$= \frac{1}{2}at^2 \therefore \frac{1}{22}t^2 = \frac{1}{2}at^2 \Rightarrow \frac{1}{a} = 11$ **Ans. 11**.

Q.13 Refer to the diagram. There is a container. A solid in the shape of inverted cone (density d_2) is floating in a liquid of density d_1. The total height of the solid cone is h_2. The height of the portion submerged in the liquid is h_1. If $\frac{h_2}{h_1} = 3$ find $\frac{d_1}{d_2}$.



Solution: By similarity, $\frac{h_1}{r_1} = \frac{h_2}{r_2}$

$$\frac{d_1}{d_2} = \frac{\text{volume of total cone}}{\text{volume of submerged cone}} = \frac{\frac{1}{3}\pi r_2^2 h_2}{\frac{1}{3}\pi r_1^2 h_1} = \frac{h_2^3}{h_1^3} = 27$$

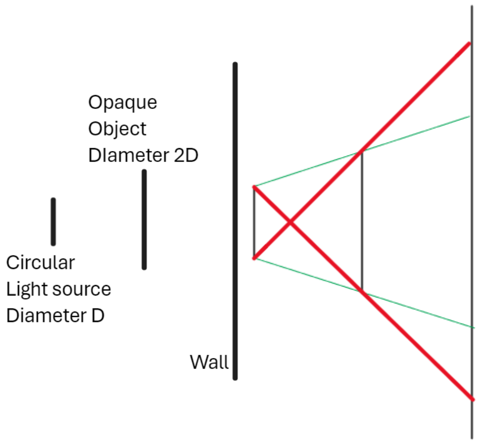
Ans. 27.

Q.14 There are three points A, B, C on a straight line L such that A is on the left side of B and C is on the right side of B. Distance between A and B is d. Distance between B and C is k times d. A static fixed charge of $+Q_A$ coulombs is at point A. A static fixed charge $+31Q_A$ coulombs is at C. A charge $-Q_B$ coulombs is at B. It is observed that the net electrostatic force on the charge at B is F_1. Now, the charge at A is placed at C and charge at C is placed at A, i.e. the charges have interchanged the positions. Now the net elctrostatic force on the charge at B is F_2. It is found that $\frac{|F_2|}{|F_1|} = \frac{125}{3}$. Calculate k -ans. 11

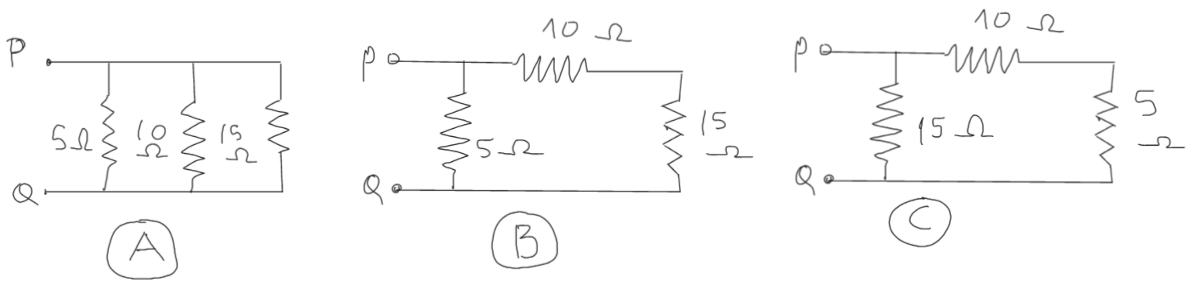
Solution: Let's denote Coulomb's constant by K.
$F_1 = K\left(\frac{Q_AQ_B}{d^2} - \frac{31Q_AQ_B}{k^2d^2}\right)$ and $F_2 = K\left(\frac{31Q_AQ_B}{d^2} - \frac{Q_AQ_B}{k^2d^2}\right)$, so $\left|\frac{125}{3}\right| = \left|\frac{31 - \frac{1}{k^2}}{1 - \frac{31}{k^2}}\right|$.
Simplifying, we get $k = 11$. **Ans. 11**.

Q.15 Refer to the diagram. It shows side view of a circular light source, a circular opaque object and a wall. The object is exactly midway between the wall and the light source. A circular umbra of diameter D_u and a circular penumbra of diameter D_p is seen on the wall. Find $9 \times \frac{D_p}{D_u}$.

Solution: Using similarity, one can easily prove that $D_u = 3D$ and $D_p = 5D$, so $9\dfrac{D_p}{D_u} = 15$
Ans. 15.



Q.16 Refer to the diagram. The equivalent resistances of the three circuits between points P and Q are R_a, R_b, R_c ohms respectively. If $R_a : R_b : R_c = L : M : N$ where L, M, N are natural numbers such that $gcd(L, M, N) = 1$ then calculate $\dfrac{L+M+N}{2}$ -ans. 95



Solution: $\dfrac{1}{R_a} = \dfrac{1}{5} + \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{11}{30}, \ \dfrac{1}{R_b} = \dfrac{1}{5} + \dfrac{1}{25} = \dfrac{6}{25}, \dfrac{1}{R_c} = \dfrac{1}{15} + \dfrac{1}{15} = \dfrac{2}{15},$
so $R_a : R_b : R_c = \frac{30}{11} : \frac{25}{6} : \frac{15}{2} = 180 : 275 : 495 = 36 : 55 : 99$ **Ans. 95**.

Q.17 A solid block of metal of mass 100 gm (specific heat 0.2 $cal/gm°C$) at temperature $660°C$, 500 gm of ice at $0°C$ and 100 gm of steam at $100°C$ are kept in an insulated container. After some time, equilibrium temperature is reached. Calculate the equilibrium temperature in degrees celcius. Assume the following values: Latent heat of fusion of water: 80 cal/gm, latent heat of vaporisation of water = 540 cal/gm, specific heat of water= 1 $cal/gm°C$.

Solution: Let the equilibrium temperature be x and let's make assumption that x is between 0 and 100°C So we get
$(660 - x) \times 100 \times 0.2 + (100)(540) + (100 - x)100 = 500 \times 80 + 500x$
$\Rightarrow 13200 - 20x + 54000 + 10000 - 100x = 40000 + 500x$
$\Rightarrow x = 60$ **Ans. 60**.

Q.18 A bullet of mass 10 gm is fired horizontally. It hits a stationary solid wooden block of mass 200 gm kept on a frictionless surface. The bullet hits the block horizontally at the speed of 42 m/sec. It gets stuck in the block and the block with the bullet starts moving. If the loss of kinetic energy in this process is E joules, write $10E$ as your answer.

Solution: Let the velocity of the block be v m/sec. Using conservation of linear momentum, we get $(10)(42) = (200 + 10)v \Rightarrow v = 2$ m/sec
$\therefore$ loss of energy $= \frac{1}{2}(0.01)(42)^2 - \frac{1}{2}(0.210)(2)^2 = 8.4$ Joules **Ans. 84**.

Q.19 An object is kept in front of a screen. The distance between the object and the screen is 360 cm. A converging lens is kept between them so that a sharp image of the object is obtained on the screen. The lens is nearer to the object than the screen. Now the lens is moved towards the screen to a position such that a sharp image of the object is obtained on the screen. The distance between the two positions of the lens is 120 cm. Calculate the focal length of the lens in centimeters.

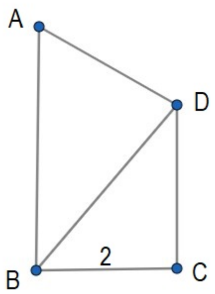
Solution: Let the focal length be f. Let the initial distance of the lens from the object be u.
So, in (case I) we have $\frac{1}{360-u} - \left(-\frac{1}{u}\right) = \frac{1}{f} \Rightarrow \frac{1}{360-u} + \frac{1}{u} = \frac{1}{f}$
In (case II) we have $\frac{1}{240-u} + \frac{1}{120+u} = \frac{1}{f}$
$\therefore \frac{1}{360-u} + \frac{1}{u} = \frac{1}{240-u} + \frac{1}{120+u} \Rightarrow u(360-u) = (240-u)(120+u) \Rightarrow u = 120$
$\therefore \quad \frac{1}{f} = \frac{1}{120} + \frac{1}{240} = \frac{1}{80}$ **Ans. 80**.

Q.20 Ajay starts running from rest at constant acceleration of $a\ m/sec^2$ for time t_1 seconds. Then he runs at constant speed for $4t_1$ seconds. Then he decelerates at constant deceleration of $\frac{a}{2}\ m/sec^2$ till he comes to rest. He has covered a total distance of s meters. Vijay starts running from rest at constant acceleration of $2a\ m/sec^2$ for time $\frac{t_1}{2}$ seconds. Then he runs at constant speed for $4t_1$ seconds. Then he decelerates at constant deceleration of $\frac{a}{k}\ m/sec^2$ till he comes to rest. He also has covered a total distance of s meters. Find k and write $10k$ as your answer.

Solution: Velocity of Ajay at t_1 seconds $= at_1$. Distance travelled in t_1 seconds $= \frac{1}{2}at_1^2$ Distance travelled between t_1 and $5t_1$ seconds $= (at_1)(4t_1) = 4at_1^2$ Since deceleration is half the acceleration, time of deceleration $= 2t$, Distance travelled in deceleration $= \frac{1}{2}\left(\frac{a}{2}\right)(2t_1)^2 = at_1^2$
$\therefore$ total distance travelled $= \frac{1}{2}at_1^2 + 4at_1^2 + at_1^2 = \frac{11}{2}at_1^2$
Velocity of Vijay at $\frac{t_1}{2} = at_1$. Distance travelled in $\frac{t_1}{2}$ seconds $= \frac{1}{2}(2a)\left(\frac{t_1}{2}\right)^2 = \frac{at_1^2}{4}$ Distance travelled in $4t_1$ seconds $= (4t_1)(at_1) = 4at_1^2$ Time taken to come to rest $= \left(\frac{at_1}{a/k}\right) = kt_1$
Distance travelled $= \frac{1}{2}\left(\frac{a}{k}\right)(kt_1)^2$ total distance travelled $= \frac{at_1^2}{4} + 4at_1^2 + \frac{akt_1^2}{2} = \frac{17at_1^2}{4} + \frac{akt_1^2}{2}$
$\therefore \frac{11}{2}at_1^2 = \frac{17at_1^2}{4} + \frac{akt_1^2}{2} \Rightarrow k = \frac{5}{2}$ **Ans. 25**.

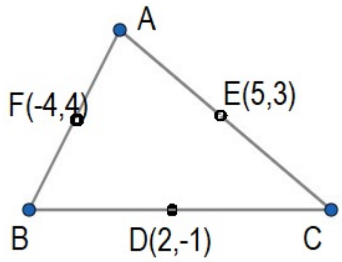
Mathematics

Q.21 In trapezium $ABCD$, $\overline{AB} \| \overline{CD}$, the angle at B is a right angle, and the diagonal BD is perpendicular to the leg AD. The length of the leg BC is 2 , and the length of the diagonal BD is $\sqrt{13}$. The area of the trapezoid $ABCD$ is Δ then 3Δ equals



Solution:
Note $\overline{AB} \| \overline{CD} \Rightarrow m\angle BCD = 90$. By Pythagoras Theorem $CD = 3$
Note $\triangle DBC \sim \triangle BAD \Rightarrow \frac{BC}{DC} = \frac{AD}{BD} = \frac{2}{3} = \frac{AD}{\sqrt{13}} \Rightarrow AD = \frac{2\sqrt{13}}{3}$
Area of $ABCD =$ Area of $BCD+$ Area of BDA
$= \frac{2 \times 3}{2} + \frac{\sqrt{13}}{2} \cdot \frac{2\sqrt{13}}{3}$
$\Delta = 3 + \frac{13}{3} = \frac{22}{3} \Rightarrow 3\Delta = 22$ **Ans. 22**.

Q.22 In $\triangle ABC$, $D(2,-1)$ is midpoint of $\overline{BC}$. $E(5,3)$ is midpoint of $\overline{CA}$. $F(-4,4)$ is midpoint of $\overline{AB}$. G is centroid of $\triangle ABC$. Find AG^2.
Solution:



Let $A(h,k) \Rightarrow x$ coordinate of $c = 10 - h$ and y coordinate of $c = 6 - k$

Similarly x coordinate of $B = 4 - (10 - h) = -6 + h$

and y coordinate of $B = -2 - (6 - k) = -8 + K$

Hence x coordinate of $A = -8 - (-6 + h) = -2 - h$

and y coordinate of $A = 8 - (-8 + k) = 16 - k$

Hence $-2 - h = h \Rightarrow 2h = -2 \Rightarrow h = -1$

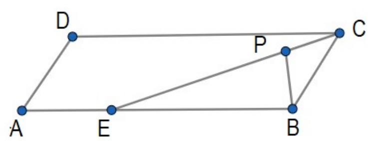
and $16 - k = k \Rightarrow 2k = 16 \Rightarrow k = 8$

Note G divides $\overline{AD}$ in $2:1$

$$AD = \sqrt{(-1-2)^2 + (8+1)^2} = \sqrt{90}$$
$$\frac{AG}{AD} = \frac{2}{3} \Rightarrow AG = \frac{2}{3} \times \Rightarrow \sqrt{90} = 2\sqrt{10}$$
$$\Rightarrow AG^2 = 40$$

Q.23 The parallelogram $ABCD$ has area K . Point E is on side AB such that $\frac{AE}{EB} = \frac{1}{2}$. The point P is on segment EC such that the area of $\triangle EPB$ is one fourth of the area of $ABCD$. If the area of $\triangle CPB$ is 7 find K.

Solution:



Construct $\overline{EF} \| \overline{AD}$ as shown.

Area of $EBCF = \frac{2}{3}K$

Area of $EBC = \frac{1}{2}\left(\frac{2}{3}k\right) = \frac{k}{3}$

Note $\triangle CPB$ and $\triangle EBC$ have collinear base and common height.

$$\Rightarrow \text{ratio of areas} = \text{ ratio of bases.}$$
$$\frac{\text{Area of } \triangle CPB}{\text{Area of } \triangle CEB} = \frac{CP}{CE} = \frac{1}{4}$$
$$\Rightarrow \text{ Area of } \triangle CPB = \frac{1}{4} \times \frac{K}{3} = \frac{K}{12} = 7$$
$$\Rightarrow K = 84$$

Q.24 Let S_i be the set of i consecutive natural numbers written in decreasing order like $S_1 = \{2024\}$, $S_2 = \{2023, 2022\}$, $S_3 = \{2021, 2020, 2019\} \cdots$ and so on. Find the middle term of S_{63}.

Solution:

Note first term in S_i is $2024 - (1 + 2 + 3 + \cdots + (i - 1))$
Hence first term is S_{63} is $2024 - (1 + 2 + \cdots + 62)$
$= 2024 - (\frac{62 \times 63}{2})$
$= 2024 - 1953$
$= 71$

Note last term in S_i is First Term $-(i - 1)$
Hence Last term in $S_{63} = 71 - (63 - 1)$
$= 71 - 62$
$= 9$

Hence middle term $= \frac{\text{First + Last}}{2} = \frac{71+9}{2} = 40$

Q.25 Let $a, b,$ and c be the roots of the equation $x^3 + 6x^2 - 52x + 8 = 0$ which means $(x - a)(x - b)(x - c) = x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = x^3 + 6x^2 - 52x + 8 = 0$, find the value of $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}$.
Solution: Note that equation $x^3 + 6x^2 - 52x + 8 = 0$ can be rearranged as

$$(x + 2)^3 = x^3 + 6x^2 + 12x + 8 = 64x$$

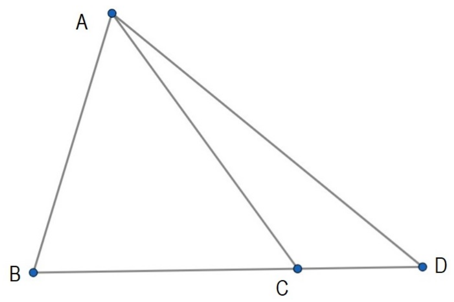
or equivalently

$$\sqrt[3]{x} = \frac{x + 2}{4}.$$

Also note that $a + b + c = -6$, therefore

$$\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = \frac{(a + 2) + (b + 2) + (c + 2)}{4} = \frac{a + b + c + 6}{4} = 0.$$

Q.26 In the diagram, $\triangle ABD$ has C on BD. If $BC = 14, CD = 7, \dfrac{AC}{AD} = \dfrac{3}{4}$, and $\cos(\angle ACB) = \dfrac{3}{5}$. Find AB.



Solution:
Let E be foot of perpendicular from A on $\overline{BD}$.
Given $\cos ACB = \frac{3}{5} \Rightarrow \frac{EC}{AC} = \frac{3}{5}$
Let $EC = 3x$ then $AC = 5x$
Given $\frac{AC}{AD} = \frac{3}{4} \Rightarrow AD = \frac{20x}{4}$

$$BC = 14 \Rightarrow BE = 14 - 3x$$
$$ED = 3x + 7$$

By Pythagoras Theorem $AE = 4x$

$$\text{In } \triangle AED \Rightarrow \left(\frac{20x}{3}\right)^2 = (4x)^2 + (3x + 7)^2$$

Solving we get $x = 3 \Rightarrow AE = 12$ and $BE = 14 - 9 = 5$

$$\Rightarrow AB = 13$$

Q.27 Let $P(x) = x^3 + Ax^2 + Bx + 10$. If $P(1) = P(2) = P(3)$, find $B - A$.
Solution:

$$\begin{aligned} &P(1) = 1 + A + B + 10 \\ &P(2) = 8 + 4A + 2B + 10 \\ &\Rightarrow P(2) - P(1) = 3A + B = -7 \cdots (1) \\ &P(3) = 27 + 9A + 3B + 10 \\ &\Rightarrow P(3) - P(2) = 5A + B = -19 \cdots (2) \\ &(2) - (1) \Rightarrow 2A = -12 \quad A = -6 \end{aligned}$$

Putting in (1) we get $3A + B = -7$

$$\begin{aligned} &\Rightarrow -18 + B = -7 \\ &\Rightarrow B = 11 \\ &\Rightarrow B - A = 11 - (-(6) = 17 \end{aligned}$$

Q.28 Let a be a positive real number such that $\frac{a^2}{a^4 - a^2 + 1} = \frac{4}{37}$. Then $\frac{a^3}{a^6 - a^3 + 1} = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $\frac{n-3}{m}$
Solution:

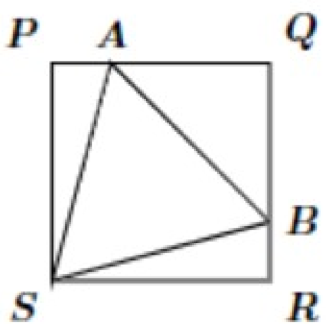
$\frac{a^2}{a^4 - a^2 + 1} = \frac{4}{37} \Rightarrow \Rightarrow \frac{a^4 - a^2 + 1}{a^2} = \frac{37}{4} \Rightarrow a^2 + \frac{1}{a^2} - 1 = \frac{37}{4}$

$\Rightarrow \left(a + \frac{1}{a}\right)^2 = \frac{37}{4} + 3 = \frac{49}{4} \Rightarrow \left(a + \frac{1}{a}\right) = \frac{7}{2}$

$\frac{a^3}{a^6 - a^3 + 1} = \frac{m}{n} \Rightarrow a^3 - 1 + \frac{1}{a^3} = \frac{n}{m} \Rightarrow \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right) - 1 = \frac{n}{m}$

$\Rightarrow \frac{343}{8} - \frac{21}{2} - 1 = \frac{n}{m} \Rightarrow \frac{343 - 92}{8} = \frac{251}{8} \Rightarrow \frac{n-3}{m} = 31$

Q.29 Given that $PQRS$ is a square and that ABS is an equilateral triangle (see the diagram), Let ratio of the area of $\triangle ABQ$ to the area of $\triangle APS$ be λ. Report λ^2.



Solution:
Let $PA = x$ and $AQ = y$ by symmetry $BR = x, QB = y$

$$\begin{aligned} PS &= x + y \\ AS^2 &= x^2 + (x + y)^2 = 2x^2 + 2xy + y^2 \\ &= AB^2 \end{aligned}$$

But $AB^2 = 2y^2$

$$\Rightarrow 2x^2 + 2xy + y^2 = 2y^2$$
$$\Rightarrow 2x^2 + 2xy = y^2$$
$$\Rightarrow 2x(x+y) = y^2$$
$$[PAS] = \frac{x(x+y)}{2} \quad [AQB] = \frac{y^2}{2}$$
$$\frac{[ABQ]}{[PAS]} = \frac{y^2}{2} \times \frac{2}{x(x+y)} = \frac{y^2}{x(x+y)} = 2$$
$$\lambda^2 = 4.$$

Q.30 α and β are the roots of quadratic equation $2024x^2 + 173x - 1 = 0$. Given $\alpha > \beta$, find the value of $\frac{1}{\alpha} + \frac{10}{\beta}$.

Solution:

$2024x^2 + 173x - 1 = 0$

$2024x^2 + 184x - 11x - 1$

$184x(11x+1) - 1(11x+1)$

$\alpha = \frac{1}{184}$ or $\beta = -\frac{1}{11}$

$\frac{1}{\alpha} + \frac{10}{\beta} = 184 - 110 = 74$