M Prakash Institute | **Solutions of XI Entrance Test 3**
30 April 2023 | **2pm to 5pm**
Each question carries five marks

Chemistry
Useful information:
Atomic Masses: H : 1, He : 4, C : 12, O : 16, Na : 23, P : 31, Cl : 35.5, Ca : 40

Q.1)
1 mole of potassium chlorate when slowly heated produces a gas and a salt. What will be the mass in grams of the gas liberated ?

Solution:
$2\ KClO_3 \longrightarrow 2\ KCl + 3\ O_2$
$\uparrow$ (2 mole) $\uparrow$ (3 mole)
$\therefore\ 6.023 \times 10^{23} = 1$ mole $KClO_3 \longrightarrow 1.5$ mole $O_2 = 48$ grams O_2

Answer = 48

Q.2) In the year 1866, scientist Newland arranged known elements in the order of their increasing atomic masses for the periodic classification starting from Hydrogen upto Thorium. According to his periodic law, chemical and physical properties of 4^{th} element would resemble with the x^{th} element. What is 'x' ?

Solution: Law of octaves, in chemistry, the generalization made by the English chemist Newlands in 1865 that, if the chemical elements are arranged according to increasing atomic weight, those with similar physical and chemical properties occur after each interval of seven elements.

Answer = 11

Q.3) x = Number of total atoms in the formula of compound potassium dichromate. Write the value of 'x' as your answer.

Solution: Formula = $K_2Cr_2O_7$; Total atoms = 11

Answer = 11

Q.4) What volume of O_2 gas (in ml) would be required to react with nitrogen monoxide gas to produce 50 ml of nitrogen dioxide gas, under similar conditions of temperature and pressure?
Given: $NO_{(g)} + O_{2\,(g)} \longrightarrow NO_{2\,(g)}$

Solution: Balanced equation = $2\ NO + O_2 \longrightarrow 2\ NO_2$
$\Rightarrow$ For 50 mL of Nitrogen Dioxide gas to be produced, we would require 25 mL of Dioxygen Gas.

Answer = 25

Q.5) What is the total number of acidic radicals among the following?
$SO_4^{2-}, NH_4^+, PO_4^{3-}, F^-, Cl^-, Br^-, Fe^{2+}, Al^{3+}, Na^+$

Solution: Acidic Radicals $\Rightarrow$ $SO_4^{2-}, PO_4{}^{3-}, F^-, Cl^-, Br^-$

Answer = 5

Q.6) Identify from the following the total number of redox reactions $\rightarrow$

a) $H_2S + SO_2 \longrightarrow S \downarrow + H_2O_{(l)}$

b) $KClO_3 \xrightarrow{\Delta} KCl_{(s)} + O_{2\,(g)}$

c) $Mg + O_2 \xrightarrow{\Delta} MgO_{(s)}$

d) $H_3C - CH_3 \xrightarrow{\Delta} H_2C = CH_2 + H_2 \uparrow$

e) $FeSO_4 + KMnO_4 + H_2SO_4 \longrightarrow MnSO_4 + K_2SO_4 + Fe_2\,(SO_4)_3 + H_2O$

f) $CaCO_3 \longrightarrow CaO + CO_2 \uparrow$
g) $NiO + H_2 \longrightarrow Ni \downarrow + H_2O_{(l)}$
h) $CaO + H_2O \longrightarrow Ca(OH)_2 + \text{Heat}$
i) $MnO_2 + HCl \longrightarrow MnCl_2 + H_2O_{(l)} + Cl_2 \uparrow$
j) $NH_{3\ (g)} + HCl_{(s)} \longrightarrow NH_4Cl \uparrow$

Solution: Following are the REDOX reactions:
a) $H_2S + SO_2 \longrightarrow S \downarrow + H_2O_{(l)}$
b) $KClO_3 \xrightarrow{\Delta} KCl_{(s)} + O_{2\ (g)}$
c) $Mg + O_2 \xrightarrow{\Delta} MgO_{(s)}$
d) $H_3C - CH_3 \xrightarrow{\Delta} H_2C = CH_2 + H_2 \uparrow$
e) $FeSO_4 + KMnO_4 + H_2SO_4 \longrightarrow MnSO_4 + K_2SO_4 + Fe_2(SO_4)_3 + H_2O$
g) $NiO + H_2 \longrightarrow Ni \downarrow + H_2O_{(l)}$
i) $MnO_2 + HCl \longrightarrow MnCl_2 + H_2O_{(l)} + Cl_2 \uparrow$

Answer = 7

Q.7) How many -(CH_2)- groups are present in the ester produced when ethanoic acid reacts with ethanol?

Solution: ⇒

$$\underset{\text{(Ethanoic Acid)}}{CH_3COOH} + \underset{\text{(Ethanol)}}{CH_3CH_2OH} \rightarrow \underset{\text{(Ethyl Ethanoate)}}{H_3C - \overset{\overset{O}{||}}{C} - CH_2 - CH_3} + \underset{\text{water}}{H_2O}$$

⇒ Ethyl Ethanoate contains only 1 -(CH_2)- group.

Answer = 1

Q.8) Study the following reactions. Identify compound (C) and write its molecular moss.
$Al_2O_3 + (A) \longrightarrow NaAlO_2 + (B)$
$CH_3COOH + (A) \rightarrow (C) + (B)$

Solution: ⇒ $Al_2O_3 + \underset{(A)}{NaOH_{(aq)}} \longrightarrow NaAlO_2 + \underset{(B)}{H_2O}$

$CH_3COOH + \underset{(A)}{NaOH_{aq}} \longrightarrow \underset{(C)}{CH_3COONa} + \underset{(B)}{H_2O}$

Molar mass of compound (C) = 82

Answer = 82

Q.9) Write the molecular mass of chemical (X) from the chemical reactions given below. (Write the last two digits of the molar mass as your answer, i.e. if molar mass is 156, write 56 as your answer.)
$(X) + H_2SO_4 \rightarrow CaSO_4 + Cl_2 + H_2O$
$Ca(OH)_2 + Cl_2 \rightarrow (X) + H_2O$
$(X) + CO_2 \rightarrow CaCO_3 + Cl_2$

Solution: ⇒ $CaOCl_2 + H_2SO_4 \rightarrow CaSO_4 + Cl_2 + HO_2$
$Ca(OH)_2 + Cl_2 \rightarrow CaOCl_2 + H_2O$
$CaOCl_2 + CO_2 \longrightarrow CaCO_3 + Cl_2$
Molar mass of $CaOCl_2$ = 127

Answer = 27

Q.10) Water of crystallization in 1 mole of Epsom salt ($MgSO_4 \cdot xH_2O$) is represented by 'x'. What is the value of 'x'?

Solution: ⇒ Epson Salt = $MgSO_4.7H_2O \rightarrow$ 7 waters of crystallization.

Answer = 7

Physics

Use $g = 10\ m/s^2$ wherever required.

Q.11 An object is thrown vertically up from a tall building with initial speed $u\ m/s$. The object fell through a distance of 35 meters in the 5th second. Find u.

Solution: Take upward direction as positive. So, we have
$s_5 - s_4 = -35 \Rightarrow (u(5) - \frac{1}{2}(10)(5^2)) - (u(4) - \frac{1}{2}(10)(4^2) = -35 \Rightarrow u = 10$

Q.12 A force of 1 N is applied in a horizontal direction on a block of mass 1 kg which is kept on a frictionless horizontal surface. The force is applied for t seconds. Then, for additional t seconds, there is no force acting on the object. After a total time of $2t$ seconds from the start, a force of 1 N is applied in the opposite direction for t seconds. Total distance travelled by the object in these $3t$ seconds is 200 meters. Find t (in seconds).

Solution: Acceleration $= \frac{F}{m} = 1\ m/s^2$. Since the motion is symmetric in the first and last t seconds, distance travelled is same in the first and last t seconds, which is $= \frac{1}{2}(a)(t^2) = \frac{t^2}{2}$, so total distance travelled in the first and last t seconds is t^2. The velocity reached at the end of t seconds is given by $v = at = t\ m/s$. So, distance travelled under this contstant velocity $= vt = t^2$, so finally we have $t^2 + t^2 = 200 \Rightarrow t = 10$ seconds.

Q.13 A wooden cube (density 0.5 g/cc) of size $40cm \times 40cm \times 40cm$ is floating in water (desnity 1 g/cc). A metal cube (density 4.5 g/cc) of size $20cm \times 20cm \times 20cm$ is now gently placed on the top of the wooden block. What is the height (in cm) of the portion of the metal block which is above water level?

Solution: Suppose the portion above water level is h. So, portion in water is the volume of the wooden cube and some part of metal cube. This is $64000 + 400(20 - h)$ cc. So, the weight of water of this volume is the buyont force. It must be same as the weight of both blocks. So, we get
$64000 + 400(20 - h) = 64000 \times 0.5 + 20^3 \times 4.5 \Rightarrow h = 10.$

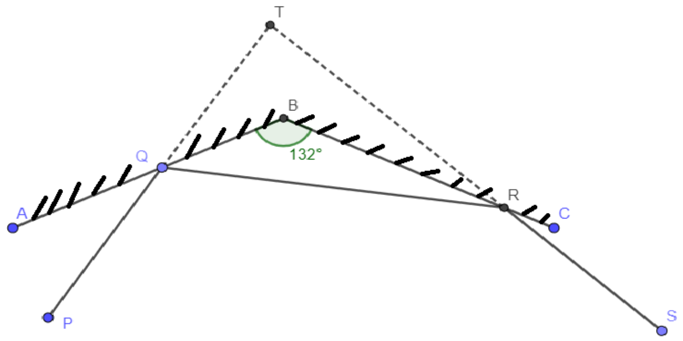
Q.14 Consider a straight line with four points A, B, C, D on it in this order, i.e. A is the leftmost point and then towards the right of A are B, C, D in this order. The points are equally spaced, i.e. $AB = BC = CD$. Static charge $+Q_1$ is present at A and $-Q_1$ at C. Charge $+Q_2$ is present at D. A static charge is at B and the net force on it due to A, C, D is zero. Calculate $\dfrac{Q_2}{Q_1}$

Solution: Suppose the charge at B is $+Q_3$. Let's treat force directed towards right (i.e. in the direction of A to B) as positive. Suppose $AB = BC = CD = d$. We have net force at B $= K\left(\dfrac{Q_1Q_3}{d^2} + \dfrac{Q_1Q_3}{d^2} - \dfrac{Q_2Q_3}{(2d)^2}\right)$ and this must be zero. So, we get (by cancelling K, Q_3 and d^2), $\dfrac{2Q_1}{1} = \dfrac{Q_2}{4} \Rightarrow \dfrac{Q_2}{Q_1} = 8$

Q.15 A water heater consists of three resistances which are R_1, R_1 and R_2 ohms. It is given that $R_2 > R_1$. When the heater is formed by connecting all of them in parallel, it takes t minutes to increase the temperature of 10 liters of water by $20°$. It takes $10t$ minutes to increase the temperature of 10 liters of water by $20°$ when all the resistances are connected in series. Calculate $\frac{R_2}{R_1}$.

Solution: Heat generated by both methods must be same. Suppose the voltage is V. Let $\frac{R_2}{R_1} = k$. When the resistances are in series, the effective resistance is $R_1 + R_1 + kR_1 = (2+k)R_1$. When they are connected in parallel, effective resistance is the reciprocal of $\frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{kR_1}$ i.e. $\dfrac{kR_1}{2k+1}$. So, we get $\dfrac{V^2}{(2+k)R_1}(10t) = \dfrac{V^2}{\frac{kR_1}{2k+1}}t$. This gives $2k^2 - 5k + 2 = 0$, i.e. $k = 2$ or $k = \frac{1}{2}$. But $R_2 > R_1$ is given. So, $k = 2$.

Q.16 As shown in the figure, AB and BC are two plane mirrors. $m\angle ABC = 132°$. A light ray PQ is reflected as ray QR, which is reflected from the second mirror as ray RS. $m\angle AQP = 35°$. The rays PQ and RS when extended backward, intersect at T. Find $m\angle PTS$. (The figure drawn here is not to scale.)



Solution: $m\angle AQP = m\angle TQB = m\angle BQR = 35° \Rightarrow m\angle QRB = 180 - m\angle BQR - m\angle QBR = 13°$. But $m\angle TRB = m\angle BRQ$, so we have $m\angle TQR = 2 \times 35 = 70°$ and $m\angle TRQ = 2 \times 13 = 26°$. So, $m\angle PTS = m\angle QTR = 180 - 70 - 26 = 84°$.

Q.17 Image of an object through a convex lens is real and half the size of the object. When the object is moved towards the lens by 10 cm, its image is real and of the same size of the object. Calculate the focal length of the lens in cm.

Solution: Let the focal length be f and initial object distance be u. So, numerically, image distance is $\frac{u}{2}$. When we consider the sign convention, we have to take object distance as $-u$, foacl length as f and image distance as $\frac{u}{2}$
So, we have $\frac{2}{u} - (-\frac{1}{u}) = \frac{1}{f} \Rightarrow 3f = u$. In the second case, we get $\frac{1}{u-10} - (-\frac{1}{u-10}) = \frac{1}{f}$. Equating the two, we get $\frac{3}{u} = \frac{2}{u-10} \Rightarrow u = 30 \Rightarrow f = 10\ cm$.

Q.18 A solid block of metal of mass 600 gm (specific heat 0.1 $cal/gm°C$) at temperature $1500°C$ is put in 950 gm of water at $40°C$. All of this is kept in an insulated container. After some time, equilibrium temperature is reached. Calculate the mass of steam present in the container in grams. Take latent heat of vaporisation of water as 540 cal/gm and specific heat capacity of water as 1 $cal/gm°C$.

Solution: Since mass of steam is asked, clearly the equilibrium temperature is either $100°C$ or more. So, water will absorb heat to reach $100°C$ which is $950 \times (100 - 40) = 57000\ cal$. Heat given away by metal to reach $100°C$ is $600 \times (1500 - 100) \times 0.1 = 84000\ cal$. So, remaining heat available is $84000 - 57000 = 27000\ cal$. If x is the mass of water getting converted to steam at $100°C$, then we have $27000 = 540x \Rightarrow x = 50$.

Q.19 Two point masses A (mass 10 kg, speed $= 1\ m/s$) and B (mass 20 kg, speed $= 4\ m/s$) are moving on a straight line in the same direction. B is behind A. At the same time, point mass C (mass 30 kg, speed $= u\ m/s$) is coming towards them on the same straight line from the opposite side. All three collide at the same instant. Their velocity after collision is zero. Calculate u in m/s.

Solution: We have $10(1) + 20(4) - 30(u) = 0 \Rightarrow u = 3$.

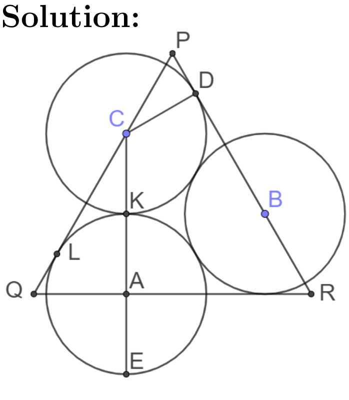
Q.20 Team A and team B are participating in a relay race. Player A_1 from team A and B_1 from team B are at the start line. Once the match starts, i.e. at $t = 0$,
A_1 starts with acceleration of $1.2\ m/s^2$. A_1 runs with this acceleration for 5 seconds and then runs with constant speed till he reaches the 75 m mark where he passes the baton to player A_2. A_2 starts from rest with acceleration of $1\ m/s^2$, maintains acceleration for 5 seconds and then runs with constant speed.
B_1 starts with $1\ m/s^2$, maintains acceleration for 6 seconds and then runs with constant speed. He hands over the baton to B_2 who is standing at the 72 m mark. B_2 starts from rest with acceleration $\frac{5}{3}\ m/s^2$, maintains the acceleration for 5 seconds and runs with constant speed thereafter. At what time in seconds after $t = 0$ do B_2 and A_2 cross each other?

Solution: Speed of A_1 after 5 seconds is $6\ m/s$. Distance travelled in these 5 seconds is $\frac{1}{2}(1.2)(5^2) = 15\ m$. So, A_1 has to travel 60 meters more to reach the 75 meters mark. He needs10 more seconds. So, A_1 reaches the 75 meter mark in 15 seconds.
Speed of B_1 after 6 seconds is $6\ m/s$. Distance travelled in these 6 seconds is $\frac{1}{2}(1)(6^2) = 18\ m$. He needs to run 54 more meters to reach the 72 meters mark. He needs 9 more seconds to travel this. So B_1 reaches the 72 meters mark in 15 seconds.
So, B_2 and A_2 start running at the same time. We don't know whether B_2 crosses A_2 while they are accelerating. Suppose, it is so, and B_2 crosses A_2 in t seconds after he starts, then at the time of crossing, he would have run 3 meters more than A_2.
So, we get, $(\frac{1}{2})(\frac{5}{3})t^2 = 3 + \frac{1}{2}(1)t^2 \Rightarrow t = 3$, so B_2 crosses A_2 after three seconds. So, the answer is $15 + 3 = 18$.

Maths

Q.21 The diagram shows three touching semicircles with radius 1 inside an equilateral triangle, which each semicircle also touches. The diameter of each semicircle lies along a side of the triangle. What is the area of square whose length of side is same as that of the equilateral triangle?

Solution:



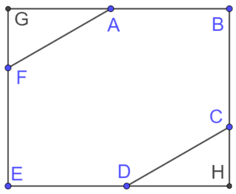
$\triangle CPD$ is $30 - 60 - 90$ triangle
$CD = 1$ So $CP = \frac{2}{\sqrt{3}}, PD = \frac{1}{\sqrt{3}}$
$QL = PD = \frac{1}{\sqrt{3}}$
$CL^2 = CK \cdot CE = 3$
$\therefore CL = \sqrt{3}$
$\therefore PQ = PC + CL + 2Q$
$= \frac{2}{\sqrt{3}} + \sqrt{3} + \frac{1}{\sqrt{3}}$
$= 2\sqrt{3}$ **Ans. 12.**

Q.22 For a given arithmetic series the sum of the first 50 terms is 1275 , and the sum of the first 148 terms is 6882 . What is the first term of the series?

Solution: Let the first term be a and common difference d.
So we have $1275 = 25(2a + 49d)$ and $6882 = 74(2a + 147d)$
$\Rightarrow 51 = 2a + 49d,\ 93 = 2a + 147d \Rightarrow 42 = 98d \Rightarrow d = \frac{3}{7} \Rightarrow a = 15$ **Ans. 15.**

Q.23 A hexagon has consecutive angle measures of $90°, 120°, 150°, 90°, 120°$ and $150°$. If all of its sides are 4 units in length, the area of the hexagon is $K\left(1 + \dfrac{1}{\sqrt{3}}\right)$. Find K?

Solution:



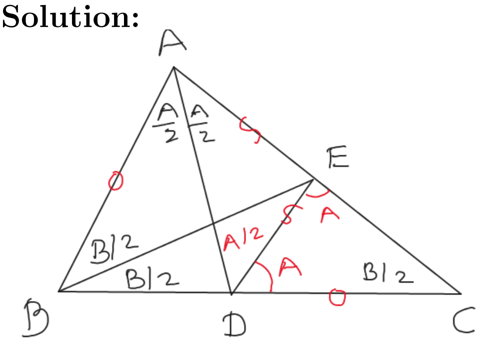
$\triangle CHD$ is a 30-60-90 triangle. We get $CH = 2$ and $HD = 2\sqrt{3}$. Similarly $GF = 2$ and $AG = 2\sqrt{3}$. So, $\square GBHE$ is a rectangle of sides 6 and $4 + 2\sqrt{3}$.Required area = Area($\square GBHE$) − 2 Area($\triangle CHD$) $= (6)(4 + 2\sqrt{3}) - 2(2\sqrt{3}) = 24 + 8\sqrt{3} = 24\left(1 + \dfrac{1}{\sqrt{3}}\right)$ **Ans. 24.**

Q.24 There are four unequal, positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

Solution: Let $S = a + b + c$. So, we get $N = 5a + 3b + 5c = (5a + 5b + 5c) - 2b = 5S - 2b$ and also $N = 4a + 5b + 4c = 4a + 4b + 4c + b = 4S + b$. This gives $5S - 2b = 4S + b \Rightarrow S = 3b \Rightarrow N = 5(3b) - 2b = 13b$. Since N is between 131 and 150 and it is divisible by 13, we get $N = 143 \Rightarrow b = 11 \Rightarrow S = 33$ **Ans. 33.**

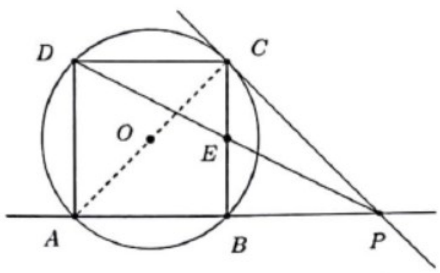
Q.25 In triangle $ABC, \angle B = 2\angle C$. Ray AD is the angle bisector of $\angle A$ and $DC = AB$. Then the measure of $\angle A$ is

Solution:



In $\triangle ABE$ and $\triangle DCE$,
$\angle ABE = \angle DCE = B/2,\ AB = DC,\ BE = CE$
$\therefore \triangle ABE \cong \triangle DCE$
$\therefore \angle BAE = \angle CDE = \angle A$ and $AE = DE$
$\therefore \angle ADE = A/2 \quad \therefore \angle DEC = \angle A \quad \therefore DE || AB$
$\therefore \angle ABD = \angle EDC \therefore \angle B = \angle A = 2\angle C$
$\therefore \angle C = 36°$ and $\angle A = \angle B = 72°$ **Ans. 72.**

Q.26 $ABCD$ is a square inscribed in a circle of radius 1 unit. The tangent to the circle at C meets AB produced at P. Find PD^2



Solution: Clearly, $\angle ACB = \angle PCB = 45°$, so $\triangle ACB \cong \triangle PCB$ so $AB = PB$. Since radius of the circle is 1 unit, sides of the square are $\sqrt{2}$. So, $PD^2 = PA^2 + AD^2 = (2\sqrt{2})^2 + (\sqrt{2})^2 = 10$. **Ans. 10.**

Q.27 If $f(x) = ax + b$ and $f(f(f(x))) = 27x + 26$ then $a + b =$

Solution: $f(f(x)) = a(ax + b) + b = a^2x + ab + b \Rightarrow f(f(f(x))) = a^2(ax + b) + ab + b = a^3x + (a^2 + a + 1)b \Rightarrow a = 3, b = 2$. **Ans. 5.**

Q.28 In $\triangle ABC, AB = 13, AC = 20$ and median $AM = \dfrac{\sqrt{697}}{2}$. Find area of $\triangle AMB$.

Solution: Using Apollonius theorem, $AB^2 + AC^2 = 2(AM^2 + BM^2)$
$\Rightarrow 169 + 400 = 2(\frac{697}{4} + BM^2)$ Simplifying, we get $BC = 21$. area of $\triangle AMB = \frac{1}{2}$area of $\triangle ABC$. Calculate area of $\triangle ABC$ using Heron's formula. **Ans. 63.**

Q.29 $A(3,7), P\left(\frac{47}{11}, \frac{91}{11}\right), B(5,9)$ are collinear and P is between A, B.

$C(-10,1), Q\left(\frac{-22}{13}, \frac{-35}{13}\right), D(-1,-3)$ are collinear and Q is between C, D.

Let $\lambda_1 = \dfrac{AP}{PB}$ and $\lambda_2 = \dfrac{CQ}{QD}$. Find $\lambda_1 \times \lambda_2$.

Solution: Since it is already given that the points are collinear, it is enough to use only x coordinate to calculate the ratios. $\dfrac{AP}{PB} \times \dfrac{CQ}{QD} = \dfrac{\frac{47}{11} - 3}{5 - \frac{47}{11}} \times \dfrac{-\frac{22}{13} + 10}{-1 + \frac{22}{13}} = \dfrac{14}{8} \times \dfrac{108}{9} = 21$
Ans. 21.

Q.30 n is a natural number. It is given that

$$(n + 1999) + (n + 2000) + \cdots + (n + 2023)$$

is a perfect square. The least value of n is

Solution: $(n+1999)+(n+2000)+\cdots+(n+2023) = 25n+\frac{25}{2}(1999+2023) = 25(n+2011)$, so $n = 14$ **Ans. 14.**

Answer Key:

1	2	3	4	5	6	7	8	9	10
48	11	11	25	5	7	1	82	27	7
11	12	13	14	15	16	17	18	19	20
10	10	10	8	2	84	10	50	3	18
21	22	23	24	25	26	27	28	29	30
12	15	24	33	72	10	5	63	21	14