M Prakash Institute **29 January 2023** **XI Entrance Test 2**
Each question carries five marks **2pm to 5pm** **Paper Type AD**

Chemistry

Useful information:
Atomic Masses: H : 1, He : 4, C : 12, O : 16, Na : 23, Al : 27, P : 31, Cl : 35.5, K : 39

Q.1 What is the molecular mass of the third homologue of ethylene?

Solution: A homologous series is a collection of compounds with the same general formula that differ only in the carbon chain length. Compounds in a homologous series often have a fixed set of functional groups, resulting in chemical and physical properties that are comparable. For example, homologous series of alkane: CH_4, C_2H_6, C_3H_8, etc. are homologous. They differ from each other by $(-CH_2)$ unit.
homologue is the every next member in a homologous series. So, the series of Alkenes have members such as follows:

IUPAC Name	*Molecular Formula*
Ethene	C_2H_4
1*st homologue* : *Propene*	C_3H_6
2*nd homologue* : *Butene*	C_4H_8
3*rd homologue* : *Pentene*	C_5H_{10}

Molar Mass of $C_5H_{10} = 70$ **Ans. 70.**

Q.2 An element 'X' belonging to third period of the modern periodic table has tendency of forming amphoteric oxide. Identify the element and write the total number of electrons in its valance shell.

Solution: When an element reacts chemically, both with a base or an acid is termed as amphoteric. As aluminium oxide Al_2O_3, react with both acids and bases, it is known as Amphoteric Oxide. The Chemical equations are given below:
(i) $Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O$.
(ii) $Al_2O_3 + 2NaOH \rightarrow 2NaAlO_2 + H_2O$.
So, the element is Aluminium.Its electronic configuration is (2,8,3). Hence total valance electrons in it = 3 **Ans. 3.**

Q.3 Calculate the mass in grams of 18.066×10^{24} molecules of H_2 gas.

Solution: 6.022×10^{23} molecules of any gas = 1 mole of that gas
Refering to the question,
18.066×10^{24} molecules of H_2 gas. = 30 mole of H_2 gas = 60 grams of H_2 gas. **Ans. 60.**

Q.4 Calculate the total number of neutrons in one atom of the element with the smallest atom from the following list:-
$_{12}(Mg)^{24}, _{13}(Al)^{27}, _{15}(P)^{31}, _{16}(S)^{32}, _{17}(Cl)^{37}$

Solution: Atomic radius goes on decreasing while going from left to right within a period. The reason behind this : While going from left to right within a period, the atomic number increases one by one, meaning the positive charge on the nucleus increases by one unit at a time. However, the additional electron gets added to the same outermost shell. Due

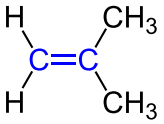
to the increased nuclear charge the electrons are pulled towards the nucleus to a greater extent and thereby the size of the atom decreases.
Hence, smallest atom among the given is Chlorine, i.e., ${}_{17}(Cl)^{37}$.
Total number of neutrons in ${}_{17}(Cl)^{37} = 37 - 17 = 20$. **Ans. 20.**

Q.5 Identify from the following list the number of elements which will not react with $H_2O_{(l)}$ in the absence of air even at high temperatures.
Sodium, Gold, Silver, Mercury, Potassium, Calcium, Helium, Lithium, Argon

Solution: Scientists have developed the reactivity series by doing many experiments of single displacement reaction. The arrangement of metals in the increasing or decreasing order of reactivity is called the reactivity series of metals. Metals are divided into the following groups according to their reactivity.
1. Highly reactive metals (Potassium, Sodium, Lithium, Calcium).
2. Moderately reactive metals (Magnesium, Aluminium, Zinc, Iron, Tin, Lead)
3. Less reactive metals (Copper, Mercury, Silver, Gold).

Hence, the elements which will not react with $H_2O_{(l)}$ in the absence of air even at high temperatures = Gold, Silver, Mercury, Helium, Argon.
Total elements = 5 **Ans. 5.**

Q.6 What is the total number of hydrogen atoms present as methyl groups of isobutylene molecule?
Solution: Here is the expanded structural formula of isobutylene:

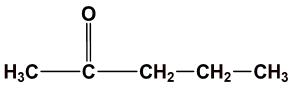


The total number of Hydrogen atoms present in methyl groups of isobutylene = 6 **Ans. 6.**

Q.7 Scientist Mendeleev kept vacant places for the elements not discovered till his times. He named the undiscovered elements as 'eka' which show resemblance with the chemical and physical properties of some elements known to him. For example, element (X) kept in modern periodic table was named as eka-Aluminium by Mendeleev. Identify 'X' and write its atomic number as your answer.

Solution: Eka-Aluminium = Gallium. Atomic Number of Gallium = 31 **Ans. 31.**

Q.8 How many $(-CH_2-)$ groups are present in pentan-2-one?
Solution: Pentan-2-one molecule is as shown. Total number of $(-CH_2-)$ groups in it are 2 **Ans. 2.**



Q.9 Element 'X' is the most electropositive element of period 4 from Modern Periodic table. What is the total number of neutrons in the cation formed by 'X' to achieve the stable configuration of its nearest Noble gas?

Solution: The most electropositive element of period 4 from the Modern Periodic table is Potassium. It forms uni-positive cation for achieving the configuration of the nearest Noble Gas which is Argon. However the total number of neutrons in this cation ; i.e.; K^+ remains unchanged.
Number of neutrons in $K^+ = 39 - 19 = 20$ **Ans. 20.**

Q.10 Identify the total number of possible chemical reactions from the following unbalanced chemical equations:
a) $Cu_{(s)} + 2\ HCl_{(dilute)} \rightarrow CuCl_{2(aq)} + H_2 \uparrow$
b) $Cu_{(s)} + ZnO_{(s)} \overset{\Delta}{\rightarrow} Zn_{(s)} + CO \uparrow$

c) $MnO_{2(s)} + HCl_{(aq)} \rightarrow MnCl_{2(aq)} + H_2O_{(l)} + Cl_2 \uparrow$
d) $CaCO_{3(s)} \xrightarrow{\Delta} CaO_{(s)} + CO_{2(g)}$
e) $Cu_{(s)} + FeSO_{4(aq)} \rightarrow Fe_{(s)} + CuSO_{4(aq)}$
f) $H_2O_{(l)} + CO_{2(g)} \rightarrow H_2CO_{3(aq)}$
h) $BaSO_4 \downarrow + ZnCl_{2(aq)} \rightarrow ZnSO_{4(aq)} + BaSO_{4(aq)}$
i) $CaO_{(s)} + H_2O_{(l)} \rightarrow Ca(OH)_{2(aq)}$+ Heat
j) $SO_{2(g)} + H_2S_{(g)} \rightarrow H_2O_{(l)} + S$
k) $H_2S_2O_7 + H_2O_{(l)} \rightarrow H_2SO_4$

Solution: The possible chemical reactions from the following unbalanced chemical equations are:

c) $MnO_{2(s)} + HCl_{(aq)} \rightarrow MnCl_{2(aq)} + H_2O_{(l)} + Cl_2 \uparrow$
d) $CaCO_{3(s)} \xrightarrow{\Delta} CaO_{(s)} + CO_{2(g)}$
f) $H_2O_{(l)} + CO_{2(g)} \rightarrow H_2CO_{3(aq)}$
i) $CaO_{(s)} + H_2O_{(l)} \rightarrow Ca(OH)_{2(aq)}$+ Heat
j) $SO_{2(g)} + H_2S_{(g)} \rightarrow H_2O_{(l)} + S$
k) $H_2S_2O_7 + H_2O_{(l)} \rightarrow H_2SO_4$
Total reactions = 6 **Ans. 6.**

Physics

Use $g = 10\ m/s^2$ wherever required.

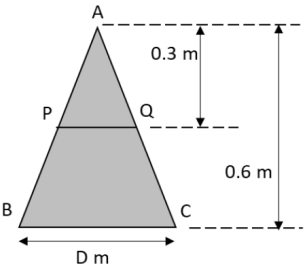
Q.11 A large weather balloon is rising vertically up at a constant speed of 8 m/s. When it was 132 m above the ground, a small steel part of mass 10 gm got detached from the balloon and fell to the ground. When the part hit the ground, the balloon was D m away from it. Find $\frac{D}{2}$ and write that as your answer. Do not consider any air resistance/wind force.

Solution: The initial velocity of the steel part is same as the balloon, i.e. 8 m/s. (upwards) The time taken by it to reach ground is given by $132 = -8t + \frac{1}{2}(10)t^2 \Rightarrow t = 6$. (convention followed is upwards negative) In six seconds, the balloon would have further gone up by $8 \times 6 = 48$ meters. So, the total distance between them when the object hits the ground is $48 + 132 = 180$ meters. **Ans. 90.**

Q.12 A block has 10 kg mass. A strong wind force of magnitude equal to half the weight of the block is acting towards east. At a certain moment, velocity of the block is u m/s towards east and it is at point P. At that moment, another force towards west started acting on the block. Magnitude of that force is 1.3 times the weight of the block. The block came to a halt at point Q which is 4 m away from point P. Calculate u and write that as your answer.

Solution: At P when the second force starts acting, the resultant force on the blcok is $(1.3 - 0.5 = 0.8)$ times the weight, i.e. 80 N. So, the acceleration of the block is 8 m/s^2 towards west, whereas the velocity is u m/s towards east. So, we have final velocity zero, distance 4 meters, so we get $0 = u^2 - 2(8)(4) \Rightarrow u = 8$ **Ans. 8.**

Q.13 Observe the diagram of the cone given below and answer the question.

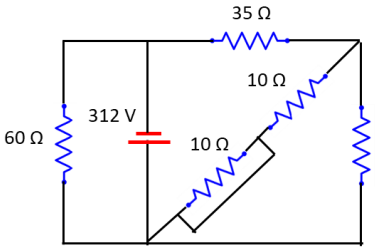


A solid metal cone ABC is stationary and it is floating in a liquid of density 16 g/cc. The axis through A is vertical and the liquid surface is along line PQ. Find density d of the metal in unit g/cc.

Solution: The mass of the cone is equal to the mass of displaced liquid. Mass of liquid $= (16)\left(\frac{1}{3}\pi\frac{D^2}{4} \times 0.6 - \frac{1}{3}\pi(\frac{D^2}{16}) \times 0.3\right)$ mass of cone $= d\left(\frac{1}{3}\pi\frac{D^2}{4} \times 0.6\right)$. Equating the two, we get $d = 14$ **Ans. 14.**

Q.14 Q is a positive number. Charge on a single electron is $-Q$ Coulomb. There are two stationary and electrically neutral particles X and Y which are $1.5Q$ m apart. Later, n electrons move from X to Y. Now, the force between the X and Y is 288 Newton. Take the value of force constant in Coulomb's law as 18×10^{-2} $N{-}m^2/Coulomb^2$. Find number n. If the force is that of attraction write $(n+1)$ as your answer. If the force is repulsion, write $(n-1)$ as your answer.

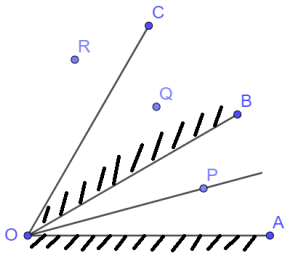
Solution: Charge on $Y = -nQ$ and charge on $X = +nQ$. So, force is that of attraction. $F = 18 \times 10^{-2}\frac{(nQ)(nQ)}{(1.5Q)^2} = 288 \Rightarrow n = 60$ since force is attractive, answer is 61. **Ans. 61.**



Q.15 Find current passing through the battery and express your answer in Ampere.

Solution: Because of shorting, one resistance of 10 ohms on the diagonal is bypassed. The other diagonal 10 ohms is in parallel with the side 10 ohms giving effective resistance 5 ohmas. It is in series with the 35 ohms, so effective resistance is 40. It is in parallel with the 60 ohms so, total effective resistance is $\frac{40\times60}{40+60} = 24$ ohms. So, current $= \frac{312}{24} = 13$ amperes. **Ans. 13.**

Q.16



As shown in the figure, OA and OB are two plane mirrors with their reflecting surfaces facing each other. $m\angle BOA = 30°$. An object P is exactly midway between the mirrors, i.e. $m\angle POA = 15°$. It is moving towards O at the uniform speed of 20 m/s. Its image in the mirror OB is Q. OC is the image of the mirror OA in mirror OB. R is the image of Q in OC. Find the speed with wchich R is moving towards P.

Solution: Observe that at every instant, $\triangle POR$ is an equilateral triangle. So, suppose at $t = 0$, P is at 40 meters from O. So, $PR = 40$. At $t = 1$, $OP = 20$ so $PR = 20$. So, speed of R towards P is also 20. **Ans. 20.**

Q.17 A convex lens is put between an object and a screen. A sharp inverted image of the object is formed on the screen. Focal length of the lens is 10 cm. Now the lens is shifted towards the object by 15 cm. Again a sharp inverted image of the object is formed on the screen. Find the initial distance (in cm) between the object and the lens.

Solution: Let u and v be the unsigned distances of the object and image from the pole of the lens. So, first equation is $\frac{1}{v} - \frac{1}{-u} = \frac{1}{10}$. Second situation gives $\frac{1}{v+15} - \frac{1}{-(u-15)} = \frac{1}{10}$. Solving, we get $u = 30$ or $u = 5$. $u5$ will give a virtual image. So $u = 30$ **Ans. 30.**

Q.18 A solid block of Cobalt of mass 225 gm (specific heat 0.1 $cal/gm°C$) at temperature $410°C$ is put on 100 gm of ice at $0°C$. All of this is kept in an insulated container. After some time, equilibrium temperature is reached. Calculate the equilibrium temperature in $°C$. Take latent heat of fusion of ice as 80 cal/gm and specific heat capacity of water as 1 $cal/gm°C$.

Solution: Let the equilibrium temp be x.
So heat given away by Cobalt $= 225(0.1)(410 - x)$
heat absorbed by ice (1) $0°$ ice to $0°$ water $= 100 \times 80 = 8000$ cal
(2) water from $0°$ to $x° = 100 \times 1 \times x$
Equating, we get, $22.5 \times 410 - 22.5x = 8000 + 100x \quad \therefore x = \frac{22.5 \times 410 - 8000}{122.5} = 10$ **Ans. 10.**

Q.19 A father of mass 70 kg is skating along a straight line at uniform speed of 3 m/s. His daughter (mass 35 kg) is skating towards him along the same from behind him at uniform speed of 6 m/s. She reaches him and holds him. They continue to move along the same line. Suppose the loss of kinetic energy in this collision is E joules. Then mark $\frac{E}{5}$ as your answer.

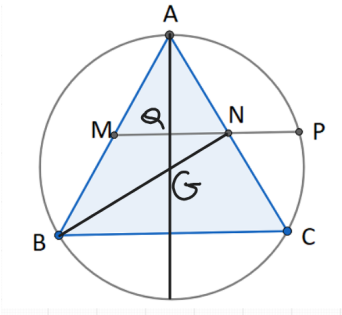
Solution: Suppose the velocity after the two start skating together is v. So, by COLM, we get $70 \times 3 + 35 \times 6 = 105v \Rightarrow v = 4$. So, loss of kinetic energy
$= \frac{1}{2}(70 \times 3^2 + 35 \times 6^2 - 105 \times (4)^2) = 105$ Joules. **Ans. 21.**

Q.20 Two cars A and B are parked on a straight road. Distance between them is 300 m. They start moving towards each other at the same time. Car A starts from rest at uniform acceleration of 1 m/sec^2 and continues with this acceleration for 5 sec. It then decelerates at 1 m/sec^2 for 5 sec. It continues moving in this cycle of acceleration and deceleration. Car B starts to move towards A with uniform acceleration of 2 m/sec^2 and continues with this acceleration for 5 sec and then decelerates at 2 m/sec^2 for 5 sec. It continues moving in this cycle of acceleration and deceleration. Find the time in seconds after which they meet.

Solution: Observe that car A travels 25 meters in the 10 seconds cycle and car B travels 50 meters in the 10 seconds cycle. So, the distance between them reduces by 75 meters every 10 seconds. So, they meet after 40 seconds. **Ans. 40.**

Maths

Q.21 In equilateral triangle ABC of side length $\sqrt{45}+3$, suppose that M and N are the mid-points of AB and AC, respectively. The triangle is inscribed in a circle. Ray MN meets the circle at P. Determine the length of the line segment NP.



Solution: Suppose G is the circumcenter. Let $GA = R \therefore AB = R\sqrt{3} \Rightarrow AM = MN = \frac{R\sqrt{3}}{2}$

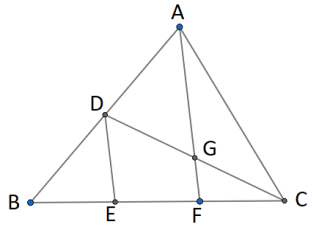
$MQ = \frac{1}{2}AM = \frac{R\sqrt{3}}{4} = QN \quad GM = \frac{R}{2} \quad \therefore GQ^2 = GM^2 - MQ^2 = \frac{R^2}{4} - \frac{3R^2}{16} = \frac{R^2}{16}$
$QP^2 = GP^2 - GQ^2 = R^2 - \frac{R^2}{16} = \frac{15R^2}{16}$
$\therefore NP = QP - QN = \frac{R\sqrt{15}}{4} - \frac{R\sqrt{3}}{4} = \frac{\sqrt{3}}{4}R(\sqrt{5}-1) = \frac{\sqrt{3}}{4}\frac{(3+\sqrt{45})(\sqrt{5}-1)}{\sqrt{3}} = 3$ **Ans. 3.**

Q.22 When $a = 2022$ and $b = 2023$, the value of

$$\frac{a\sqrt{a} + b\sqrt{b}}{(\sqrt{a}+\sqrt{b})(a-b)} + \frac{2\sqrt{b}}{\sqrt{a}+\sqrt{b}} - \frac{\sqrt{ab}}{a-b}$$

Solution: Let $\sqrt{a} = u, \sqrt{b} = v$. So, given expression is
$\frac{u^3+v^3}{(u+v)(u^2-v^2)} + \frac{2v}{u+v} - \frac{uv}{u^2-v^2} = \frac{u^2-uv+v^2+2uv-2v^2-uv}{u^2-v^2} = 1$ **Ans. 1.**

Q.23 In the adjoining diagram D is the midpoint of AB.A line DE is drawn to cut BC at E.AF is parallel to DE. It is given that $EF = FC$ and area of $\triangle BDE$ is 12, the area of $\triangle AGD$ is



Solution: $\frac{\triangle ABF}{\triangle DBE} = 4$ because D is midpoint of AB and $DE \parallel AF \therefore [\triangle ABF] = 48$
$\therefore [\square DEFA] = 36 \quad DE = \frac{1}{2}AF \therefore [AEF] = 2[DEA] \therefore [AEF] = 24 \quad [DEA] = 12$
$[AEF] = [AFC] = 24 \therefore [ABC] = 48 + 24 = 72 \therefore [ADC] = \frac{1}{2}[ABC] = 36$
Because $EF = FC$ and $DE \| AF$ we have $DG = GC \therefore [ADG] = \frac{1}{2}[ADC] = 18$ **Ans. 18.**

Q.24 Two sides of a triangle are 10 and 5 in length and the length of the median to the third side is $6\frac{1}{2}$. The area of the triangle is $6\sqrt{x}$. The value of x is

Solution: Suppose the triangle is $\triangle ABC$ with $AB = 10, AC = 5, AD = \frac{13}{2}$, then by Apollonius theorem, $AB^2 + AC^2 = 2(BD^2 + AD^2) \Rightarrow 100 + 25 = 2\left(\frac{169}{4} + BD^2\right) \Rightarrow BC = 9$. So, using Heron's formula, area of the triangle is $6\sqrt{14}$. **Ans. 14.**

Q.25 The 12 numbers $a_1, a_2, \ldots, a_{12}$ are in arithmetical progression. The sum of all these numbers is 354 . Let $P = a_2 + a_4 + \cdots + a_{12}$ and $Q = a_1 + a_3 + \cdots + a_{11}$. If the ratio $P : Q$ is 32 : 27, the common difference of the progression is

Solution: $P = 192,\ Q = 162$. Note that $P - Q = 6d \Rightarrow d = 5$ **Ans. 5.**

Q.26 Line $45x - 9y = 67$ divides segment joining $A(-1, 7)$ and $B(2, -3)$ in the ratio $\frac{m}{n}$ where m, n are coprimes. Find $m + n$

Solution: Suppose the point of intersection is C. So, using section formula, its coordinates are $\left(\frac{-n+2m}{m+n}, \frac{7n-3m}{m+n}\right)$. It should satisfy the equation of the line, so we get
$45\left(\frac{-n+2m}{m+n}\right) - 9\left(\frac{7n-3m}{m+n}\right) = 67 \Rightarrow \frac{m}{n} = \frac{7}{2}$ **Ans. 9.**

Q.27 $ABCD$ is a rectangle. Through C a variable line is drawn so as to cut AB at X and DA produced at Y. If area of rectangle $ABCD$ is 99, then $BX \times DY$ is

Solution: $\triangle DCY \sim \triangle BXC \therefore \frac{DY}{BC} = \frac{DC}{BX} \therefore BX \times DY = BC \times DC = 99$ **Ans. 99.**

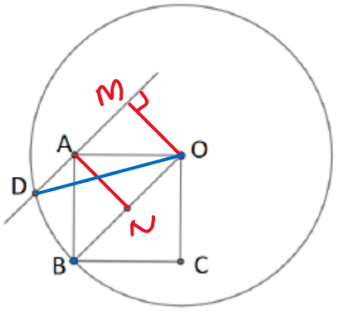
Q.28 x, y, z are distinct real numbers such that

$$x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$$

The value of $x^2y^2z^2$ is

Solution: Let each be k. So, $xy + 1 = ky \Rightarrow 1 = y(k - x) \Rightarrow y = \frac{1}{k-x}$
Similarly $z = \frac{1}{k-y}$ and $x = \frac{1}{k-z}$. Using y and z in the equation for x, we get
$x = \frac{1}{k-\frac{1}{k-\frac{1}{k-x}}} = \frac{1}{k-\frac{k-x}{k^2-kx-1}} = \frac{k^2-kx-1}{k^3-k^2x-k-k+x} = \frac{k^2-kx-1}{k^3-x(k^2-1)-2k}$
$\Rightarrow k^3x - x^2(k^2-1) - 2kx = k^2 - kx - 1 \Rightarrow x^2(k^2-1) + kx(1-k^2) + k^2 - 1 = 0$
So, either $k = \pm 1$ or $x^2 - kx + 1 = 0 \Rightarrow k = x + \frac{1}{x}$ but $k = x + \frac{1}{y}$ so this will mean $x = y$.
But x, y, z are distinct numbers. So, this is not possible. So, $k = \pm 1$.
Case I: $k = 1 \Rightarrow y = \frac{1}{1-x}$ and $z = \frac{1}{1-y} = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x} \Rightarrow xyz = -1$
Case II: $k = -1 \Rightarrow y = \frac{1}{-1-x}$ and $z = \frac{1}{-1-y} = \frac{1}{-1-\frac{1}{-1-x}} = \frac{1+x}{-x} \Rightarrow xyz = +1$ **Ans. 1.**

Q.29 In the following Figure O is the centre of the circle. $ABCO$ is a square with B on the circle. Through A a line parallel to OB is drawn to cut the circle at D nearer to B. Then $\angle AOD =$



Solution: $AN = OM = ON = \frac{R}{2} \quad OD = R \therefore \triangle OMD$ is a $30 - 60 - 90\triangle$
$\angle DOM = 60^\circ \quad \angle AOM = 45^\circ \therefore \angle AOD = 15^\circ$ **Ans. 15.**

Q.30 a, b, c are digits of a 3 -digit number such that $64a + 8b + c = 403$, then the value of $a + b + c$ is

Solution: $403 = 64(6) + 8(2) + 3$. **Ans. 11.**