M Prakash Institute **Solutions of 26 November 2023** **XI Entrance Test 1**
Each question carries five marks **10 am to 1 pm**

Key:

Q.No.	1	2	3	4	5	6	7	8	9	10
Answer	71	71	56	60	5	15	60	75	24	50
Q.No.	11	12	13	14	15	16	17	18	19	20
Answer	30	17	20	30	45	14	75	20	20	25
Q.No.	21	22	23	24	25	26	27	28	29	30
Answer	1	15	25	24	63	32	11	45	52	10

Chemistry

Atomic numbers:
H:1,, C:6, O:8, Na:11, Mg:12, Al:13, S:16, Cl:17, ,Ca:20, Fe:26, Cu:29, Br; 35 ,I:53 Hg: 80
Atomic masses:
H:1, C:12, O:16, Na:23, Mg:24, Al:27, Cl:35.5, S:32, Ca:40, Fe:56, Cu:63.5, Br:80 , I:127, Hg:200
Avogadro Number : 6×10^{23}

Q.1 In a chemical reaction of regular fire extinguisher a salt is produced. Assume that molar mass of the salt is "X" gram, then enter the value of X/2 in the bubble sheet.

Solution: $2NaHCO_3 \ + \ H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O + 2CO_2 \uparrow$
MM of $Na_2SO_4 = 142$ **Ans. 71**.

Q.2 Determine the amount of gas produced in grams when 1 mole of CO_2 is completely treated with bleaching powder.

Solution: $CaOCl_2 \ + \ CO_2 \rightarrow CaCO_3 + Cl_2 \uparrow$
1 Mole of Cl_2 is produced. **Ans. 71**.

Q.3 Write the sum of total number of molecules of water of crystallisation in green vitriol, blue vitriol, crystalline alum, crystalline Washing soda and Glauber's salt, together.

Solution: 7+5+24+10+10 =56 **Ans. 56**.

Q.4 Determine the mass of NaOH in grams in 2.5 litre of 0.6 M NaOH solution.

Solution: $n = 0.6 \times 2.5 = 1.5$. $\therefore$ Mass $= 1.5 \times 40 = 60$. **Ans. 60**.

Q.5 Count the number of substances from following list that have pH more than 7.
Limewater, tomato juice, vinegar, toothpaste, blood, black coffee, milk, milk of magnesia, solution of baking soda.

Solution: Limewater, toothpaste, blood, milk of magnesia, solution of baking soda are the substances having pH more than 7. **Ans. 5**.

Q.6 From the compounds given below, enter the sum of the valencies of all the metals only in the bubble sheet. $Cu_2O, FeO, MgCl_2, Fe_2O_3, MgCl_2, AlBr_3, HgI_2$

Solution: $1 + 2 + 2 + 3 + 2 + 3 + 2 = 15$ **Ans. 15**.

Q.7 The following reaction takes place in a blast furnace at a temperature of about 1500°C to produce pig iron (impure iron) from iron ore.
$Fe_2O_3 \ + \ 3CO \rightarrow 2Fe \ + \ 3CO_2$
Find the amount (in g) of Fe_2O_3 required to produce 0.75 mol of Iron.

Solution: $Fe_2O_3 \ + \ 3CO \rightarrow 2Fe \ + \ 3CO_2$
1 mol Fe_2O_3 (160 gm) gives 2 mol Fe (112 gm)
So, say, (x) gm Fe_2O_3 gives 0.75 mol (42 gm) Fe.
Calculating, $x = 60gm$. **Ans. 60**.

Q.8 1.125 mol CO_2 gas is produced on thermal decomposition of 150 gm of impure $CaCO_3$. The purity of $CaCO_3$ is ....%.

Solution: 1 mol $CaCO_3$ gives 1 mol CO_2

150 gm ; i.e. 1.5 mol $CaCO_3$is given and CO_2 produced is 1.125 mol.
Thus 1.125 mol; i.e. 1.125 gm of $CaCO_3$ is used. 1.125 g = x% of 1.5 , $x = 75\%$ **Ans. 75**.

Q.9 Three straight chain hydrocarbons; one alkane, one alkene and one alkyne are given. If each hydrocarbon has exactly 4 carbon atoms in it, write the total number of Hydrogen atoms in these 3 compounds together.

Solution: The compounds are C_4H_{10} (n- butane), C_4H_8 (butene), C_4H_6 (butyne)

Total no. of H atoms = 24. **Ans. 24**.

Q.10 9 $\times 10^{25}$ atoms are present in .... mole of water.

Solution: $3 \times 6 \times 10^{23}$ atoms = 1 mol water
$\therefore 9 \times 10^{25} = x$ mol water.
Solving, $x = 50$ mol. **Ans. 50**.

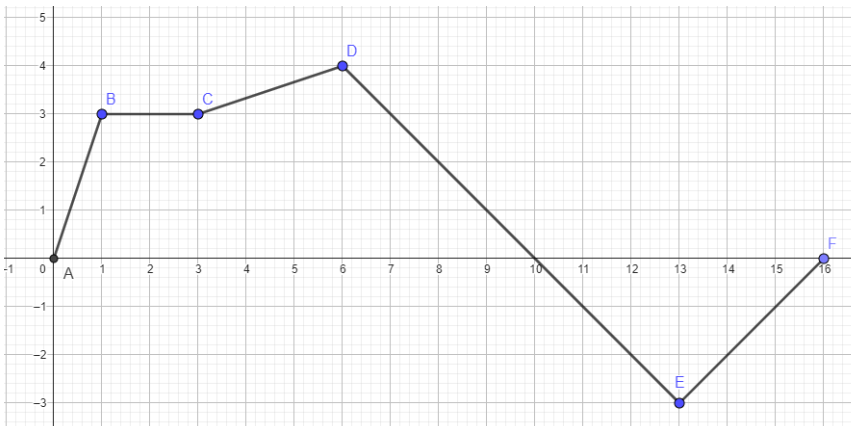
Physics

Use $g = 10\ m/s^2$ wherever required.

Q.11 An object A is dropped from the top of a cliff at $t = 0$ (with zero initial velocity.) At the same instant an object B is thrown vertically upwards with some initial velocity $u\ m/sec$. At $t = 1$, another object C is dropped from the top of the cliff (with initial velocity zero). Objects A and C are in the same vertical line, but object B is not in the same vertical line, i.e. B will not collide with A or C. At $t = 2$ seconds, A and B cross each other. At $t = 2.75$ seconds, objects B and C cross each other. What is the initial velocity of B in meter per second?

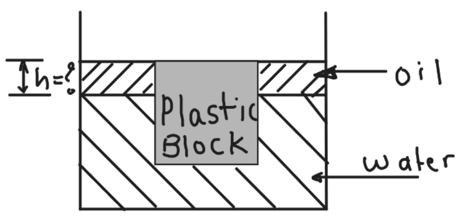
Solution: Let the height of the cliff be h meters. So, when A and B cross, the sum of their displacement is h. Also, when B and C cross, the sum of their displacement is h. So, we have $h = \frac{1}{2}g(2)^2 + (u(2) - \frac{1}{2}g(2)^2) \Rightarrow h = 2u$
Also, for B and C crossing, we have $h = \frac{1}{2}g(1.75)^2 + u(2.75) - \frac{1}{2}g(2.75)^2$. Using $h = 2u$ and simplifying, we get $u = 30\ m/sec$ **Ans. 30**.

Q.12 Refer to the diagram. It is a velocity−time graph with velocity (in m/sec) on Y axis and time in seconds on X axis. Calculate the net displacement of the particle (in meters) from $t = 0$ to $t = 16$.



Solution: Net displacement is equal to the area under the graph. Area below X axis is negative. **Ans. 17**.

Q.13 Refer to the diagram. There is a container. Water is filled in it to a level. There is a layer of oil above water. A cuboid block of plastic is floating in it. The block size is $40\ mm \times 40mm \times 40mm$. The top surface of the block coincides with the top surface of oil. The densities are: water: 1 gm/cc, oil:0.5 gm/cc, plastic: 0.75 gm/cc. Find the height of layer of oil in mm.

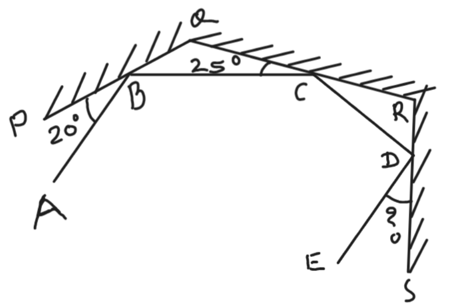


Solution: We have $(40-h)+0.5(h) = 40(0.75) \Rightarrow h = 20$
Ans. 20.

Q.14 There are three points A, B, C on a straight line L such that A is on the left side of B and C is on the right side of B. A static fixed charge of $+Q_A$ coulombs is at point A. A static fixed charge $+Q_C$ coulombs is at C. A charge $-Q_B$ coulombs is at B and it is observed that the net elctrostatic force on the charge at B is zero. $AB = 10cm,\ BC = 20cm$. Now, the charge at A is replaced by a negative charge of the same magnitude, i.e. the charge at A is $-Q_A$ coulombs. The static charge $-Q_B$ is moved at point D which is on the same line L, but is on the left side of A such that the net electrostatic force on the static charge $-Q_B$ is zero. Calculate distance AD in centimeres.

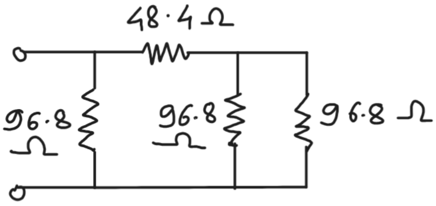
Solution: Since the net force is zero, in the first case we have $K\dfrac{Q_AQ_B}{AB^2} = K\dfrac{Q_BQ_C}{BC^2} \Rightarrow \dfrac{Q_C}{Q_A} = 4.$
In the second case, since net force is zero, we get $K\dfrac{Q_AQ_B}{AD^2} = K\dfrac{Q_BQ_C}{DC^2} \Rightarrow \dfrac{CD}{DA} = 2 \Rightarrow AD = 30$
Ans. 30.

Q.15 Refer to the diagram. PQ, QR, RS are three plane mirrors. Ray AB is reflected as ray BC on mirror PQ. Ray BC is reflected as ray CD on mirror QR. Ray CD is reflected as ray DE on mirror RS. It is found that lines AB and DE are parallel to each other. If $\angle ABP = 20°$ and $\angle BCQ = 25°$ then find $\angle EDS$ in degrees.



Solution: If incident angle at a mirror is θ then the ray deviates from original direction by 2θ. Here, since the final ray is parallel to original ray, total deviation is $180°$. So, we have $2(20 + 25 + \angle EDF) = 180 \Rightarrow \angle EDF = 45°$
Ans. 45.

Q.16 A water heater consisting of resistances as shown in the diagram is connected to a 220 $Volts$ supply. Calculate the time required (in minutes) to raise temperature of 20 liters of water through $10°\ C$ using this heater. Take specific heat capacity of water as $1\ cal/gm°\ C$ and $4.2J = 1\ cal$.



Solution: Amount of energy required to heat the water is $20000 * 10 * 4.2$ joules. Equivalent resistance of the circuit is 48.4 ohms. So, using formula $\frac{V^2}{R}t$, we get $\frac{220^2}{48.4}t = 840000 \Rightarrow t = 840$ seconds , i.e. 14 minutes. **Ans. 14**.

Q.17 A solid block of metal of mass 2205 gm (specific heat 0.1 $cal/gm°C$) at temperature $1100°C$ is put in 1000 gm of ice at $0°C$. All of this is kept in an insulated container. After some time, equilibrium temperature is reached. Calculate the mass of steam present in the container in grams. Take latent heat of vaporisation of water as 540 cal/gm, specific heat capacity of water as 1 $cal/gm°C$ and latent heat of fusion of water as 80 cal/gm.

Solution: Since the question asks for the mass of steam present, it is clear that the final equilibrium temperature is $100°$. So, let's calculate the amount of heat given away by the metal block when it cools from $1100°$ to $100°$.
Heat given= $(2205)(1100-100)(0.1) = 220500$ calories. Suppose m is the mass of steam present. So, we have $220500 = (1000)(80) + (1000)(100) + m(540) \Rightarrow m = 75$. **Ans. 75**.

Q.18 A 10 gm object (A) is moving in a straight line at 1 m/sec. Another object (B) is coming towards A in the same straight line at speed $u\ m/sec$. After collision, A travels with same speed but in the opposite direction. B also travels with same speed but with opposite direction. Now B collides with a stationary object C of mass $m\ gm$. After collision, B starts travelling with the same speed but in the opposite direction. Object C starts moving with speed 1 m/sec. Find the mass of object C in grams.

Solution: Suppose the masses are m_B and m_C.
For the first collision, we have $(10)(1) - (m_B)(u) = -(10)(1) + (m_B)(u) \Rightarrow (m_B)(u) = 10$.
For the second collision, we have $(m_B)(u) + 0 = -(m_B)(u) + (m_C)(1)$.
Using $(m_B)(u) = 10$, we get $m_C = 20$. **Ans. 20**.

Q.19 Image of an object through a convex lens is real and twice the size of the object. When the object is moved towards the lens by 20 cm, its image is virtual and twice the size of the object. Calculate the focal length of the lens in cm.

Solution: For the first case, we have $v = 2u$ and since the image is real, we have v positive and u negative, so we have $\frac{1}{2u} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{3f}{2} = u$
In the second case, now, object distance becomes $u - 20$. $v = 2(u - 20)$, but because the image is virtual, it is negative. So, we have $-\frac{1}{2(u-20)} + \frac{1}{u-20} = \frac{1}{f} \Rightarrow 2(u - 20) = f$. Using $\frac{3f}{2} = u$, we get $f = 20$. **Ans. 20**.

Q.20 A police car is at rest on a straight road. A thief travelling in a car at the speed of 36 km/hr crosses the police. At the instant the thief is crossing the police, the police starts accelerating at a constant rate. The police car reaches a speed of 48 km/hr in 25 seconds and then travels at the same constant speed for t_2 seconds and catches the thief. Find t_2 in seconds.

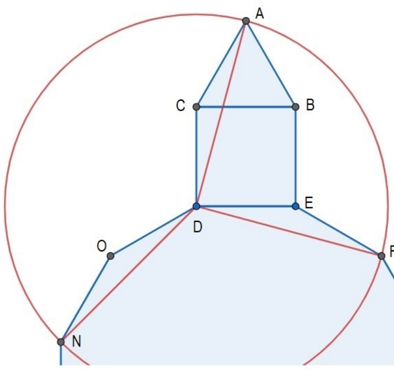
Solution: When the police catch the thief, distance travelled by both is same. First, let's convert the speeds to m/sec. $36\ km/hr = 10\ m/sec$ and $48\ km/hr = \frac{40}{3}\ m/sec$.
Also, we know that for a constant acceleration travel, average velocity is $\frac{u+v}{2}$, so average velocity of the police car in the first 25 seconds is $\frac{20}{3}\ m/sec$.
So, we have $10(25 + t_2) = (\frac{20}{3})(25) + (\frac{40}{3})(t_2) \Rightarrow t_2 = 25$. **Ans. 25**.

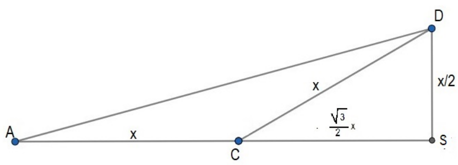
Maths

Q.21 Centroid of the triangle formed by lines
$AB : 13x - 5y + 1 = 0, BC : 6x + 3y + 4 = 0, CA : 2x + 24y - 37 = 0$ is (h, k).
Find the value of $9h + 6k$.

Solution: Solving AB and AC together, we get coordinates of $A\left(\frac{1}{2}, \frac{3}{2}\right)$. Similarly $B\left(-\frac{1}{3}, -\frac{2}{3}\right)$ and $C\left(-\frac{3}{2}, \frac{5}{3}\right)$ Hence coordinates of centroid are $(h, k) = \left(\frac{\frac{1}{2} - \frac{1}{3} - \frac{3}{2}}{3}, \frac{\frac{3}{2} - \frac{2}{3} + \frac{5}{3}}{3}\right) = \left(-\frac{4}{9}, \frac{5}{6}\right)$
$\Rightarrow 9h + 6k = 9\left(-\frac{4}{9}\right) + 6\left(\frac{5}{6}\right) = 1$ **Ans. 1**.

Q.22 Outside to 12 sided regular polygon $DEFGHIJKLMNO$, square $BCDE$ is constructed. Outside to square $BCDE$ equilateral triangle ABC is drawn. If side of equilateral triangle is $\frac{15}{\sqrt{2+\sqrt{3}}}$ then circumradius of $\triangle AFN$ is.





Solution: $\triangle ACD, \triangle DEF$ and $\triangle DON$ are Congruent. They are isosceles with vertex angle $150°$ and side $= \frac{15}{\sqrt{2+\sqrt{3}}}$. Applying Pythagoras Theorem to $\triangle ADS$ we have square of side opposite to $150°$ is $\left((1 + \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2\right)\left(\frac{15}{\sqrt{2+\sqrt{3}}}\right)^2 = 225$. **Ans. 15**.

Q.23 Set A_i has i elements which follow the pattern,
$A_1 = \{1\}, A_2 = \{3, -5\}, A_3 = \{7, -9, 11\}, A_4 = \{13, -15, 17, -19\},$
$A_5 = \{21, -23, 25, -27, 29\}, A_6 = \{31, -33, 35, -37, 39, -41\}$ and so on.
S denotes the sum of elements of the set A_{25}. Find $\sqrt{S}$.

Solution: Last term of A_{25} will be $\frac{25\times 26}{2} = 325^{th}$ term of AP. So $t_{325} = 2(325) - 1 = 649$. Last term $= 649$. Note that 24 terms before that add up to -24 Hence sum of 25 terms of A_{25} $= 649 - 24 = 625 = S, \Rightarrow \sqrt{S} = 25$. **Ans. 25**.

Q.24 $ABCDEF$ is regular hexagon. Point P is in the interior of $\triangle ACD$ such that area of $\triangle PBC = 5$ and area of $\triangle PAD = 6$. Find the area of hexagon.

Solution: Note $ABCD$ is isosceles trapezium $AD = 2BC$.
Let $BC = x \Rightarrow AD = 2x$.
Let height of $\triangle PBC$ from $P = h_1$.
Let height of $\triangle PAD$ from $P = h_2$.
Area of $PBC = [PBC] = 5 = \frac{x\cdot h_1}{2} \Rightarrow x \cdot h_1 = 10$
Area of $PAD = [PAD] = 6 = \frac{2x\cdot h_2}{2} \Rightarrow x \cdot h_2 = 6 \Rightarrow x\,(h_1 + h_2) = 16$.
Note $h_1 + h_2$ is height of $ABCD$.
Area of $ABCD = \frac{(BC+AD)(height)}{2} = \frac{(x+2x)(h_1+h_2)}{2} = \frac{3x(h_1+h_2)}{2} = \frac{3\times 16}{2} = 24$ **Ans. 24**.

Q.25 Suppose that x and y are integers such that $x \geq 5, y \geq 3$,
and $\sqrt{x-5} + \sqrt{y-3} = \sqrt{x+y}$. Find the minimum possible value of xy.

Solution: Squaring both sides, $(x-5) + (y-3) + 2\sqrt{(x-5)(y-3)} = x + y$
$\Rightarrow \sqrt{(x-5)(y-3)} = 4 \Rightarrow (x-5)(y-3) = 16$.
16 can be factored in 5 ways. $(1, 16), (2, 8), (4, 4), (8, 2), (16, 1)$
Now checking value of xy for all cases,
$(x-5) = 1 \Rightarrow x = 6, y - 3 = 16 \Rightarrow y = 19 \Rightarrow xy = 19 \times 6 = 114$
$(x-5) = 2 \Rightarrow x = 7, y - 3 = 8 \Rightarrow y = 11 \Rightarrow xy = 7 \times 11 = 77$
$(x-5) = 4 \Rightarrow x = 9, y - 3 = 4 \Rightarrow y = 7 \Rightarrow xy = 9 \times 7 = 63$
$(x-5) = 8 \Rightarrow x = 13, y - 3 = 2 \Rightarrow y = 5 \Rightarrow xy = 13 \times 5 = 65$
$(x-5) = 16 \Rightarrow x = 21, y - 3 = 1 \Rightarrow y = 4 \Rightarrow xy = 21 \times 4 = 84$ Hence minimum value $= 63$.
Ans. 63.

Q.26 In $\triangle ABC, \cos\angle A = \frac{2}{3}, \cos\angle B = \frac{1}{9}$, and $BC = 24$. Find the length AC.

Solution: Let $\overline{CF}$ be perpendicular from C on AB. $\cos B = \frac{1}{9} = \frac{BF}{BC} = \frac{BF}{24} \Rightarrow BF = \frac{8}{3}$
Applying Pythagoras Theorem to $\triangle CBF$, $CF^2 = BC^2 - BF^2 = 24^2 - \left(\frac{8}{3}\right)^2 = \frac{5120}{9}$
$\cos A = \frac{2}{3} = \frac{AF}{AC} \Rightarrow$ If $AF = 2k$ then $AC = 3k$
Applying Pythagoras Theorem to $\triangle ACF$, $(3k)^2 - (2k)^2 = \frac{5120}{9}$
$\Rightarrow 5k^2 = \frac{5120}{9} \Rightarrow k^2 = \frac{1024}{9} \Rightarrow k = \frac{32}{3} \Rightarrow AC = 32$ **Ans. 32**.

Q.27 $x^4 - 4x^3 + Ax^2 + 12x + B$ is a perfect square of some polynomial with integer coefficients. Find $B - A$.

Solution: As given polynomial is of degree 4. Hence required polynomial will have degree 2. Note coefficient of x^4 is 1. Hence we can assume the required polynomial as $x^2 + ax + b$.
Squaring we get $\left(x^2 + ax + b\right)^2 = x^4 + 2ax^3 + \left(a^2 + 2b\right)x^2 + 2abx + b^2$
Now comparing coefficients we get
$2a = -4 \Rightarrow a = -2$
$2ab = 12 \Rightarrow -4b = 12 \Rightarrow b = -3$
$\Rightarrow A = a^2 + 2b = (-2)^2 + 2(-3) = -2$ and $B = b^2 = (-3)^2 = 9 \Rightarrow B - A = 9 - (-2) = 11$
Ans. 11.

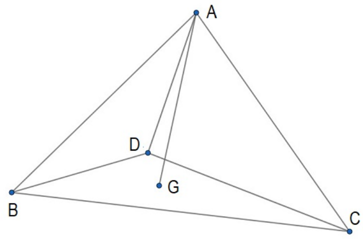
Q.28 For some constant k the polynomial $P(x) = 3x^2 + kx + 117$ has the property that $P(1) = P(10)$. Evaluate $P(3)$

Solution: $P(x) = 3x^2 + kx + 117$
$\Rightarrow P(1) = 3(1)^2 + k(1) + 117 = k + 120$
$\Rightarrow P(10) = 3(10)^2 + k(10) + 117 = 10k + 417$
$\Rightarrow 9k = -297 \Rightarrow k = -33$
$\Rightarrow P(x) = 3x^2 - 33x + 117$
$\Rightarrow P(3) = 27 - 99 + 117 = 45$. **Ans. 45**.

Q.29 X, Y, Z are on circle with center O such that Z and O are on opposite sides of chord XY. A, B are on $\overline{XY}$ such that $OA = OB = ZA = ZB = 5$. If $XY = 12$ then AB^2 equals.

Solution: Note OAZB is rhombus,
$\Rightarrow OZ$ is perpendicular bisector of AB.
$\Rightarrow OZ$ is perpendicular bisector of XY.
$\Rightarrow OXZY$ is rhombus with $OX = OY = OZ$
$\Rightarrow \Delta OXZ$ and $\triangle OYZ$ are equilateral. If radius of circle $= R$ and $XY = 12$
$\Rightarrow \sqrt{3}R = 12 \Rightarrow R = 4\sqrt{3}$,
If M is midpoint of OZ then $OM = 2\sqrt{3}$. Note $\triangle OMX$ is $30 - 60 - 90$.
By Pythagoras theorem $AM^2 = OA^2 - OM^2 = 25 - (2\sqrt{3})^2 = 13$
$\Rightarrow AB^2 = 4AM^2 = 4 \times 13 = 52$ **Ans. 52**.

Q.30 $ABCD$ is regular tetrahedron, 3 dimentional figure with all four faces as equilateral triangles. Find distance of A from face BCD if $AB = 5\sqrt{6}$.



Solution: Note G is centroid of $\triangle BCD$.
Using properties of 30-60-90 triangle if side $= 5\sqrt{6}$, $\quad BG = 5\sqrt{2}$
Applying Pythagoras Theorem to $\triangle ABG$ we get
$AB^2 = BG^2 + AG^2 \Rightarrow (5\sqrt{6})^2 = (5\sqrt{2})^2 + AG^2 \Rightarrow AG^2 = 150 - 50 = 100 \Rightarrow AG = 10$. **Ans. 10**.